Electricity & Magnetism

A paraphrased version of

Important Articles

of the Textbook by

Resnick/Halliday/Krane

for

B. Sc. Students

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CONTENTS

Electric Charge & Coulomb's Law

Electromagnetism

The study of electricity and magnetism with inter related phenomena.

Electron

Smallest unit of negative electric charge.

Maxwell's Equations

A series of classical equations that govern the behaviour of electromagnetic waves in all practical situations. They connect vector quantities applying to any point in a varying electric or magnetic field.

The equations are:

1- $\oint \vec{E} \cdot d\vec{A} = q/\epsilon_0$ Gauss' law for electricity

2- $\oint \vec{B} \cdot d\vec{A} = 0$ Gauss' law for magnetism

3- $\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$ Faraday's law of induction

4- $\oint \vec{B} \cdot d\vec{s} = \mu_0 i$ Ampere's law.

Maxwell's four equations play the same role in electromagnetism as Newton's laws in classical mechanics, or Schrodinger's wave equation in Quantum Mechanics, or the laws of thermodynamics in the study of heat.

Electromagnetic Waves (or Electromagnetic radiation)
Transverse waves in space having an electric component and a magnetic component, each being perpendicular to each other and both perpendicular to the direction of propagation. These do not require any medium for its motion.
[See Appendix A]

Radio Waves (or Radio Frequency)
The frequency of electromagnetic radiation within the range used in radio, i.e. from 3×10^7 to 3×10^9 Hertz.

Table-1:
Characteristics of the Four Fundamental Forces

Force	Relative Strength	Range	Importance
Strong	1	10^{-15} m	Holds nucleus together
Electromagnetic	10^{-2}	Infinite	Friction, tensions, etc
Weak	10^{-5}	10^{-15} m	Nuclear decay
Gravitational	10^{-39}	Infinite	Organizes universe

The role of weak force is in natural radioactivity. This is also short range force i.e. up to the size of a nucleus. Yet the weak force causes some stars ultimately to explode.
Dr. Abdus Salam proposed a theory in 1967, according to which the weak and electromagnetic forces can be regarded as part of a single force, called the electroweak force. Glashow, Salam and Weinberg shared the 1979 Nobel Prize for it. Other new theories, called grand unification theory, have been proposed which combine the strong and electroweak forces into single frame work.

Electric charge

Quantity of electricity; Flow of electrons in a conductor.

The process of rubbing transfers a tiny amount of charge from one body to the other, which makes a charge imbalance.

Charges of the same sign repel each other, and charges of the opposite sign attract each other.

Industrial applications

Electrical forces between charged bodies have many applications.

1- Electrostatic paint spraying and powder coating
2- Fly-ash precipitation.
3- Non impact ink-jet printing.
4- Photocopying.

Conductor

A material through which an electric charge is readily transferred.

In a typical conductor, there are about 10^{23} conduction electrons per cm^3.

Examples of conductors are copper, metals, tap water and human body.

Conduction electrons

Mobile free electrons in a conductor.

Insulator

A material through which an electric charge is not readily transferred.
In an insulator at room temperature, we are on the average unlikely to find even 1 conduction electron per cm^3.
Examples of insulators are glass, pure water and plastics.

Semiconductor

A material that has a resistivity midway between that of conductors and that of insulators.
In a typical semiconductor there are about 10^{10} to 10^{12} conduction electrons per cm^3.
Examples are silicon and germanium.

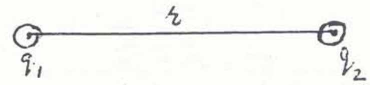
Hall Effect

When an electric current is passed through a conductor and a magnetic field is applied at right angles, a potential difference is produced between two opposite surfaces of the conductor. The direction of the potential gradient is perpendicular to both the current direction and the field direction. It is caused by deflection of the moving charge carriers in the magnetic field. The size and direction of the potential difference gives information on the number and type of charge carriers.

Coulomb's Law

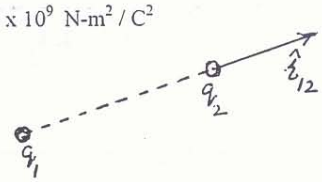
Statement:

"The magnitude of the force between two point charges is directly proportional to the product of the magnitudes of the charges and inversely proportional to the square of the distance between them". Mathematically



$$F_e \propto q_1 q_2 / r^2$$

or $$\boxed{F_e = k \frac{q_1 q_2}{r^2}}$$ where $k = 1/4\pi\epsilon_0 = 9 \times 10^9$ N-m^2/C^2

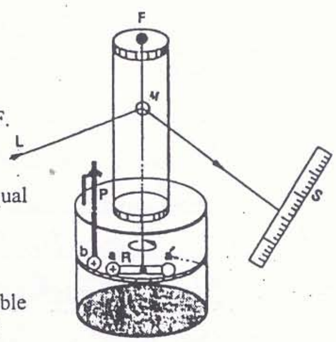


in vector form

$$\boxed{\vec{F}_e = k \frac{q_1 q_2}{r^2_{12}} \hat{r}_{12}}$$

Coulomb's Torsion balance:

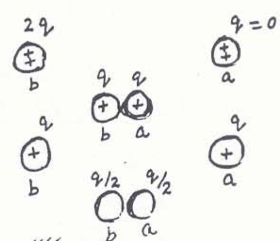
As shown in the figure, the torsion balance consists of a horizontal insulated rod R carrying two metallic spheres *a* & *a'* at the ends of the rod. The rod is suspended from middle with a fibre supported from F. A small mirror M is attached to the fibre and a beam of light reflected from this mirror falls on the scale. Another insulated rod P, carrying a small sphere *b*, equal in size a, can be brought near the small sphere *a*.



Experimental details:

The sphere *b* is charged by rubbing it with some suitable material. When sphere *b* is touched with sphere *a*, it shares its charge so that both have equal charges,

$$q_1 = q_2 = q$$

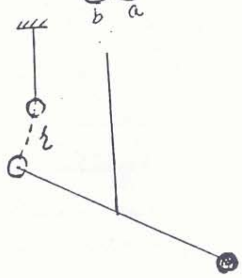


The charge on *b* can be further divided into q/2, q/4, so on, by touching it with the spheres of equal sizes.
By keeping the distance *a* and *b* fixed, we find

$$F \propto q_1 q_2$$

On varying the distance between two spheres with constant charge, it found that

$$F \propto 1/r^2$$



Combining the above two relations, we get

$$\boxed{F \propto q_1 q_2 / r^2}$$

which is Coulomb's Law.

