Additional Questions

of the Textbook

PHYSICS XI

(Chapters 1 & 2)

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Chapter 1

Q.1 ***What is Physics? Name four branches of modern physics.***

Ans. Physics is the branch of science, which deals with the study of matter and energy and of the relationship between them. Among many branches, the four branches of modern physics are:

i) Nuclear physics, ii) Particle physics,
iii) Relativistic mechanics, iv) Solid state physics.

Q.2 ***Distinguish between biological science & physical science. What are three main frontiers of fundamental science?***

Ans. Biological science deals with living things.
And Physical science deals with non-living things. Physics is an important and basic part of physical sciences besides its other disciplines, such as chemistry, astronomy, etc.
Three main frontiers are: i) the world of extremely large, the universe itself, ii) the world of extremely small & iii) the world of complex matter.

Q.3 ***Which areas the study of physics involves? Describe briefly.***

Ans. The study of physics involves:

i) investigating such things as the laws of motion,
ii) structure of space and time,
iii) nature and type of forces that hold different materials together,
iv) interaction between different particles,
v) interaction of electromagnetic radiation with matter,
vi) modern physics whose some branches are;
   a) nuclear physics,
   b) atomic physics,
   c) particle physics,
   d) relativistic mechanics

Q.4 ***Write any four inter-disciplinary areas of Physics.***

Ans. The overlapping of physics and other fields gave birth to new branches such as physical chemistry, biophysics, astrophysics, health physics, engineering physics geophysics and medical physics.

Q.5 ***What is relativistic mechanics?***

Ans. Relativistic Mechanics is based on theory of relativity, which leads to four-dimensional space-time concept.
Two important consequences of Einstein's relativity theory are the equivalence of mass and energy and the limiting velocity of the speed of light for material objects. Relativistic mechanics describes the motion of objects with velocities that are appreciable fractions of the speed of light.
Even more important is the relation between the mass m and energy E.
They are coupled by the relation $E = mc^2$.

Q.6 ***Write down at least four sentences on the information technology.***

Ans. Information media and fast means of communications have brought all parts of the world into a global village. Science and technology are a potent force to change the outlook of mankind. Computer networks are products of chips developed from the basic ideas of physics.

Q.7 *What is a "Physical quantity"?*

Ans. Physical quantity is the quantity, in terms of which, the laws of physics are expressed, e.g. mass, length, and time, velocity, force, temperature, and electric current, etc.

Physical quantities are often divided into two categories: base quantities and derived quantities.

Q.8 *What are base quantities? How they are measured? Explain the meaning of the term base unit.*

Ans. Base quantities are certain physical quantities such as length, mass and time. These quantities are not defined in terms of other physical quantities. The base quantities are the minimum number of those physical quantities in terms of which other physical quantities can be defined.

b) The measurement of a base quantity involves two steps: first the choice of a standard, and second, the establishment of a procedure for comparing the quantity to be measured with the standard. The ideal standard should be accessible and invariable.

c) Base unit is a reference value of a quantity used to express other values of the same quantity. It is not derived from other units. There are seven base units for various physical quantities namely: length, mass, time, temperature, electric current, luminous intensity and amount of substance.

Q.9 *What do you mean by a unit? Sometimes length of objects is measured in units of hands. Why this is poor standard of length?*

Ans. Unit is a reference value of a quantity used to express other values of the same quantity. It is used for the measurement of a physical quantity.

b) The size of human hand varies from person to person, so the length measured will be different for different hands.

Q.10 *How many different kinds of units are in SI system? Write down names of seven base units.*
[Alt. *Which quantities are measured in terms of the base units of SI units? Name these quantities along with their units of measurement.*]

Ans. SI system is built up from three kinds of units: i) base units, ii) supplementary units, & iii) derived units.

The seven base units for various physical quantities in SI system are: metre (for length), kilogram (for mass), second (for time), ampere (for elecrric current), Kelvin (for thermodynamic temperature), candela (for intensity of light) and mole (for amount of substance).

[Length, mass, time, electric current, temperature, intensity and amount of substance are measured as base units]

Q.11 *What is meant by derived units? Define radian and steradian. Are they the base units of system international?*

Ans. Derived units are the units of physical quantities expressed in terms of fundamental units. Examples are velocity, acceleration, force, etc.

b) Radian is the plane angle subtended at the centre of a circle by an arc equal in length to its radius.

Steradian is the solid angle (three-dimensional angle) subtended at the center of a sphere by an area of its surface equal to the square of radius of the sphere.

For the time being, radian and steradian are not base units of SI system.

Q.12 ***Write down some points which should be kept in mind while using units.***

Ans. Some <u>conventions</u> for indicating units are:

i) The symbol of unit named after a scientist has initial capital letter.
ii) The prefix should be written before the unit without any space.
iii) A combination of base units is written each with one space apart.
iv) Compound prefixes are not allowed.
v) Measurement in practical work should be recorded immediately in the most convenient unit.

Q.13 ***Name three units which are after the names of the scientists and express them in terms of base units.***

Ans. i) N (Newton), unit of force.

$F = ma$ or $1N = 1kg \times 1ms^{-2} = kg\ ms^{-2}$

ii) W (Watt), unit of power.

$\text{Power} = \frac{\text{work}}{\text{time}}$ or $1\text{ watt} = \frac{1\text{joule}}{1\text{second}} = Js^{-1}$

iii) J (Joule), unit of energy

$\text{Work} = F \times d$ or $1\text{ joule} = 1\text{ newton} \times 1\text{ metre}$ or $1\ J = N\ m$

Q.14 ***Which quantity is defined in the unit of light year? Define light year and convert it into SI unit of that quantity. Are there more micro seconds in a year than there are seconds in a year?***

Ans. <u>Large distance</u> is measured in the unit of <u>light year</u>. It is the distance that light travels through space in one year; equal to 9.46 x 10^{15} meters.

$S = vt$ or $1\text{ light year} = 3\times10^{8}\times(1\times60\times60\times24\times365\text{ sec}) = 9.46\times10^{15}$ m

b) Yes, there are <u>more micro seconds</u> than seconds in a year.

Number of seconds in a year $= 1\times60\times60\times24\times365\text{ sec} = 31.54\times10^{6}\text{ sec}$

No. of micro-seconds in a year $= 1\times60\times60\times24\times365\times10^{6}\text{ sec} = 31.54\times10^{12}\ \mu\text{ sec}$

Q.15 ***Calculate the amount of energy equivalent to one kilogram of mass, according to Einstein's equation. Can weight be measured in terms of units of mass? If not why?***

Ans. From Einstein's mass-energy equation:

$E = mc^2 = 1\text{ kg}\times(3\times10^{8}ms^{-1})^2 = 9\times10^{16}\text{ J}$

b) Weight is the force by which a mass is attracted to the Earth.
Mathematically $w = mg = kgms^{-2} = N$ (newton)
The <u>weight has derived units</u>, but mass has basic unit. So weight <u>cannot</u> be measured in units of mass.

Q.16 ***How many nanometers are in a kilometer and kilo metres in a nanometer? Also express joule in terms of base units.***

Ans. $1\text{ km} = 10^3\text{m} = 10^3\frac{10^{-9}}{10^{-9}} = 10^{12}\times10^{-9}\text{m} = 10^{12}\text{ nm}$ &

$1\text{ nm} = 10^{-9}\text{m} = 10^{-9}\times\frac{10^3}{10^3} = 10^{-12}\times10^3\text{m} = 10^{-12}\text{ km}$

$1\text{ Joule} = 1\text{ newton x 1 metre} = 1kg\times1ms^{-2}\times1m = 1kgm^2s^{-2}$

Q.17 ***State the possible causes of error in a measurement.***

Ans. The error may occur due to:

i) negligence or inexperience of a person,
ii) the faulty apparatus,
iii) inappropriate method or technique.

Q.18 ***State the possible causes of uncertainty in a measurement.***

Ans. The uncertainty may occur due to;

i) inadequacy or limitation of an instrument,
ii) natural variations of the object being measured,
iii) natural imperfections of a person's senses.

Q.19 ***Distinguish between a random error and a systematic error in the measurement of a physical quantity.***

Ans. Random error is the error due to fluctuations in the measured quantity. It is said to occur when repeated measurements of the quantity, give different values under the same conditions.

Systematic error is the error due to incorrect design or calibration of the measuring device. It refers to an effect that influences all measurements of a particular quantity equally. It produces a consistent difference in readings.

Q.20 ***What is random error and its cause. How can it be reduced?***

[Alt. ***Why do we a take a large number of observations while performing experiment?*** **]**

Ans. Random error is the error due to fluctuations in the measured quantity. It is said to occur when repeated measurements of the quantity, give different values under the same conditions.

To reduce the effect of random error, we should repeat the measurement several times and take an average of these values.

Q.21 ***What is systematic error?. How can it be reduced?***

Ans. Systematic error is the error due to incorrect design or calibration of the measuring device. It refers to an effect that influences all measurements of a particular quantity equally. It produces a consistent difference in readings. Systematic error can be reduced, by comparing the instruments with another that is known to be more accurate.

Q.22 ***How readings are taken in practical work? Are systematic and random errors are actually experimental values?***

Ans. Measurement in practical work should be recorded immediately in the most convenient unit.

Yes, both random error and systematic errors are actually experimental values.

Q.23 ***What do the terms random error and systematic error mean?***

Ans. Random errors are minor experimental or accidental errors due to judgment and cannot be controlled. They can be reduced by repeating the measurement several times and take an average of these values.

Systematic errors are constant errors, which cause all results to be incorrect by roughly the same amount. They can be reduced, by comparing the instruments with another that is known to be more accurate.

Q.24 ***What is scientific notation? What is the advantage of improving the quality of a measuring instrument and techniques?***

Ans. Numbers are expressed in standard form called scientific notation, which employs powers of ten. The accepted practice is that there should be only one non-zero digit left of decimal. Example: 2.32×10^{-3}

b) By doing so, we can measure the result up to more significant figures and so improved the experimental accuracy of the result.

Q.25 ***What are significant figures and what information we get? How many significant figures are there in 62.3 cm, 62.4 cm and 62.5 cm?***

Ans. In any measurement, the accurately known digits and the first doubtful digit are called significant figures. These are known to be reasonably reliable. The information is, the significant figures tell us how accurate measurement can be made.
There are three significant figures in each of the given terms.

Q.26 ***What is meant by measurement? How can we indicate uncertainty or accuracy? Does a change in units of a measurement have any effect upon its significant figure?***

Ans. It is a combination of numerical value and the name of the unit.
Counting the number of significant figures indicates uncertainty or accuracy.
No, change of units has no effect upon the significant figure.

Q.27 ***What are the rules in deciding the significant figures ?***

Ans. a) All nine digits (1,2,3,45,6,7,8,9) are significant.
b) Zero is significant between two significant figures.
c) Zeros to the left of significant figures are not significant.
d) Zero to the right of a significant figure may or may not be significant.
  i) In decimal fraction, zeros to the right are significant
  ii) In integers, the number of significant zeros depends upon the accuracy of the instrument.
e) Figures other than the powers of ten are significant.

Q.28 ***A sphere has diameter equal to 2.25 cm as measured by vernier calipers of least count 0.01 cm. What will be its volume with the uncertainty in its measurements?***

Ans. The volume V will be:

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\times\pi\times(\frac{D}{2})^3 = \frac{4}{3}\times 3.14\times(\frac{2.27}{2})^3 = 6.13\text{cm}^3$$

$$\& \quad \%\text{ age uncertainty} = \frac{\text{least count}}{\text{measurement}}\times\frac{100}{100} = \frac{0.01}{6.13}\times 100\times\text{per cent} = 0.16\%$$

Q.29 ***In rounding off a figure, what rules are followed? Round off the figures 2.85, 2.38, 2.35 & 2.74 upto one decimal point.***

Ans. The following rules are followed.
a) If first digit dropped < 5, no change in last digit.
b) If first digit dropped > 5, the last digit is increased by one.
c) If dropping digit is 5;
  i) increased by one, for odd previous digit,
  ii) retained as such, for even previous digit.

Rounded off figures will be: 2.8, 2.4, 2.4 & 2.7.

Q.30 ***What will be the uncertainty in a time period T for a time of 30 vibrations is 51.3 s recorded by 1/10th of a second stopwatch?***

Ans. The uncertainty in time period T will be:

$$\text{uncertainty in T} = \frac{\text{Least count of timing device}}{\text{No. of vibs.}} = \frac{0.1}{30} = 0.003 \text{ s}$$

& quoted as T = time period ± uncertainty $= \frac{51.3}{30} = 1.71 \pm 0.003 \text{ s}$

Q.31 ***In calculations, what rules are followed for rounding off the result?***

Ans. In the result, for multiplication and division, keep the number contained in the least accurate factor.

And for addition and subtraction, the number of decimal places retained in the answer should equal smallest number of decimal places in any of the quantities.

Q.32 ***Three numbers are given i.e., 6.435, 2.347 and 0.7. Add and multiply them, and round off the result.***

Ans. Addition will give:

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 6.435
 2.347
 0.7
 9.482
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The answer will be rounded off to one decimal place as 9.5.

& Multiplication will give:

6.435 x 2.347 x 0.7 = 10.572

The result will be rounded off as 10.6.

Q.33 ***How uncertainty of many values are determined? Calculate mean deviation of the readings 2.10 mm, 2.12 mm & 2.23 mm.***

Ans. The uncertainty is determined by taking mean deviation, which is the mean of each deviation from average value.

$$\text{Mean Deviation} = \frac{\Sigma \text{ deviation from average}}{\text{No. of readings}}$$

We have, Average $= \frac{2.10 + 2.12 + 2.23}{3} = 2.15$

Each Deviation: (2.15 –2.10) , (2.15 – 2.12) & (2.23 – 2.15) is: 0.05, 0.03 & 0.08

Mean Deviation: $\frac{0.05 + 0.03 + 0.08}{3} = 0.05$

So Uncertainty in the reading is: 2.15 ± 0.05 mm

Q.34 ***What is difference between precision and accuracy of a measurement?***

Ans. A precise measurement is the one, which has less absolute uncertainty.

Precision (or absolute uncertainty): In measurements considering the magnitude of error. The less magnitude of error gives more precise measurement; it is equal to the least count of the measuring instrument.

An accurate measurement is the one, which has less fractional uncertainty.

Accuracy: In measurements considering the relative error. The less relative error gives more accurate result.

Precision depends upon instrument and accuracy depends upon fractional error.

Q.35 ***What do you mean by a unit, dimensions and dimensional formula of a physical quantity? Which one is easy to work with, SI units or in dimensions?***

Ans. A unit is a standard, which is used for the measurement of a physical quantity. The dimensions of a physical quantity show the relation between that quantity to the base quantity. It is a measurement of any sort; especially length, height and width. Each base quantity is considered a dimension denoted by a specific symbol written within square brackets.

Dimensional formula is derivation of a relation by correct guessing of factors on which physical quantity depends.

It is easier to work in SI units than to work in dimensions.

Q.36 ***What is dimensional analysis in a physical quantity?***

Ans. Using the method of dimensions called the dimensional analysis.

We can check the correctness of a given formula or an equation and can also derive it. Dimensional analysis makes use of the fact that expression of the dimensions can be manipulated as algebraic quantities.

Dimensional analysis is a technique whose main uses are:

a) to test the probable correctness of an equation between physical quantities,
b) to provide a safe method of changing the units in a physical quantity,
c) to solve partially a physical problem whose direct solution cannot be achieved by normal methods.

Q.37 ***Is it possible to have two quantities with the same dimensions but different units? Give examples to support your answer.***

Ans. Yes, two quantities can have same dimensions but different units, e.g.,

i) Work & torque: Work = $W=\vec{F}\cdot\vec{d}=Fd=[F]\times[L]=[MLT^{-2}]\times[L]=[ML^2T^{-2}]$

& torque = $\tau=\vec{r}\times\vec{F}=r\times F=[L]\times[MLT^{-2}]=[ML^2T^{-2}]$

ii) momentum & impulse: Momentum = $p=mv=[M]\times[v]=[M]\times[LT^{-1}]=[MLT^{-1}]$

& Impulse = $I=\vec{F}\times t=F\times t=[F]\times[t]=[MLT^{-2}]\times[T]=[MLT^{-1}]$

Q.38 ***When an equation is dimensionally correct? And if it is dimensionally correct, does it mean that equation is necessarily correct?***

Ans. When dimensions of both sides of the equation are the same, then the given equation is dimensionally correct.