Coulomb's Law & Newton's Gravitation Law

Resemblance

Both are inverse square laws.

$$F_g = G\frac{m_1 m_2}{r^2} \Rightarrow F \propto 1/r^2$$

$$F_e = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2} \Rightarrow F \propto 1/r^2$$

Differences

1 – Gravitational forces are always attractive, but electrostatic forces can be repulsive or attractive.

2 – In law of gravitation we determine G from the known masses.
In Coulomb's Law we first define k to have a particular value.

Some Characteristics of Coulomb's Law

1 – We can apply 'the principle of superposition' to electrical forces to calculate the sum in vector form.

2 – Coulomb's law remain valid in quantum limit. It holds also for fundamental particles such as electrons and quarks. But Newton's law of gravitation is useful for everyday approximation of general theory of relativity.

3 – At relativistic speeds instead of Coulomb's law, Maxwell's equations are applied for more complete analysis.

4 – When incorporated into the structure of quantum physics, correctly describes;

i) electrical forces that bind electrons with nucleus.
ii) the forces that bind atoms to form molecules.
iii) intermolecular forces to form solids & liquids.

The Electric Field

Field

The concept of a field was introduced to explain the interaction of particles or bodies through space.

Electric field

A region in which a force having magnitude and direction would be exerted upon a unit positive charge.

Electric field strength (or Electric Intensity) **E**:

A measure of the strength of an electric field at a point, defined as the force per unit charge on an electric charge placed at that point. Mathematically

$$\vec{E} = \vec{F}/q_0$$

Scalar field

A field such as temperature field or pressure field describing the spatial variation.

Vector field

A field such as a gravitational field or magnetic field in which the magnitude and direction of the vector quantity are one-valued functions of position.

Static field

A field in which quantities such as temperature, pressure or flow velocity, do not vary with time.

Time-Varying field:

A field in which the quantities vary with time.

Interaction of Forces

Two theories explain that how forces are transmitted from one body to another or from one charge to another charge.

i) Action at a distance

It means that the force between two masses (or charged bodies) is conveyed directly with no time delay between the two bodies.

$$\text{charge} \rightleftharpoons \text{charge}$$

This view violates the special theory of relativity which limits the speed to c (velocity of light)

ii) Field theory

This modern interpretation based on field concept and now an essential part of the general theory of relativity.

Represented as

$$\text{charge} \rightleftharpoons \text{field} \rightleftharpoons \text{charge}$$

in which each charge (or mass) interacts not directly with the other but instead with the gravitational field established by the other.

Point charge

A small uniform sphere of charge rather than a true mathematical point.

Electric dipole

A system of two equal and opposite charges placed at a very short distance apart.

Electric dipole moment (p)

The product of either of the charges and the distance between them. Symbolically

$$p = qd.$$

Electric quadrupole

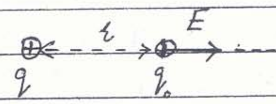
A distribution of charge equivalent to two equal electric dipoles arranged very close together and set in opposite directions.

E due to point charge q

The magnitude of force acting on q_0 is

$$F = \frac{1}{4\pi\epsilon_0}\frac{q q_0}{r^2}$$

so $$E = \frac{F}{q_0} = \frac{1}{4\pi\epsilon_0}\frac{q q_0}{r^2}\times\frac{1}{q_0}$$

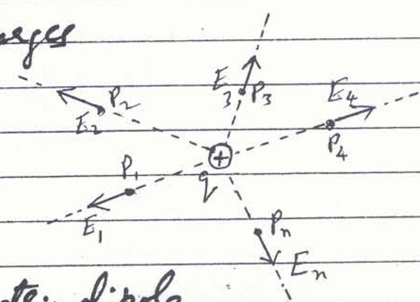


or $$\boxed{E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}}$$

E for a group of N point charges

From the application of principle of superposition

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots + \vec{E}_n$$



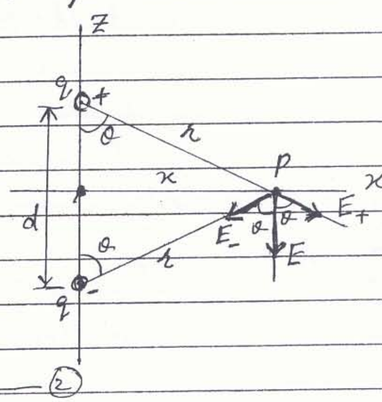
E at point P due to an electric dipole

The total electric field at P is

$$\vec{E} = \vec{E}_+ + \vec{E}_-$$

or $$E_+ = E_- = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

or $$E_+ = \frac{1}{4\pi\epsilon_0}\frac{q}{x^2+(d/2)^2} \quad \text{—— (1)}$$



The x-component of total field is

$$E_+\sin\theta - E_-\sin\theta = 0$$

so total field has only z component

$$E = E_+\cos\theta + E_-\cos\theta = 2E_+\cos\theta \quad \text{—— (2)}$$

from eqs (1) & (2)

$$E = 2\frac{1}{4\pi\epsilon_0}\frac{q}{x^2+(d/2)^2}\frac{d/2}{\sqrt{x^2+(d/2)^2}} \qquad \left[\cos\theta = \frac{d/2}{\sqrt{x^2+(d/2)^2}}\right.$$

or $$\boxed{E = \frac{1}{4\pi\epsilon_0}\frac{qd}{[x^2+(d/2)^2]^{3/2}}} \quad \text{—— (3)}$$

or $$E = \frac{1}{4\pi\epsilon_0}\frac{p}{[x^2+(d/2)^2]^{3/2}}$$ where $qd = p$ = electric dipole moment from eq. (3)

$$E = \frac{1}{4\pi\epsilon_0} \frac{qd}{\left[x^2\left(\frac{x^2+(d/2)^2}{x^2}\right)\right]^{3/2}}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{p}{(x^2)^{3/2}\left[1+\left(\frac{d}{2x}\right)^2\right]^{3/2}}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{p}{x^3} \left[1+\left(\frac{d}{2x}\right)^2\right]^{-3/2}$$

Binomial theorem

$$\left[(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots\cdots\right]$$

or $E = \frac{1}{4\pi\epsilon_0} \frac{p}{x^3} \left[1 + \frac{3}{2}\frac{d^2}{4x^2} + \cdots\cdots\right]$

For $x >> d$, neglecting higher powers

$$\boxed{E = \frac{1}{4\pi\epsilon_0} \frac{p}{x^3}}$$

A Comparison.

Due to point charge we have

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \Rightarrow E \propto 1/r^2$$

but for a dipole

$$E = \frac{1}{4\pi\epsilon_0} \frac{p}{r^3} \Rightarrow E \propto 1/r^3$$

i.e. for a dipole, E varies much rapidly comparing with point charge.

& For an electric quadrupole, we have

$$E \propto 1/r^4$$

Electric lines of force (or Electric field line)

i) A line so drawn that a tangent to it at any point indicates the orientation.

ii) The path of a free positive charge takes when it is placed in an electric field.

Properties of lines of force

1- The lines of force give the direction of the electric field at any point.

2- The lines of force originate on positive charges and terminate on negative charges.

3- The lines of force are drawn so that the number of lines per unit cross-sectional area (perpendicular to the lines) is proportional to the magnitude of the electric field.

4- Two lines of force cannot cross at a point.

5- The lines of force are perpendicular to the surface of a conductor.

Principle of superposition (of waves)

When two (or more) waves of the same type pass through the same region, the amplitude of vibration at any point is the algebraic sum of the individual amplitudes.

Principle of superposition (in electric fields)

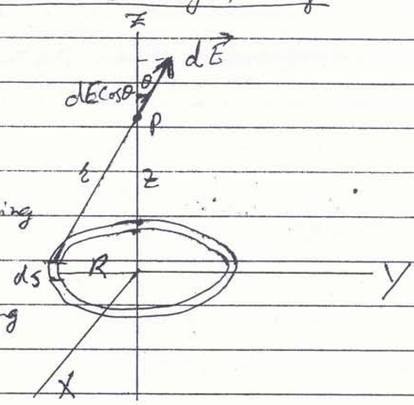
At a given point the electric fields due to separate charge distributions simply add up (vectorially) or superimpose independently.

E due to continuous charge distribution

i) Ring of charge

To find $\vec{E}$ at point P due to the ring of charge

Let



Z = distance P from the ring

R = radius of the ring

λ = linear charge density

ds = length of small element of the ring

$dq = \lambda ds$ = charge on ds

&

$q = \lambda(2\pi R)$ = total charge on the ring

We have

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

For differential element of length ds

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2} \qquad \left[\begin{array}{l} dq = \lambda ds \\ r^2 = z^2 + R^2 \end{array}\right.$$

or $$dE = \frac{1}{4\pi\epsilon_0}\frac{\lambda ds}{(z^2+R^2)} \quad \text{--- (1)}$$

& $$\cos\theta = \frac{z}{r} = \frac{z}{(z^2+R^2)^{1/2}} \quad \text{--- (2)}$$

From the symmetry of the problem, E will have no x or y components, all the elements will be paired, so

$E_x = E_y = 0$

the total E must be parallel to z-axis

The z component of dE is $dE\cos\theta$, from eqs (1) & (2)

$$dE\cos\theta = \frac{1}{4\pi\epsilon_0}\frac{\lambda ds}{(z^2+R^2)}\frac{z}{(z^2+R^2)^{1/2}} = \frac{z\lambda ds}{4\pi\epsilon_0(z^2+R^2)^{3/2}} \quad \text{--- (3)}$$

or $$E_z = \int dE\cos\theta = \int \frac{z\lambda ds}{4\pi\epsilon_0(z^2+R^2)^{3/2}}$$

As same value for all charge elements

$$E_z = \frac{z\lambda}{4\pi\epsilon_0(z^2+R^2)^{3/2}}\int ds = \frac{z\lambda(2\pi R)}{4\pi\epsilon_0(z^2+R^2)^{3/2}} = \frac{qz}{4\pi\epsilon_0(z^2+R^2)^{3/2}}$$

or $$\boxed{E_z = \frac{qz}{4\pi\epsilon_0(z^2+R^2)^{3/2}}} \quad \text{--- (4)}$$

For $Z >> R$, eq (2) gives

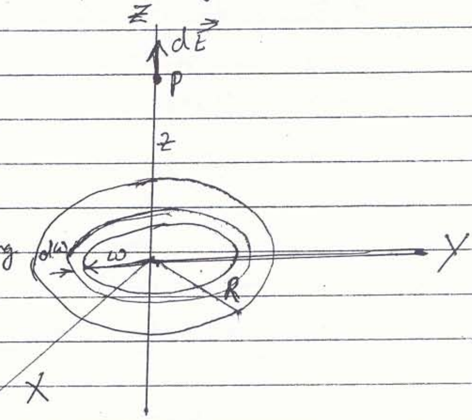
$$E_z = \frac{1}{4\pi\epsilon_0}\frac{q}{Z^2}$$

i.e. for large enough distances, the ring would appear as a point charge.