An equation, which is dimensionally correct, may not be necessarily correct because of difference of numerical value on other side.

Q.39 ***What is the principle of homogeneity ?***

Ans. According to principle of homogeneity, the dimensions of the quantities on both sides of the equation should be same, irrespective of the form of the formula. It is used to check the correctness of an equation.

Q.40 ***Write two drawbacks of dimensional analysis.***

Ans. i) The numerical value of the constant cannot be determined by dimensional analysis.

ii) It cannot be applied to physical quantities involving the trigonometric and logarithmic functions.

Q.41 ***How do you check a formula for dimensional consistency? State two ways in which an equation that is dimensionally consistent may be physical incorrect.***

Ans. The success for deriving a formula for a physical quantity depends on the correct guessing of various factors on which the physical quantity depends. For it, reduce both sides of the formula to the base units and compare.

b) The two ways are: i) The constants cannot be detected as they are dimensionless.
ii) Some quantities are dimensionless, e.g., refractive index n & angle θ.

Q.42 ***The dimensions of torque are the same as that of energy. Explain why it would nevertheless be inappropriate to measure torque in joules. State an appropriate unit.***

Ans. Energy is a scalar quantity and its units are of work units, as the displacement of which is measured in the same direction as the line of action of the force applied.
Torque is a vector quantity and its unit is Nm, as it is the product of force and perpendicular distance from the axis of rotation. So it is inappropriate to measure torque in joules.

Q.43 ***What is the dimension of angle?***

Ans. The angle is dimensionless.
From $S = r\theta$, we have

$$\theta = \frac{S}{r} \text{ or } [\theta] = \frac{[L]}{[L]} = 1$$

It shows that angular displacement is a ratio between linear distance and angular distance, so it is dimensionless.

Q.44 ***Write the dimensional formula for power. If force times speed equals power, write dimensional formula for the force.***

Ans. $\text{Power} = P = \frac{\text{Work}}{\text{time}} = \frac{Fd}{t} = \frac{Nm}{s} = \frac{kgms^{-2} \times m}{s} = \frac{[MLT^{-2}] \times [M]}{[T]} = [ML^2T^{-3}]$

b) We have, Power $= P = \vec{F} \cdot \vec{v} = Fv$

or force $= F = \frac{P}{v} = \frac{\text{Work}/\text{time}}{\text{distance}/\text{time}} = \frac{J/s}{m/s} = \frac{J}{m} = \frac{Nm}{m} = N = kgms^{-2} = [MLT^{-2}]$

Q.45 ***What is the ratio of dimensions of volume 'v' and area 'A' in terms of three fundamental quantities, mass, length and time?***

Ans. Taking the ratio between volume V and area A:

$$\frac{V}{A} = \frac{m^3}{m^2} = m = [L] = [M^0LT^0]$$

Q.46 ***Prove that the equation*** $x = \frac{1}{2}at^2$ ***is dimensionally correct.***

Ans. Applying dimensional analysis to the equation, $x = \frac{1}{2}at^2$

the dimensions of left side of the equation are:

$$x = m = [L] \quad \text{..... (1)}$$

& the dimensions of right side of the equation are:

$$\frac{1}{2}at^2 = ms^{-2} \times s^2 = ms^{-2+2} = m = [L] \quad \text{..... (2)}$$

from equations (1) & (2), we conclude that the equation is dimensionally correct.

Q.47 ***Show that the equation*** $T = 2\pi\sqrt{l/g}$ ***is dimensionally correct.***

Ans. Applying dimensional analysis to the equation, $T = 2\pi\sqrt{l/g}$

the dimensions of left side of the equation are:

$T = [T]$ ..... (1)

& the dimensions of right side of the equation are:

[2π = constant & $g = a = \frac{\text{meter}}{\text{sec}^2} = \frac{L}{T^2}$

$$2\pi\sqrt{l/g} = \left[\sqrt{l/g}\right] = \left[\sqrt{\frac{L}{L/T^2}}\right] = \left[\sqrt{\frac{L \times T^2}{L}}\right] = \left[\sqrt{T^2}\right] = [T] \quad .... \ (2)$$

from equations (1) & (2), we conclude that the equation is dimensionally correct.

Q.48 ***Show that the equation*** $S = v_i t + \frac{1}{2}at^2$ ***is dimensionally correct.***

Ans. Applying dimensional analysis to the equation, $S = v_i t + \frac{1}{2}at^2$

the dimensions of left side of the equation are:

$S = [L]$ ..... (1)

& the dimensions of right side of the equation are:

$$v_i t + \frac{1}{2}at^2 = \left[LT^{-1}T\right] + \left[\frac{1}{2}LT^{-2}T^2\right] = [L] + [L] = [L] \quad ..... \ (2)$$

from equations (1) & (2), we conclude that the equation is dimensionally correct.

Q.49 ***Show that the equation*** $F = ma$ ***is dimensionally correct.***

Ans. Applying dimensional analysis to the equation, F = ma

the dimensions of left side of the equation are:

F = N = kg m s^{-2} = [MLT^{-2}] ..... (1)

& the dimensions of right side of the equation are:

$ma = \text{kg m s}^{-2} = \left[MLT^{-2}\right]$ ..... (2)

from equations (1) & (2), we conclude that the equation is dimensionally correct.

Q.50 ***Calculate the dimensions of*** $\left[\frac{F \times l}{M}\right]^{1/2}$***, and the dimensions of nuclear energy.***

Ans. i) The dimensions will be,

$$\left[\frac{F \times l}{M}\right]^{1/2} = \left([F] \times [l] \times \left[M^{-1}\right]\right)^{1/2} = \left(\left[MLT^{-2}\right] \times [L] \times M^{-1}\right)^{1/2} = \left[MLT^{-2}\right]^{1/2} \times [L]^{1/2} \times \left[M^{-1}\right]^{1/2}$$

$$= \left[M^{1/2}L^{1/2}T^{-1}\right] \times \left[L^{1/2}\right] \times \left[M^{-1/2}\right] = \left[LT^{-1}\right] \quad \text{Ans.}$$

ii) The energy of any type is referred as work, so

Work = $W = \vec{F} \cdot \vec{d} = Fd = [F] \times [L] = [MLT^{-2}] \times [L] = [ML^2T^{-2}]$

Chapter 2

Q.1 ***What is vector quantity?***

Ans. Vector or Vector quantity is a physical quantity that requires both a magnitude in proper units and a direction for its complete description; Graphically it is represented by a straight line, length of which is equal to its magnitude and arrow-head shows its direction.

Q.2 ***Is it possible for two vectors of different magnitudes to have zero resultant? If not, can three vectors be so combined? Also find resultant of equal vectors to get same vector.***

Ans. No, two vectors having different magnitudes cannot give a zero resultant.

b) Yes, three vectors can be combined to get resultant equal to zero. According to head-to-tail rule, if tail of the first vector joins with the head of the third (last) vector, then the resultant of all three vectors will be zero.

c) For two equal vectors, take three sides of a triangle. The resultant of two vectors will be the third side of the triangle.

Q.3 ***When do two vectors have maximum and minimum resultant? What is the range for two vectors of same magnitude equal to A?***

Ans. The resultant will be maximum when two vectors are in the same direction (parallel) and it is minimum when lie in opposite direction (anti-parallel).

b) For parallel combination, the magnitude will be double (2A), and for anti-parallel it will be zero. So the range is between zero to 2A.

Q.4 ***Define component of a vector. Can a vector have a component greater than the vector's magnitude?***

Ans. Component of a vector is the effective value of the vector in a given direction. And rectangular components are the components along mutually perpendicular directions.

b) No, a vector cannot have a component greater than the vector's magnitude, because all components are part of original vector and are less than the vector.