For $Z = 0$, $E_z = 0$

i.e. a test charge at the centre of the ring would experience no net force.

ii) A Disk of Charge



To find $\vec{E}$ at point P

Let

Z = distance P from the disk

$d\omega$ = width of one of the concentric ring

ω = the radius of that ring

σ = surface charge density

$dA = (2\pi\omega)d\omega$ = differential area of the ring

$dq = \sigma dA$ = charge on the ring

From previous knowledge [eq(3)], component of E along z direction

$$dE_z = \frac{dq\, z}{4\pi\epsilon_0 (z^2+\omega^2)^{3/2}} = \frac{z\sigma\, 2\cancel{\pi}\omega\, d\omega}{4\cancel{\pi}\epsilon_0 (z^2+\omega^2)^{3/2}}$$

$$\text{or } dE_z = \frac{\sigma z}{4\epsilon_0}(z^2+\omega^2)^{-3/2}(2\omega)\,d\omega$$

$$\text{or } E_z = \int dE_z = \int \frac{\sigma z}{4\epsilon_0}(z^2+\omega^2)^{-3/2}(2\omega)\,d\omega = \frac{\sigma z}{4\epsilon_0}\int_0^R (z^2+\omega^2)^{-3/2}\, 2\omega\, d\omega$$

$$\text{or } E_z = \frac{\sigma z}{4\epsilon_0}\left[\frac{(z^2+\omega^2)^{-3/2+1}}{-3/2+1}\right]_0^R \qquad \left[\int x^m dx = \frac{x^{m+1}}{m+1}\right]$$

$$= \frac{\sigma z}{4\epsilon_0}\left[\frac{z^{2\times -1/2}}{-1/2} - \frac{(z^2+R^2)^{-1/2}}{-1/2}\right] = \frac{\sigma z}{4\epsilon_0}\left[\frac{-2}{z} + \frac{2}{\sqrt{z^2+R^2}}\right]$$

$$= \frac{\sigma z}{4\epsilon_0}\times\frac{-2}{z}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right] = -\frac{\sigma}{2\epsilon_0}\left[1-\frac{z}{\sqrt{z^2+R^2}}\right]$$

taking +ve direction of E

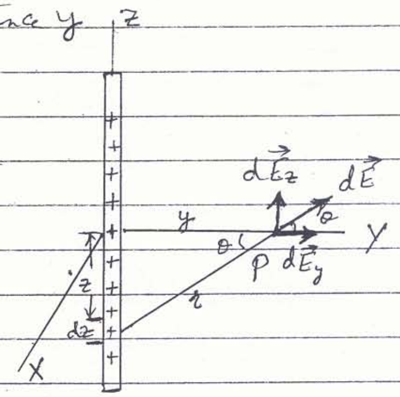
$$\boxed{E_z = \frac{\sigma}{2\epsilon_0}\left[1-\frac{z}{\sqrt{z^2+R^2}}\right]}$$

For $R >> z$, $E_z = \frac{\sigma}{2\epsilon_0}$ — the case of infinite sheet

For $z >> R$ — reduces to the field of a point charge

iii) Infinite line of Charge

To find $\vec{E}$ at point P, a distance y from the line of charge



Let

y = distance P from the line of charge

dz = length of small element

z = distance of dz from centre

λ = linear charge density

$dq = \lambda dz$ = charge on dz

We have

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}$$

or $$dE = \frac{1}{4\pi\epsilon_0}\frac{\lambda dz}{(y^2+z^2)} \quad \text{——— (1)}$$

~~$E_z = 0$, as for every E_+ there is corresponding E_-~~

~~So $+E_z - E_z = 0$~~

From cylindrical symmetry, rotation of point P in x-y plane can give either E_y or E_x. So there is no idea to calculate separately E_x.

So $$E = E_y = \int dE_y = \int_{z=-\infty}^{z=+\infty} dE\cos\theta$$

For infinite length, both halves are equal

So $$E = 2\int_0^{z=+\infty} \cos\theta\, dE = 2\int_0^{\infty}\cos\theta \frac{1}{4\pi\epsilon_0}\frac{\lambda dz}{(y^2+z^2)}$$

or $$E = \frac{\lambda}{2\pi\epsilon_0}\int_0^{\infty}\cos\theta\frac{dz}{y^2+z^2}$$

$\left[\begin{array}{l} z = y\tan\theta \\ dz = y\sec^2\theta\, d\theta \end{array}\right.$ $\left[\frac{d}{dx}\tan x = \sec^2 x\right.$

or $$E = \frac{\lambda}{2\pi\epsilon_0}\int_{\theta=0}^{\theta=\pi/2}\cos\theta\frac{y\sec^2\theta\, d\theta}{y^2+y^2\tan^2\theta}$$

$$= \frac{\lambda}{2\pi\epsilon_0 y}\int_0^{\pi/2}\cos\theta\frac{\sec^2\theta}{1+\tan^2\theta}d\theta$$

$$\left[\frac{\sec^2\theta}{1+\frac{\sin^2\theta}{\cos^2\theta}} = \frac{\sec^2\theta}{\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}} = \frac{\sec^2\theta\cos^2\theta}{\sin^2\theta+\cos^2\theta} = 1\right.$$

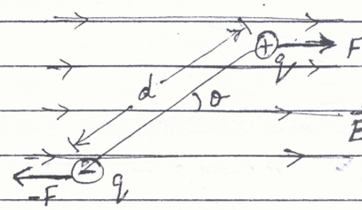
or $$E = \frac{\lambda}{2\pi\epsilon_0 y}\int_0^{\pi/2}\cos\theta\, d\theta = \boxed{\frac{\lambda}{2\pi\epsilon_0 y}}$$

As line of charge has cylindrical symmetry

So $\boxed{E = \frac{\lambda}{\ldots}}$

A dipole in an electric field

In the figure, a dipole is in an external field $\vec{E}$.



To find the net torque about the center of the dipole due to two forces.

$\tau = F\frac{d}{2}\sin\theta + F\frac{d}{2}\sin\theta = Fd\sin\theta$

$[F=qE]$ $[p=qd]$ $[\vec{A}\times\vec{B} = AB\sin\theta]$

or $\tau = qEd\sin\theta = qdE\sin\theta = pE\sin\theta = \vec{p}\times\vec{E}$

or $\boxed{\tau = \vec{p}\times\vec{E} = pE\sin\theta}$ —— (1)

To find the potential energy for dipole moment in $\vec{E}$

We define: Potential energy (ΔU) as "the amount of work done against the field in displacing the charge from one point to other".

Mathematically $\Delta U = -W$ —— (2)

$[s = r\theta]$

taking $W = \vec{F}\cdot\vec{s} = Fs\cos\theta = F\cos\theta\, s = F\cos\theta\, r\theta$

as $F\cos\theta$ is $\perp$ to r (and $\parallel$ to s)

so $W = F_\perp r\theta$ $[\tau = rF\sin\theta = rF_\perp = F_\perp r]$

or $W = \tau\theta$

or $dW = \tau\, d\theta$ —— (3)

Considering work done by the electric field in turning the dipole through an initial angle θ_0 to final angle θ.

$W = \int dW = \int_{\theta_0}^{\theta} \tau\, d\theta$

As τ tends to decrease θ, i.e. τ & $d\theta$ are in opposite directions

so $W = \int_{\theta_0}^{\theta} -\tau\, d\theta$ —— (4)

from (1) & (4), $W = \int_{\theta_0}^{\theta} -pE\sin\theta\, d\theta = -pE\int_{\theta_0}^{\theta}\sin\theta\, d\theta$

or $W = pE(\cos\theta - \cos\theta_0)$ —— (5) $[\int\sin x\,dx = -\cos x]$

From eqs (2) & (5) $\Delta U = U(\theta) - U(\theta_0) = -W = -pE(\cos\theta - \cos\theta_0)$

taking θ_0 as reference angle to $90°$ & choosing $U(\theta_0) = 0$

then $U = -pE\cos\theta$ or $\boxed{U = -\vec{p}\cdot\vec{E}}$ —— (6) $[\vec{A}\cdot\vec{B} = AB\cos\theta]$

~~Thus~~ U is minimum when $\vec{p}$ & $\vec{E}$ are parallel.

Thus the motion of a dipole in a uniform electric field can be interpreted either from the perspective of force or energy.

Gauss' Law

Electric Flux (Φ_E)

i) Electric lines of force in an electric field considered collectively.

ii) The scalar product of the electric field and the area. Mathematically,

$$\Phi_E = \oint \vec{E} \cdot d\vec{A}$$

Gauss' Law

i) The flux through any closed surface times ϵ_0 is equal to the total charge enclosed in it.

ii) For any closed surface drawn in an electric field the integral $\int \vec{E} \cdot d\vec{A}$ of the normal component of the electric field $\vec{E}$ over the surface is equal to the total charge within the surface.

Mathematically

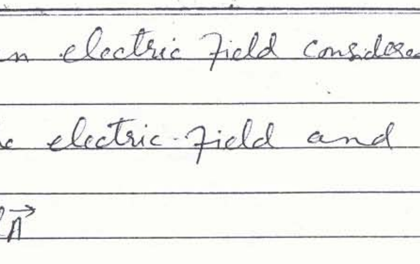
$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = q$$

To obtain Coulomb's Law from Gauss' Law

Consider an isolated point charge q.

Choosing a spherical surface of radius r.

Applying Gauss' Law.



$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = q$$

or $\epsilon_0 \oint E\, dA = q$

or $\epsilon_0 E \oint dA = q$ or $\epsilon_0 E \cdot 4\pi r^2 = q$

or $\boxed{E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}}$ —— (1)

This eq (1) gives the magnitude of $\vec{E}$ at distance r, due to a point charge q.

Now we have $E = F/q_0$ or $F = q_0 E$ —— (2)

from eqs (1) & (2) $\boxed{F = \frac{1}{4\pi\epsilon_0} \frac{q q_0}{r^2}}$ —— (3) which is Coulomb's Law.

Characteristics

1- Gauss' Law is more general, as it also covers moving charges.

2- From it Coulomb's Law can be obtained.

3- Gauss' Law is one of the fundamental equation of Maxwell's equations.

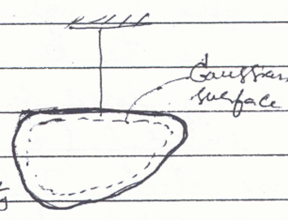
4- The factor $\frac{1}{4\pi\epsilon_0}$ in Coulomb's Law permits simple formulation for Gauss' Law.

Charged Isolated Conductor

Theorem:

An excess charge placed on an isolated conductor moves entirely to the outer surface of the conductor. None of the excess charge is found within the body of the conductor.

If $E \neq 0$, E would exert force on conduction electrons, & internal current would set up. From experiments we know that there is no such enduring currents in isolated conductor.

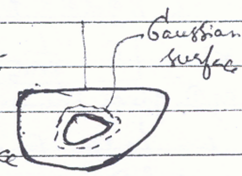


So under equilibrium condition, $E = 0$ inside the conductor, then flux is also zero,
So $\therefore$ from the Gauss's law,

$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = q$$

the net charge must also be zero, i.e. no excess charge would be within the body of the conductor and it is entirely to the outer surface. Hence the theorem is proved.

Isolated conductor with a Cavity



Draw a Gaussian surface around the cavity
Because $E = 0$ inside the conductor
there can be no flux this this Gaussian surface
From Gauss' Law

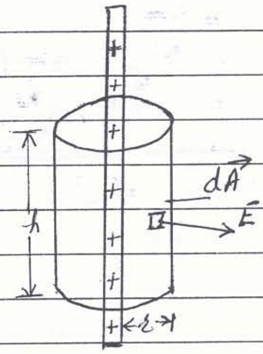
$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = q$$

this cavity surface can enclose no net charge
So there is no charge on the cavity walls,
it remains on the outer surface of the conductor.

Applications of Gauss' Law

Infinite line of charge

To find the electric field E at a distance r from the line. Choose a circular cylinder of radius r and length h, closed at each end by plane caps normal to the axis.



From Gauss' law

$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = q$$

or $\epsilon_0 \oint E\, dA = q$

as E is constant over the cylindrical surface

so $\epsilon_0 E \oint dA = q$ $\quad [\oint dA = 2\pi r h]$

or $\epsilon_0 E (2\pi r h) = q$

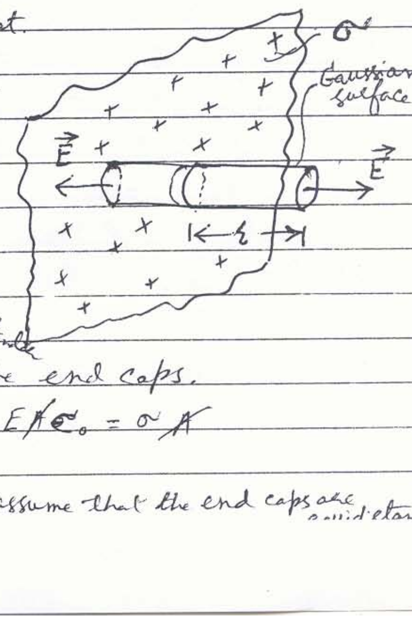
or $E = \dfrac{q}{2\pi\epsilon_0 r h} = \dfrac{\lambda h}{2\pi\epsilon_0 r h}$ $\quad [\lambda = \dfrac{dq}{ds} = \dfrac{q}{h}$

or $\boxed{E = \dfrac{\lambda}{2\pi\epsilon_0 r}}$

We see that due to cylindrical symmetry, calculations become simple. Although Gauss' law holds for all closed surfaces.