Q.5 ***If*** $\vec{A}+\vec{B}=\vec{0}$***, what can you say about the components of the two vectors? A force of 10N makes an angle 30° with x-axis. What will be its x-component?***

Ans. It shows that one vector is –ve of the other vector. As $\vec{A}+\vec{B}=\vec{0} \Rightarrow \vec{A}=-\vec{B}$, So their rectangular components are in opposite direction.

b) The x-component will be:

$$F_x = F\cos\theta = 10\times\cos 30^0 = 10\times 0.866 = 8.66\text{N}$$

Q.6 ***If one of the components of a vector is not zero can its magnitude be zero? Explain.***
[Alt. ***Will a vector be zero if one of its components is zero?*** **]**

Ans. If one of the components is not zero, its magnitude cannot be zero.

As the magnitude is given by: $A=\sqrt{A_x^2+A_y^2}$

In the formula, if $A_x = 0$ & $A_y \neq 0$, then $A=\sqrt{A_y^2}=A_y$

& if $A_y = 0$ & $A_x \neq 0$, then $A=\sqrt{A_x^2}=A_x$. It proves the result.

Q.7 ***Under what circumstances would a vector have components that are equal in magnitude?***

Ans. If the vector $\vec{A}$ is drawn in such a way that it makes an angle of 45^0 (or 135^0, 225^0, 315^0) with +ve x-axis, & if we draw rectangular components of the vector, then their magnitudes are equal.

Since $A_x = A\cos 45^0 = \frac{A}{\sqrt{2}}$ & $A_y = A\sin 45^0 = \frac{A}{\sqrt{2}}$ $\Rightarrow A_x = A_y$

This means that $\theta = 45^0$, for the angle between the rectangular components.

Q.8 ***What is position vector? The rectangular components of a point in space are (4, 6, 0). Write down its position vector and find its magnitude.***

Ans. The <u>position vector</u> $\vec{r}$ is a vector that describes the location of a particle with respect to the origin.

Position vector : $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$; [(4,6,0) means x = 4, y = 6, z = 0]

$\therefore \vec{r} = 4\hat{i} + 6\hat{j} + 0\hat{k} = \underline{4\hat{i} + 6\hat{j}}$ is the required <u>position vector</u>

& its <u>magnitude</u> is: $|\vec{r}| = r = \sqrt{x^2 + y^2 + z^2} = \sqrt{4^2 + 6^2 + 0^2} = \sqrt{52} = \underline{7.2}$

Q.9 ***Two forces $\vec{F}_1$ and $\vec{F}_2$ acting at right angle to each other. Find the expression for resultant force.***

Ans. Suppose F_1 is along x-axis and F_2 is along y-axis.

$F_x = F_{1x} + F_{2x} = F_1\cos 0^0 + F_2\cos 90^0 = F_1$

& $F_y = F_{1y} + F_{2y} = F_1\sin 0^0 + F_2\sin 90^0 = F_2$

so the resultant: $F = \sqrt{F_x^2 + F_y^2} = \sqrt{F_1^2 + F_2^2}$

Q.10 ***Two forces $\vec{F}_1 = 3N$ and $\vec{F}_2 = 5N$ act on an object in opposite direction. What is the resultant force? Under what circumstances direction of resultant will coincide?***

Ans. The forces are acting in opposite direction to a body so net resultant force is:

$\vec{F} = \vec{F}_2 - \vec{F}_1 = 5 - 3 = 2N$

b) It happens when the two forces are acting parallel to each other.

Q.11 ***Two forces act together on an object. Suggest the angle at which the magnitude of resultant force is least.***

Ans. For anti-parallel forces, i.e., <u>θ = 180</u>, the resultant will be minimum.

$F = \vec{F}_1 + (-\vec{F}_2) = \vec{F}_1 - \vec{F}_2$

for equal forces: $F_1 = F_2 = F = 0$

& for unequal forces, let $F_1 > F_2$ so $F = F_1 - F_2$

Q.12 ***Define resultant vector. Can the resultant of two vectors have a magnitude smaller than the magnitude of either?***

Ans. The <u>resultant</u> of a number of similar vectors is the single vector, which would have the same effect as all the original vectors taken together.

b) Yes, when $\theta > 90^0$, the resultant will be smaller than the magnitude of either of the similar vector.

Q.13 ***What is Rectangular coordinate system? Will the value of a vector quantity change if the reference axes are changed?***

Ans. Rectangular coordinates or Cartesian coordinates referred to three mutually perpendicular straight lines.
In x-y plane, two lines drawn at right angles to each other are known as coordinate axis and their point of intersection is known as origin.

b) No, the value of a vector quantity does not change if the reference axes are changed. A vector constitutes a direction and the magnitude. By changing reference axis, the orientation of the vector will remain same. Also the magnitude of the vector will be unaltered.

Q.14 ***Explain head to tail rule of vector addition.***

Ans. Draw the representative lines on a suitable scale of the vectors to be added. Addition of vectors (by head-to-tail rule) is obtained by drawing these representative lines in such a way that tail of first vector coincides with the head of the second vector and so on. Join tail of first vector with the head of last vector. This joining vector will represent the vector sum.

Q.15 ***What is the effect on a vector $\vec{A}$ when it is multiplied by a number n when i) $n > 0$, ii) $n < 0$.***

Ans. i) When $\vec{A}$ is multiplied by a +ve number ($n > 0$), the magnitude will be nA and its direction will be same as of $\vec{A}$.

ii) When $\vec{A}$ is multiplied by a -ve number ($n < 0$), the magnitude will be nA but its direction will be opposite to that of $\vec{A}$.

Q.16 ***What are the characteristics of two equal vectors? When is the magnitude of their resultant is same as that of either vector?***

Ans. Two vectors $\vec{A}$ and $\vec{B}$ are said to be equal if they have the same magnitude and direction, regardless of the position of their initial points.

b) Let θ be the angle between the two equal vectors (say) $\vec{A}$.
Resultant R is also equal to A, so R = A + A = A
The magnitude of the resultant will be:

$$A^2 = (A+A)^2 = A^2 + A^2 + 2AA\cos\theta = 2A^2 + 2A^2\cos\theta = 2A^2(1+\cos\theta)$$

or $A^2 = 2A^2(1+\cos\theta)$ or $\frac{1}{2} = 1 + \cos\theta$

or $\cos\theta = -\frac{1}{2} \Rightarrow \theta = 120^0$

Thus for the equal vectors, of the resultant of same magnitude, the angle between them is 120^0.

Q.17 ***Vector $\vec{A}$ lies in the xy-plane. For what orientation will both of its rectangular components be negative? For what orientation will its components have opposite signs?***

Ans. If the vector $\vec{A}$ lies in the 3^{rd} quadrant then both of its rectangular components will be negative.

b) If the vector $\vec{A}$ lies in the 2^{nd} or 4^{th} quadrant then the signs of its rectangular components will be opposite to each other.

Q.18 ***When is the relationship*** $\vec{A}+\vec{B}=\vec{A}-\vec{B}$ ***correct? If*** $\vec{A}$ and $\vec{B}$ ***are the vectors lying in yz-plane, find their resultant*** $\vec{R}$ **.**

Ans. It is correct, when $\vec{B}=\vec{0}$ that is $\vec{B}$ is a null vector, in the given relation.
or when $\vec{A}=\vec{0}$ & $\vec{B}=-\vec{B}$ i.e. $\vec{A}$ is null vector & $\vec{B}$ in opposite direction

b) In y-z plane $\vec{A}$ and $\vec{B}$ are expressed as:

$$\vec{A}=A_y\hat{j}+A_z\hat{k} \quad \& \quad \vec{B}=B_y\hat{j}+B_z\hat{k}$$

so resultant: $\vec{R}=\vec{A}+\vec{B}=(A_y\hat{j}+A_z\hat{k})+(B_y\hat{j}+B_z\hat{k})=(A_y+B_y)\hat{j}+(A_z+B_z)\hat{k}$

Q.19 ***What are coplanar forces? Can a vector directed along x-axis have y - component?***

Ans. The forces that lie in a single plane are called coplanar forces.

b) No, a vector directed along x-axis have no y-component.
As the angle between the vector and x-axis, $\theta = 0$.
So $F_y = F\sin\theta = F\sin 0^0 = 0$

Q.20 ***What is the significance of unit vector? Can a unit vector have units and magnitude?***

Ans. Unit vector is a vector in a given direction with magnitude one in that direction. It is used to indicate the direction of any given vector. Mathematically,
it is obtained by dividing a given vector by its magnitude; $\hat{A}=\frac{\vec{A}}{A}$;

Unit vectors $\hat{i}$, $\hat{j}$ & $\hat{k}$ are along x-, y- & z- axis respectively.

b) Yes, a unit vector have units, and it has magnitude equal to unity.