Infinite sheet of charge

To calculate $\vec{E}$ at points near the sheet. Choose a Gaussian surface of a closed cylinder of cross-sectional area A, arranged to pierce the plane.



Applying Gauss' law

$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = q \quad \left[\sigma = \frac{q}{A} \text{ or } q = \sigma A\right.$$

or $\epsilon_0 \oint E\, dA = \sigma A$

The flux through curved walls of the cylinder is zero, the only contribution is from the end caps.

$$\epsilon_0 (EA + EA) = \sigma A \quad \text{or} \quad 2E\cancel{A}\epsilon_0 = \sigma \cancel{A}$$

or $\boxed{E = \dfrac{\sigma}{2\epsilon_0}}$

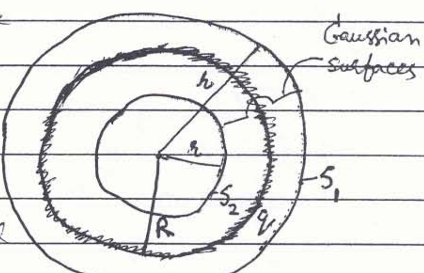
Please note that there is no need to assume that the end caps are equidistant.

Shell Theorems

1- "A uniform spherical shell of charge behaves, for external points, as if all its charge were concentrated at its center".

2- "A uniform spherical shell of charge exerts no electrostatic force on a charged particle placed inside the shell."

The spherical shell of charge is surrounded by two concentric spherical Gaussian surfaces S_1 & S_2.
From the symmetry argument the field can have only radial component.



Applying Gauss' law to surface S_1, for which $r > R$

$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = q$$

$$\text{or} \quad \epsilon_0 E \oint dA = q \quad \text{or} \quad \epsilon_0 E (4\pi r^2) = q$$

$$\text{or} \quad \boxed{E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}}$$

this expression is same as for a point charge. So we conclude our first theorem, i.e. the uniformly charged shell behaves like a point charge for all points outside the shell.

Now applying Gauss' law to surface S_2, for which $r < R$

$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = q$$

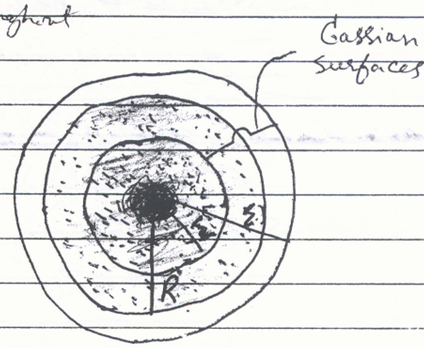
$$\text{or} \quad \epsilon_0 \oint E \cdot dA = 0 \quad [q = 0$$

$$\Rightarrow E = 0$$

So electric field vanishes inside a uniform shell of charge, and a test charge placed anywhere inside would feel no electric force. This proves the second shell theorem.

Spherically Symmetric Charge Distribution

The charge is distributed throughout the spherical volume. So volume charge density, ρ is not constant, but making restriction that it has spherical symmetry i.e. ρ depends on r.
This can be regarded as nest of concentric thin shells.



To calculate E at points for $r > R$

According to first shell theorem, each concentric shell, with charge dq, contribute a radial component dE to the electric field.

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}$$

so total field E will be the sum of all such components

$$E = \int dE = \int \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}$$

since r is constant

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} \quad \text{———} \; (1)$$

where q is the total charge of the sphere.
so for points $r > R$, E has the value that it would have if the charge were concentrated at its centre.

Now for points inside the charge distribution, $r < R$
Applying Gauss' law; $\epsilon_0 \oint \vec{E} \cdot d\vec{A} = q'$ [q' is charge within r]

$$\text{or } \epsilon_0 \oint E\, dA = q' \text{ or } \epsilon_0 E \oint dA = \epsilon_0 E\, 4\pi r^2 = q'$$

$$\text{or} \quad E = \frac{1}{4\pi\epsilon_0}\frac{q'}{r^2} \quad \text{———} \; (2)$$

where q' is part of q contained within the sphere r.
According to 2nd shell theorem, part of q lies outside this sphere makes no contribution to E at radius r.

Now charge is proportional to volume

$$\frac{q'}{q} = \frac{4/3 \pi r^3}{4/3 \pi R^3}$$

$$\text{or } q' = q\left(\frac{r}{R}\right)^3 \quad \text{——— (3)}$$

from eqs (2) & (3)

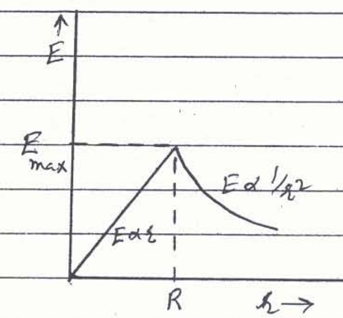
$$E = \frac{1}{4\pi\epsilon_0} \frac{q(r/R)^3}{r^2}$$

$$\text{or } \boxed{E = \frac{1}{4\pi\epsilon_0} \frac{q r}{R^3}} \quad \text{——— (4)}$$

(For uniform sphere $r < R$)

From eq (1), the case of $r > R$ gives $E \propto 1/r^2$ ——— (5)

& eq (4), the case of $r < R$ gives $E \propto r$ ——— (6)



The relations (5) & (6) are shown in the figure.
We see that for $r > R$ applies to <u>any</u> spherically symmetric charge distribution,
but for $r < R$ applies <u>only</u> to a uniform distribution of charge.

Electric Potential Energy (ΔU)

We define: i) The energy involved in bringing it to its current state from some reference state.

ii) "The amount of work done against the field in displacing the charge from one point to other."

Mathematically

$$\Delta U = -W_{ab} \quad \text{——} \ (1)$$

~~eq (1) applies only if the force is conservative.~~

This potential energy is defined only for conservative forces.

Electric Potential or Potential difference (ΔV)

We define i) The quantity which determines flow of electricity.

ii) Potential energy per unit test charge.

iii) The amount of work done against the field in displacing a unit test charge from one point to other.

Mathematically

$$\Delta V = \frac{\Delta U}{q} \quad \text{or} \quad \Delta U = q\,\Delta V \quad \text{——} \ (2)$$

also

$$\Delta V = \frac{\Delta U}{q_0} = \frac{-W_{ab}}{q_0} = \frac{1}{q_0}\int_a^b \vec{F}\cdot d\vec{s} = -\frac{1}{q_0}\int_a^b q_0\vec{E}\cdot d\vec{s}$$

$$\text{or} \quad V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{s}$$

Choosing point 'a' as reference point at ∞, where V_a taken to be zero, & taking point 'b' as any arbitrary point

$$\boxed{V = -\int \vec{E}\cdot d\vec{s}} \quad \text{——} \ (3)$$

To get the concept more clear we can take the following analogies. As "temperature" determines flow of heat, "pressure" determines flow of air; level determines flow of water, corresponding quantity in electricity is "potential".

In electricity potential of a conductor is measured relative to earth. So earth is taken as a reference of potential and assigned the value zero.

Characteristics of Electric potential

1- The potential near an isolated positive charge is positive

2- The potential near an isolated negative charge is negative.

3- A potential of zero at a point does not necessarily mean that the electric field is zero at that point.

4- The SI unit of potential is joule/coulomb, which is often represented by special unit, the <u>Volt</u>.

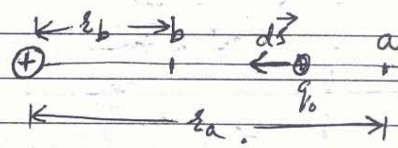
$$1 \text{ volt} = 1 \text{ joule/coulomb}.$$

The unit of potential energy, from eq ②, is electron-volt (eV).

Potential due to a point charge

To compute the potential between two points, a & b

Let



A positive test charge q_0 moves along a radial line from a to b

$$\Delta V = \frac{\Delta U}{q} = \frac{-W_{ab}}{q} = -\frac{1}{q}\int \vec{F}\cdot d\vec{s} = -\frac{1}{q}\int q\vec{E}\cdot d\vec{s}$$

$$\text{or } V_b - V_a = -\frac{1}{q}\int_a^b q\vec{E}\cdot d\vec{s} = \frac{1}{q}\times q\int_{r_a}^{r_b}\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\,dr$$

$$\text{or } V_b - V_a = \frac{q}{4\pi\epsilon_0}\left[\frac{r^{-2+1}}{-2+1}\right]_{r_a}^{r_b} \qquad \left[\int x^m dx = \frac{x^{m+1}}{m+1}\right]$$

$$\text{or } V_b - V_a = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

taking reference point 'a' at infinity, so $1/r_a = 0$ & defining $V_a = 0$

$$V_b = \frac{1}{4\pi\epsilon_0}\frac{q}{r_b}$$

if r be separation at the final point b,

$$\boxed{V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}}$$

which is <u>absolute potential</u>. And we define it as "the amount of work done in moving a unit test charge from infinity to that point against the electrical forces".

Equipotential surfaces

Definition:

"A surface over which the potential has the same value for all points lying on it".

Characteristics

1- For a uniform electric field the equipotential surfaces are planes.

2- The equipotential surfaces of a point charge form a family of concentric spheres.

3- The equipotential surfaces of a point charge are equally spaced.

4- The equipotential surfaces for a dipole are complicated.

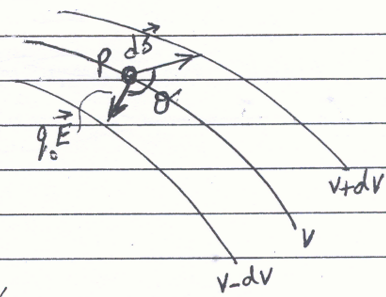
5- The equipotential surfaces are always at right angles to the lines of force and thus to $\vec{E}$.

6- The density of equipotential surfaces can give an indication of the electric field $\vec{E}$.

Calculating the Field from the Potential

To calculate E from V

in the figure equipotential surfaces are shown with differing in potential by dV.



In the fig. $\vec{E}$ at point P is perpendicular to the equipotential surface.

Let a test charge q_0 move from P through displacement $d\vec{s}$ up to $V+dV$

The work done is $= \quad \Delta U = -W \qquad [\Delta U = q\Delta V]$

$$\text{or} \quad dW = -q_0\, dV \quad \text{——} ①$$

Alternately $\quad dW = \vec{F}\cdot d\vec{s} \qquad [W = \text{force} \times \text{displ.}$

$$\text{or} \quad dW = q_0\vec{E}\cdot d\vec{s} \qquad [F = qE$$

$$\text{or} \quad dW = q_0 E\, ds \cos\theta \text{ ——} ② \qquad [\vec{A}\cdot\vec{B} = AB\cos\theta$$

from eqs ① & ② $\quad -\cancel{q_0}\, dV = \cancel{q_0}\, E\, ds\cos\theta$

$$\text{or} \quad E\cos\theta = -\frac{dV}{ds}$$

For maximum value of E

$$E = -\frac{dV}{ds}$$

$$\text{or} \quad \boxed{\vec{E} = -\frac{dV}{d\vec{s}}} \text{ ——} ③$$

where dV/ds is called potential gradient.

Potential gradient: "The change in potential per unit distance"

eq ③ can be written as:- $\vec{E} = -\text{grad}\, V$

which shows that $\vec{E}$ may be defined as "the negative gradient of electrostatic potential V".

In eq ③, $\vec{V}$ depends upon the coordinates x, y & z of the position of the test charge, we get by Calculus, three components of $\vec{E}$ at any point:- $E_x = -\partial V/\partial x$, $E_y = -\partial V/\partial y$, $E_z = -\partial V/\partial z$

If $V(x,y,z)$ for all points of space is known, the components of $\vec{E}$ and $\vec{E}$ itself can be found by taking derivatives.