Q.21 ***What is the unit vector in the direction of vector*** $\vec{A}=4\hat{i}+3\hat{j}$ ***? Also calculate its magnitude.***

Ans. The unit vector in the direction of $\vec{A}$ will be:

$$\hat{A}=\frac{\vec{A}}{A}=\frac{4\hat{i}+3\hat{j}}{\sqrt{4^2+3^2}}=\frac{4\hat{i}+3\hat{j}}{\sqrt{25}}=\frac{4\hat{i}+3\hat{j}}{5}=\frac{4}{5}\hat{i}+\frac{3}{5}\hat{j}$$

& its magnitude: $|\hat{A}|=\left|\frac{4}{5}\hat{i}+\frac{3}{5}\hat{j}\right|=\sqrt{\left(\frac{4}{5}\right)^2+\left(\frac{3}{5}\right)^2}=\sqrt{\frac{16}{25}+\frac{9}{25}}=1$

Q.22 ***What is the angle between the unit vectors*** $\hat{i},\hat{j}$ and $\hat{k}$ ***? What is indicated by the angles*** α, β and γ ***?***

Ans. The unit vectors $\hat{i},\hat{j}$ and $\hat{k}$ are mutually perpendicular to each other. So the angle between any two of the given unit vectors is 90^0.
The angles α, β and γ are the angles made by a three dimensional vector with x-, y-, & z-axis respectively.

Q.23 ***What will be the orientation of the two vectors*** $\vec{A}$ and $\vec{B}$, ***if the magnitude of their resultant is i)*** $\vec{A}+\vec{B}$, ***ii) zero.***

Ans. i) The vectors should be parallel, if the resultant is A + B.
ii) The vectors should be equal in magnitude but opposite in direction.

Q.24 Does a vector having zero length have a direction? Give an example.

Ans. Yes, zero length (magnitude) can have arbitrary direction. Such a vector is called null vector.

b) When a vector is added with its negative vector, we get a vector called null vector.

Q.25 Can we talk of a vector of zero magnitude? Is it justified to define such a vector?

Ans. Yes, we can talk about a vector of zero magnitude. That vector is called null vector.

b) Yes, it is justified to define such a vector. According to vector addition, when two vectors are added, we get a vector quantity. If two vectors $\vec{A}$ and $\vec{A}$ are equal, then $\vec{A}+(-\vec{A})=\vec{A}-\vec{A}=\vec{0}$

where R.H.S. is a null vector. If we write a scalar zero, we are equating a vector with a scalar which is not justified.

Q.26 How can you represent a vector graphically, when you are given the direction but not the magnitude and if magnitude but not direction?

Ans. By drawing a straight line of arbitrary magnitude in the given direction, we can represent that vector graphically.

b) By drawing a circle of radius equal to the magnitude of that vector. The length of the radius will be the magnitude and direction will be anywhere between 0 and 360^0.

Q.27 Can the magnitude of the resultant of two vectors be greater than the sum of the magnitude of the individual vectors?

Ans. No, the magnitude of the resultant of two vectors cannot be greater than the sum of the individual vectors.

The resultant of two vectors has maximum value for parallel vectors ($\theta = 0$)

In this case the resultant will be equal to individual vectors. (maximum value), Even here, either of the vector will have less magnitude.

Q.28 What will happen when a vector $\vec{A}$ is multiplied by a negative number –3 ?

Ans. When $\vec{A}$ is multiplied by –3, we get $-3\vec{A}$.

The magnitude of the vector is increased by a factor 3, but the direction is reversed.

Q.29 A body of mass m is moving in the downward direction to an inclined plane making an angle θ to the horizontal. Find the magnitude of the resultant force.

Ans. The resultant force R will be:

$$R = mg\sin\theta - f$$

where mg is the weight of the body, θ is the angle made by inclined plane & f is the frictional force.

mg sinθ is the component of the force (weight, mg) along the direction of motion which is parallel to the inclined plane.

Q.30 Two vectors of magnitudes 3N and 5N act in opposite directions. What is the resultant of these two vectors? If the same two vectors happen to act in the same direction, what will be the resultant then?

Ans. When vectors 3N and 5N act in opposite directions, the resultant will be:

5N – 3N = 2N , in the direction of 5N vector.

When vectors 3N and 5N act in same direction, the resultant will be:

5N + 3N = 8N , in the same direction of the vectors.

Q.31 ***Two cars are moving with velocity v ms^{-1} due east and north respectively. What is the difference between their velocities?***

Ans. Let $\vec{A} = v\ ms^{-1}$ due East & $\vec{B} = v\ ms^{-1}$ due North

& let $\vec{R}$ be the difference between the velocities.

since $|\vec{R}| = R = \sqrt{R_x^2 + R_y^2}$

& $R_x = A + B\cos 90^0 = A = v$ & $R_y = A\sin 0^0 + B\sin 90^0 = B = v$

so $R = \sqrt{A^2 + B^2} = \sqrt{v^2 + v^2} = \sqrt{2v^2} = \sqrt{2}\ v\ ms^{-1}$

& $\tan\theta = \dfrac{R_y}{R_x} = \dfrac{v}{v} = 1 \Rightarrow \theta = 45^0$

Q.32 ***Work out the angle between $\vec{a}$ and $\vec{b}$ if $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$.***

If $\vec{a}\cdot\vec{b} = 0$, can it be concluded that $\vec{a}$ & $\vec{b}$ are perpendicular to each other?

Ans. Taking $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$

Squaring both sides,

$|\vec{a} + \vec{b}|^2 = |\vec{a} - \vec{b}|^2$ or $(\vec{a} + \vec{b})(\vec{a} + \vec{b}) = (\vec{a} - \vec{b})(\vec{a} - \vec{b})$

or $a^2 + b^2 + 2\vec{a}\cdot\vec{b} = a^2 + b^2 - 2\vec{a}\cdot\vec{b}$ or $4\vec{a}\cdot\vec{b} = 0$

or $\vec{a}\cdot\vec{b} = 0 \Rightarrow \theta = 90^0$

b) since $\vec{A}\cdot\vec{B} = AB\cos\theta$ or $\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{AB} = \dfrac{\vec{a}\cdot\vec{b}}{ab} = 0 \Rightarrow \theta = 90^0$

and $\theta = 90^0$ shows that $\vec{a}$ & $\vec{b}$ are perpendicular to each other.

Q.33 ***Define scalar or dot product. Show that scalar product is commutative.***

Ans. The Scalar or dot product of vectors $\vec{A}$ and $\vec{B}$ is the scalar quantity obtained by multiplying the product of the magnitudes of the vectors by the cosine of the angle between them. Mathematically, $\vec{A}\cdot\vec{B} = AB\cos\theta$

b) According to commutative law: a * b = b * a,

For the vectors $\vec{A}$ and $\vec{B}$, applying the law,

$$\vec{A}\cdot\vec{B} = AB\cos\theta = BA\cos\theta = \vec{B}\cdot\vec{A}$$

or $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$, proves the commutative law.

Q.34 ***What are the possible cases for $\vec{A}\cdot\vec{B} = 0$?***

[Alt. ***Is it possible to have dot product of vectors zero when neither of them is null vector?***]

[Alt. ***Under what condition the dot product of two vectors will be equal to zero if both the vectors itself are non-zero vectors?***

Ans. The possible cases are:

a) $\vec{A}\cdot\vec{B} = AB\cos\theta = 0 \Rightarrow \theta = 90^0$,

shows $\vec{A}$ & $\vec{B}$ are perpendicular to each other.

b) $\vec{A}\cdot\vec{B} = \vec{A}\cdot\vec{0} = 0$, shows that $\vec{B}$ is null vector.

c) $\vec{A}\cdot\vec{B} = \vec{0}\cdot\vec{B} = 0$, shows that $\vec{A}$ is null vector.

d) $\vec{A}\cdot\vec{B} = \vec{0}\cdot\vec{0} = 0$, shows both $\vec{A}$ and $\vec{B}$ are null vectors.

[The dot product of two non-zero vectors will be equal to zero only when they are perpendicular to each other and in that case $\theta = 90^0$]

Q.35 ***Suppose $\vec{A}$ is a non zero vector. If $\vec{A}\cdot\vec{B}=0$ and also $\vec{A}\times\vec{B}=0$, where $\vec{B}$ is an unknown vector. What can be concluded about $\vec{B}$?***

[Alt. ***Suggest a condition to satisfy that both products are zero.*** **]**

Ans. Taking $\vec{A}\cdot\vec{B}=AB\cos 90^0 = 0 \Rightarrow \vec{A}\ \&\ \vec{B}$ are perpendicular to each other.

& $\vec{A}\times\vec{B}=AB\sin\theta\,\hat{n}=AB\sin 0^0=0 \Rightarrow \vec{A}\ \&\ \vec{B}$ are parallel

$\vec{A}$ and $\vec{B}$ cannot be perpendicular and parallel at the same time,

so we conclude that $\vec{B}$ is a null vector.

Q.36 ***Write the different possibilities that could make $\vec{A}_1\times\vec{A}_2=0$?***

[Alt. ***If $\vec{A}\times\vec{B}=\vec{0}$ does it follow that $\vec{A}$ and $\vec{B}$ are necessarily parallel to each other? Is its reverse true?*** **]**

[Alt. ***If the cross product of two non-zero vector $\vec{A}$ and $\vec{B}$ is zero, then what will be the angle between the vectors $\vec{A}$ and $\vec{B}$?*** **]**

Ans. The cross product of two vectors can be expressed as:

a) $\vec{A}_1\times\vec{A}_2=A_1A_2\sin 0^0\,\hat{n}=0 \Rightarrow$ both vectors are parallel $[\theta=0^0]$

b) $\vec{A}_1\times\vec{A}_2=A_1A_2\sin 180^0\,\hat{n}=0 \Rightarrow$ both are anti-parallel $[\theta=180^0]$

c) $\vec{A}_1\times\vec{A}_2=(0)A_2\sin\theta\,\hat{n}=0 \Rightarrow A_1$ is zero

d) $\vec{A}_1\times\vec{A}_2=A_1(0)\sin\theta\,\hat{n}=0 \Rightarrow A_2$ is zero

Q.37 ***If $\vec{A}\times\vec{B}$ has the maximum magnitude, then what is the angle between $\vec{A}$ and $\vec{B}$. What is the direction of the resultant of $\vec{A}\times\vec{B}$.***

Ans. We have $\vec{A}\times\vec{B}=AB\sin\theta\,\hat{n}$

For maximum value of $\sin\theta$ is 1 which corresponds to $\theta = 90^0$

So $\vec{A}\times\vec{B}=AB\sin 90^0\,\hat{n}=AB\,\hat{n}$

By applying right-hand-rule, the direction of the resultant of $\vec{A}\times\vec{B}$ will perpendicular to the plane containing $\vec{A}$ and $\vec{B}$,

Q.38 ***Can a scalar product of two vectors be negative? If so, give an example.***

[Alt. ***Show that work done against frictional force is negative.*** **]**

Ans. Yes, scalar product of two vectors can be negative.

When $\theta=180^0$, both the vectors will be in opposite directions.

$\vec{A}\cdot\vec{B}=AB\cos 180^0=AB(-1)=-AB$

b) Example:

Work done by force of friction. As ' f ' always act opposite to displacement.

$\text{Work}=\vec{F}\cdot\vec{d}=Fd\cos\theta=Fd\cos 180^0=-Fd$

Q.39 ***Find the scalar product of i) two parallel vectors, ii) two anti-parallel vectors, iii) two similar unit vectors.***

Ans. The following are given different cases:

i) for two parallel vectors: $\vec{A}\cdot\vec{B}=AB\cos\theta=AB\cos 0^0=AB$

ii) for two anti-parallel vectors: $\vec{A}\cdot\vec{B}=AB\cos\theta=AB\cos 180^0=AB(-1)=-AB$

iii) for two similar unit vectors: $\hat{i}\cdot\hat{i}=|\hat{i}||\hat{i}|\cos 0^0=1\times1\times1=1$

Q.40 ***A force $\vec{F} = 4\hat{i} + 3\hat{j}$ units has its points of application moved from the point A (2,3) to the point B (8,7). Find the work done. Can you name a cross product of two vectors, which has same units & dimensions as work?***

Ans. Work is product of force and component of displacement in the direction of force.

We have $\vec{d} = \vec{r}_2 - \vec{r}_1$ $\quad [r_1 = 2\hat{i} + 3\hat{j} \;\&\; r_2 = 8\hat{i} + 7\hat{j}\;]$

or $\vec{d} = (8\hat{i} + 7\hat{j}) - (2\hat{i} + 3\hat{j}) = 6\hat{i} + 4\hat{j}$

& work done = W = $\vec{F}\cdot\vec{d} = (4\hat{i} + 3\hat{j})\cdot(6\hat{i} + 4\hat{j}) = 4\times 6 + 3\times 4 = 36$ units

b) Torque has the same units and dimension as of work.

Work = $W = \vec{F}\cdot\vec{d} = Fd = Nm = [F]\times[L] = [MLT^{-2}]\times[L] = [ML^2T^{-2}]$

torque = $\tau = \vec{r}\times\vec{F} = r\times F = Nm = mN = [L]\times[MLT^{-2}] = [ML^2T^{-2}]$

Q.41 ***Find the scalar product of any two vectors lying perpendicular to each other. What is the scalar product of a vector with itself? What about its angle?***

Ans. The scalar product of any two vectors lying perpendicular to each other will be:

$\vec{A}\cdot\vec{B} = AB\cos\theta = AB\cos 90^0 = AB(0) = 0$

b) The scalar product of a vector with itself is the self-product of a vector $\vec{A}$

& is $\vec{A}\cdot\vec{A} = AA\cos 0^0 = A^2$

It is equal to square of its magnitude.

c) The angle between the self scalar product 0^0.

Q.42 ***Prove that*** $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

Ans. The self-product is equal to square of its magnitude.

$\hat{i}\cdot\hat{i} = ii\cos 0^0 = i^2(1) = 1^2(1) = 1 \quad [|\hat{i}| = i = 1]$

similarly $\hat{j}\cdot\hat{j} = jj\cos 0^0 = j^2(1) = 1^2(1) = 1 \quad [|\hat{j}| = j = 1]$

& $\hat{k}\cdot\hat{k} = kk\cos 0^0 = k^2(1) = 1^2(1) = 1 \quad [|\hat{k}| = k = 1]$

The above relations proves: $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

Q.43 ***Name some physical quantities which are i) scalar product of two vectors, ii) vector products of two vectors.***

Ans. Physical quantity is the quantity, in terms of which, the laws of physics are expressed, e.g. mass, length, and time, etc.

i) Some of the physical quantities making scalar product are:

Work $= \vec{F}\cdot\vec{d}$; $PE_{gravitational} = \vec{w}\cdot\vec{d} = mgh$; electric pot. difference $= \Delta V = \vec{E}\cdot\Delta\vec{r}$;

and magnetic flux $= \phi_B = \vec{B}\cdot\vec{A}$ & electric flux $= \phi_{electric} = \vec{E}\cdot\vec{A}$

ii) Some of the physical quantities making vector product are:

torque $= \vec{\tau} = \vec{r}\times\vec{F}$; angular momentum $= \vec{L} = \vec{r}\times\vec{p}$

& force on a moving charge: $\vec{F}_{moving\ charge} = q\vec{v}\times\vec{B}$

Q.44 ***Define vector or cross product. Show that*** $\vec{A}\times\vec{B}=-\vec{B}\times\vec{A}$ ***& hence show that it does not obey commutative property?***

Ans. The <u>vector product</u> of two vectors (say) $\vec{A}$ and $\vec{B}$ are perpendicular $\vec{A}$ and $\vec{B}$ is defined to be a vector such that:

i) its magnitude is A B sin θ , θ being the angle between $\vec{A}$ and $\vec{B}$

ii) its direction is perpendicular to the plane of $\vec{A}$ and $\vec{B}$ and can be determined by right-hand-rule. Mathematically, $\vec{A}\times\vec{B}=AB\sin\theta\,\hat{n}$

b) taking $\vec{A}\times\vec{B}=AB\sin\theta\,\hat{n}=\vec{C}$

from right-hand-rule, direction of resultant will be (say) upward.

& $\vec{B}\times\vec{A}=BA\sin\theta\,\hat{n}=-\vec{C}$

from right-hand-rule, direction of resultant will be (say) downward.

It proves that $\vec{A}\times\vec{B}=-\vec{B}\times\vec{A}$

c) According to commutative law: a * b = b * a,

For the vectors $\vec{A}$ and $\vec{B}$, applying the law,

We see that $\vec{A}\times\vec{B}\neq\vec{B}\times\vec{A}$

It show that cross product does not obey commutative law.

[Illustration with the help of diagram is more understandable]

Q.45 ***Write kinetic energy, K.E. = ½ mv²***, ***in terms of scalar product of two vectors. If*** $\vec{A}=2\hat{i}+3\hat{j}$, ***what is square of this vector?***

Ans. Kinetic Energy is the energy possessed by a body due to its motion.

$$K.E.=\frac{1}{2}mv^2=\frac{1}{2}m(\vec{v}\times\vec{v}) \qquad [\vec{v}\times\vec{v}=v^2 \text{ --called self product }]$$

b) The scalar product of a vector with itself is the <u>self-product</u> of a vector $\vec{A}$

& is $\vec{A}\cdot\vec{A}=AA\cos 0^0=A^2$

so $\vec{A}\cdot\vec{A}=(2\hat{i}+3\hat{j})\cdot(2\hat{i}+3\hat{j})=4(\hat{i}\cdot\hat{i})+6(\hat{i}\cdot\hat{j})+6(\hat{j}\cdot\hat{i})+9(\hat{j}\cdot\hat{j})$

or $A^2=4(1)+6(0)+6(0)+9(1)=13$

Q.46 ***If*** $\vec{F}_1=3\text{ cm}$ and $\vec{F}_2=6\text{ cm}$. ***Let*** $\vec{F}_1$ ***is at angle of 30°*** ***while*** $\vec{F}_2$ ***is lying at an angle of 120° with respect to x-axis respectively, then find their dot product. Also find*** $\vec{A}\cdot\vec{B}$, ***if*** $\vec{A}=A_x\hat{i}+A_y\hat{j}$ and $\vec{B}=B_x\hat{i}+B_y\hat{j}$.

Ans. Calculating the dot product,

$\vec{F}_1\cdot\vec{F}_2=F_1F_2\cos\theta=(3)(6)\cos 90^0=18(0)=0 \quad [\theta=\theta_2-\theta_1=120^0-30^0=90^0]$

b) Calculating $\vec{A}\cdot\vec{B}$, we have

$$\vec{A}\cdot\vec{B}=(A_x\hat{i}+A\hat{j})\cdot(B_x\hat{i}+B_y\hat{j})=A_xB_x(\hat{i}\cdot\hat{i})+A_xB_y(\hat{i}\cdot\hat{j})+A_yB_x(\hat{j}\cdot\hat{i})+A_yB_y(\hat{j}\cdot\hat{j})$$

or $\vec{A}\cdot\vec{B}=A_xB_x(1)+A_xB_y(0)+A_yB_x(0)+A_yB_y(1)=A_xB_x+A_yB_y$

Q.47 ***If*** $\vec{A}=5\hat{i}$ and $\vec{B}=2\hat{j}$, ***find*** $\vec{A}\cdot\vec{B}$ and $\vec{A}\times\vec{B}$.

Ans. i) we have $\vec{A}\cdot\vec{B}=(5\hat{i})(2\hat{j})=(5)(2)(\hat{i}\cdot\hat{j})=10(0)=0 \qquad [\hat{i}\cdot\hat{j}=0,\text{ as }\theta=90^0]$

ii) we have $\vec{A}\times\vec{B}=(5\hat{i})\times(2\hat{j})=(5)(2)(\hat{i}\times\hat{j})=10(\hat{k})=10\hat{k}$

Q.48 ***How one can find value of the angle made by the two vectors in case of dot product? Find the angle between the vectors*** $\vec{A}$ and $\vec{B}$ ***having values:***
$\vec{A} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ ***and*** $\vec{B} = 5\hat{i} + 4\hat{j} + 3\hat{k}$.

Ans. Since $\vec{A}\cdot\vec{B} = AB\cos\theta$ or $\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB}$ or $\theta = \cos^{-1}\frac{\vec{A}\cdot\vec{B}}{AB}$

b) To determine the angle, using

or $\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB} = \frac{(3\hat{i}+4\hat{j}+5\hat{k})\cdot(5\hat{i}+4\hat{j}+3\hat{k})}{\sqrt{9+16+25}\times\sqrt{25+16+9}} = \frac{15+16+15}{\sqrt{50}\times\sqrt{50}} = \frac{46}{50} = 0.92$

or $\theta = \cos^{-1}(0.92) = 23^0$

Q.49 ***Find the projection of vector*** $\vec{A} = 4\hat{i} - 8\hat{j} + 2\hat{k}$ ***in the direction of*** $\vec{B} = 4\hat{i} - 3\hat{j} - 12\hat{k}$.

Ans. By definition $\vec{A}\cdot\vec{B} = AB\cos\theta$

& the projection of $\vec{A}$ on $\vec{B}$ = A cosθ,

$$A\cos\theta = \frac{\vec{A}\cdot\vec{B}}{B} = \frac{(4\hat{i}-8\hat{j}+2\hat{k})\cdot(4\hat{i}-3\hat{j}-12\hat{k})}{\sqrt{4^2+(-3)^2+(-12)^2}} = \frac{(4\times4)+(-8\times-3)+(2\times-12)}{\sqrt{169}}$$

or $A\cos\theta = \frac{16+24-24}{13} = \frac{16}{13} = 1.23$

Q.50 ***If*** $\vec{A} = \vec{A}_1\hat{i} + \vec{A}_2\hat{j}$ & $\vec{B} = \vec{B}_1\hat{i} + \vec{B}_2\hat{j}$, ***then what will be the direction of the cross product of the two vectors*** $\vec{A}$ and $\vec{B}$ ***?***

Ans. Since both vectors $\vec{A}$ and $\vec{B}$ lie in x and y plane along the direction of $\hat{i}$ and $\hat{j}$ therefore by right-hand-rule, the direction of vector $\vec{A}\times\vec{B}$ must lie along z-axis by a unit vector $\hat{k}$. It shows that the cross product always lies perpendicular to the plane of the two given vector.

Q.51 ***How can the cross product of*** $\vec{A}$ and $\vec{B}$ ***be expressed in determinant form if the vectors are three-dimensional? What is vector area?***
[Alt. ***Express the following cross product in determinant form.***
$\vec{A}\times\vec{B} = \hat{i}(A_yB_z - A_zB_y) + \hat{j}(A_zB_x - A_xB_z) + \hat{k}(A_xB_y - A_yB_x)$]

Ans. Determinant is mathematical notation consisting of a square array of numbers or other elements between two vertical bars; the value of the expression is determined by its expansion according to certain rules.

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

b) An area having imaginary unit normal on its surface, which gives the orientation of the area, is called vector area.

Area of a parallelogram will be $\vec{A}\times\vec{B}$, and its direction is normal to the surface.

Q.52 ***What is meant by center of gravity of a body & center of mass? What will happen if the line of action of the weight of an object lies outside the base of the object?***

Ans. Center of gravity of a body is that single point through which the line of action of the weight passes, at which all the force of gravity can be considered to be acting.

b) Centre of mass is a point in a body at which whole mass of the body may be considered to act.
When an object is suspended, it will come to rest with its center of mass directly below the point of suspension.

c) If the line of action of the weight of an object lies outside the base of the object, then the object will tend to fall over because it is unstable.

Q.53 ***What is torque? Give its SI units & dimension. When it is positive or negative ? What are the three conditions when torque is zero?***

Ans. Torque is the product of the force and its moment arm. It is a vector quantity.
It is given by: $\tau = \vec{r} \times \vec{F} = r \times F = Nm = mN = [L] \times [MLT^{-2}] = [ML^2T^{-2}]$

b) SI units of torque is N m & its dimension is $[ML^2T^{-2}]$

c) The torque which tends to rotate a body in the anti-clockwise direction is taken as positive and the clockwise torque is taken as negative.

d) The three conditions when torque is zero:
   i) If the body is at rest
   ii) If the body is rotating with uniform angular velocity.
   iii) When the line of action of applied force passes through the axis of rotation.

Q.54 ***Compare the torque and force as each other's counterpart.***

Ans. Torque is the product of the force and its moment arm. It is a vector quantity.
It is given by: $\vec{\tau} = \vec{r} \times \vec{F}$ [A turning force]
Force is that which produces or prevents motion, or has a tendency to do so.
It is given by: $\vec{F} = m\vec{a}$ [linear acceleration corresponds linear force]
Torque is the counterpart of force for rotational motion. Just as force determines the linear acceleration produced in a body, the torque acting on a body determines its angular acceleration.

Q.55 ***For angular acceleration to be zero, what should be the net torque acting on the body? Under what condition the torque acting on a system is zero?***

[Alt. ***Is it possible that a force directed to a particular point produces the torque about the same point?***]

Ans. If the net torque acting on a body is zero, then the angular acceleration will be zero.

b) The torque acting on a system will be zero, when the line of action of the force passes through axis of rotation, i.e., when moment arm is zero.

Q.56 ***Can a central force give rise to a torque about that point?***

[Alt. ***What is the torque of a force about the point lying on the axis of rotation.***]

Ans. Central force is a force that acts on any affected object along a line to an origin.
No, central force cannot produce any torque, because in this case, the force is directed towards the same point. So $\vec{F}$ and $\vec{r}$ are anti-parallel to each other.
The magnitude of the torque will be:
$|\vec{\tau}| = \tau = rF\sin\theta = rF\sin 180^0 = rF(0) = 0$ [$\theta = 180^0$ – being anti-parallel]

Q.57 ***What is meant by a couple (or torque of a couple)? Give two examples of couple.***

Ans. Couple is a pair of equal parallel forces in opposite directions and not acting through a single point.

It is the product of one of the forces and the perpendicular distance between the two equal forces that act in opposite directions.

Their linear resultant is zero, but there is a net turning effect or torque.

b) The two examples of couple are:
   i) The force applied on the handle of a bicycle to turn.
   ii) The couple produced in Cavendish experiment for determination of G.

Q.58 ***Why brake drums of vehicles are of large diameters? A force of 20N is applied at the edge of a wheel of radius 40 cm. Find the magnitude of torque acting on the wheel.***

Ans. It is because of the moment arm. Since $\vec{\tau} = \vec{r} \times \vec{F}$.

Brake drums of large diameter have large moment arm so torque will be greater and brakes are more effective than with a brake drum of small diameter and moment arm.

b) The magnitude of the torque will be:

$$|\vec{\tau}| = \tau = rF\sin\theta = (0.4\text{m})(20\text{N})\sin 90^0 = (0.4\text{m})(20\text{N})(1) = 8.0 \text{ Nm} \quad [\,\theta = 90^0\,]$$

Q.59 ***What happened when some of all the forces acting on a body is zero?***
How are equilibrium and resultant of a number of vectors is related to each other?

[Alt. ***What is the first condition of equilibrium?***]

Ans. First condition of equilibrium is the sum of all the force acting on a body along x-axis and along y-axis should be equal to zero. Mathematically

$$\Sigma\,\vec{F} = 0$$

or $\quad$ i) $\Sigma\,F_x = 0$ & ii) $\Sigma\,F_y = 0$

If 1[st] condition is satisfied, there is no linear acceleration and the body will be in translational equilibrium. In this case the first condition of equilibrium is satisfied, provided linear acceleration is zero.

b) Equilibrium and resultant of a number of vectors are equal and opposite to each other.

Q.60 ***State what is meant by equilibrium? When a body will be in a complete equilibrium? What can be said about acceleration of a body, which is in complete equilibrium?***

[**Alt.** ***Write down the conditions for a body to be in complete equilibrium.***]

[Alt. ***State conditions for equilibrium of a body, which is acted upon by a number of forces.***]

Ans. If a body, under the action of a number of forces, is at rest or moving with uniform velocity, it is said to be in equilibrium.

For a body in complete equilibrium the sum of all the forces and torques acting on the body must be equal to zero. That is, when there is no linear and rotational acceleration in a body, then the body is said to be in complete equilibrium.

i.e. $\quad$ 1) $\Sigma\,F_x = 0$, $\Sigma\,F_y = 0$ & 2) $\Sigma\,\vec{\tau} = 0$

b) For a body to be in complete equilibrium, both linear and angular accelerations should be zero.

Q.61 ***Write conditions for i) translational equilibrium, ii) rotational equilibrium, iii) static equilibrium and iv) dynamic equilibrium.***
A body in static equilibrium is also in translational equilibrium. Is it true? Is its reverse true?

Ans. i) A body will be in translational equilibrium if the vector sum of all the forces acting on a body equal to zero.

ii) A body is in rotational equilibrium if the sum of all the torques acting on it equals to zero.

iii) A body is in a state of rest is said to be in static equilibrium.

iv) When a body is moving with a uniform velocity or rotating with a uniform angular velocity, it is said to be in dynamic equilibrium.

b) No, a body in static equilibrium cannot be in translational equilibrium. However the reverse is true.

Q.62 ***Explain why a particle experiencing only one force cannot be in equilibrium.***
A particle is acted upon by a single force $\vec{F} = 2N$***. Is it in the state of equilibrium?***

Ans. From conditions of equilibrium,

i.e. or 1) $\Sigma F_x = 0$, $\Sigma F_y = 0$ & 2) $\Sigma \vec{\tau} = 0$

It shows that pair of action and reaction forces acts a body. The equilibrium can only exist when sum of all the forces acting on a body is zero.
When one force is acting on a particle, equilibrium cannot exist, that's why a particle experiencing only one force cannot be in equilibrium.

b) This particle having a single force, $\vec{F} = 2N$ will not be in equilibrium, as it does not satisfy the following the two conditions of equilibrium.

Q.63 ***Give an example of a body, which is in motion, yet is in equilibrium.***

Ans. Motion is continuous change of location of a body with respect to its surroundings.

i) A body moving with a constant velocity.

ii) A paratrooper moving downward, (after jumping from an aeroplane).
The weight of the paratrooper is balanced by the upward reaction of air.
Thus, the parachute falls with nearly uniform velocity under equilibrium

Q.64 ***Can a force directed north balance a force directed east?***

Ans. No, a force directed north cannot balance a force directed east.
Let F_1 be the force directed east which is along x-axis and F_2 be the force directed north which is along y-axis, the angle θ between them will be 90^0.

Then $\Sigma (F_{1x} + F_{2x}) = F_x = F_1 \cos 0^0 + F_2 \cos 90^0 = F_1$

or $\Sigma (F_{1y} + F_{2y}) = F_y = F_1 \sin 0^0 + F_2 \sin 90^0 = F_2$

So the resultant magnitude will be: $\Sigma F = F = \sqrt{F_x^2 + F_y^2} = \sqrt{F_1^2 + F_2^2}$

The above calculations show that $\Sigma F \neq 0$
i.e. summation of all the forces is not equal to zero, but have certain value, so they cannot balance each other.