So we have two methods for calculating $\vec{E}$ for continuous charge distribution. One is based on integrating Coulomb's Law & Other is based on differentiating V.

Energy Storage in an Electric Field

A charged capacitor store in it an electrical potential energy U, equal to the work W done by the external agent as the capacitor is charged.

Let at a time t

a charge transferred from one plate to other is q' & potential difference between the plates be V'

An increment of charge transferred is dq'

resulting small change in potential energy $= dU$

then $dU = V' dq'$ $\quad$ [$\Delta U = q \Delta V$

or $dU = \frac{q'}{C} dq'$ $\quad$ [$q = CV$ or $V = q/C$

If the process is continued until total charge q has been transferred

So total potential energy is

$$U = \int dU = \int_0^q \frac{q'}{C} dq'$$

$$\text{or } U = \frac{1}{C}\int_0^q q' dq' \qquad \left[\int x\,dx = \frac{x^2}{2}\right.$$

$$\text{or } U = \frac{1}{C}\left[\frac{q^2}{2}\right]$$

$$\text{or } \boxed{U = \frac{q^2}{2C}}$$

$$\text{or } \boxed{U = \frac{1}{2}CV^2} \qquad [q = CV$$

It can be supposed that the energy stored in a capacitor resides in the electric field between its plates.

Energy density u

"The amount of stored energy per unit ~~volume~~ volume"
To calculate u in a parallel-plate capacitor we have

$$u = \frac{U}{Ad} = \frac{\frac{1}{2}CV^2}{Ad} \qquad [U = \tfrac{1}{2}CV^2$$

putting values of C & V from the side note

$$u = \frac{\frac{1}{2}\left(\frac{\epsilon_0 A}{d}\right)(Ed)^2}{Ad}$$

or $$u = \frac{\epsilon_0}{2}E^2$$

or $$\boxed{u = \frac{1}{2}\epsilon_0 E^2}$$

$\epsilon_0 \oint \vec{E}\cdot d\vec{A} = q$

or $\epsilon_0 EA = q$ or $E = \dfrac{q}{\epsilon_0 A}$ — (1)

& $\Delta V = \dfrac{-\Delta U}{q} = \dfrac{-W}{q} = \dfrac{-1}{q}\int q\vec{E}\cdot d\vec{s}$

or $V = E\int ds = Ed$ — (2)

From (1) & (2)

$V = \dfrac{qd}{\epsilon_0 A}$ — (3)

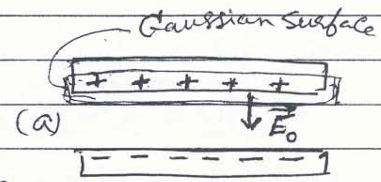
Now

$C = \dfrac{q}{V} = \dfrac{\epsilon_0 A}{d}$ — (4)

So in general capacitors
If an electric field $\vec{E}$ exists at any point in space, we can think of that point as the site of stored energy equal to u.

Dielectrics & Gauss' Law

To apply Gauss' law to a parallel-plate capacitor. Assume the charge on the plates be q.
Drawing Gaussian surfaces as shown in figs. (a) without dielectric & Fig (b) with dielectric.



Applying Gauss' law in fig(a)

$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = \epsilon_0 E_0 A = q$$

$$\text{or} \quad E_0 = \frac{q}{\epsilon_0 A} \quad \text{——} (1)$$

Now with dielectric, applying Gauss' law in fig(b)

$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = \epsilon_0 E A = q - q' \quad \text{——} (2)$$

$$\text{or} \quad E = \frac{q}{\epsilon_0 A} - \frac{q'}{\epsilon_0 A} \quad \text{——} (3)$$

where $-q'$ is the induced surface charge.

The dielectric reduces E by a factor K_e

$$E = \frac{E_0}{K_e} = \frac{q}{K_e \epsilon_0 A} \quad \text{——} (4)$$

from eqs (3) & (4)

$$\frac{q}{K_e \epsilon_0 A} = \frac{q}{\epsilon_0 A} - \frac{q'}{\epsilon_0 A}$$

$$\text{or} \quad q' = q\left(1 - \frac{1}{K_e}\right) \quad \text{——} (5)$$

This shows that the induced surface charge q' is always less in magnitude than the free charge q.

from eqs (2) & (5)

$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = q - \left[q\left(1 - \frac{1}{K_e}\right)\right]$$

$$\text{or} \quad \epsilon_0 \oint \vec{E} \cdot d\vec{A} = \cancel{q} - \cancel{q} + \frac{q}{K_e}$$

$$\text{or} \quad K_e \epsilon_0 \oint \vec{E} \cdot d\vec{A} = q$$

$$\text{or} \quad \boxed{\epsilon_0 \oint K_e \vec{E} \cdot d\vec{A} = q} \quad \text{——} (6)$$

which is the expression for Gauss' law when dielectrics are present. Applicable for capacitors in general.

Two points to note =

1- The flux integral now deals with $K_e\vec{E}$ instead $\vec{E}$.

2- The charge q contained within the Gaussian surface is taken to be the free charge only.

Current density (j)

"The electric current per unit area of conductor cross-section".

The current i of a conductor is a macroscopic quantity. And related microscopic quantity is current density j.

It is a vector and is characteristic of a point inside a conductor rather than of the conductor as a whole.

We have $\quad j = i/A$

$$\text{or } i = jA \quad \text{——— (1)}$$

we know that

$$\text{total charge} = \text{total Vol.} \times \text{No. of electrons}$$

$$\text{or} \quad q = ALne$$

Now **Drift velocity** (v_D) is "the velocity gained by free electrons in an electrical conductor upon the application of electric field, it is of the order of 10^{-3} m/s."

The charge passed in the segment of wire during time t, $\quad t = \frac{L}{v_D} \quad [s = vt]$

The current is the charge passed per unit time t,

$$i = \frac{q}{t} = \frac{ALne}{L/v_D} = nAev_D$$

$$\text{or} \quad v_D = \frac{i}{nAe} \quad \text{——— (2)}$$

from eqs (1) & (2)

$$v_D = \frac{j}{ne} \quad \text{——— (3)}$$

$$\text{or} \quad j = nev_D$$

or in the vector form: $\boxed{\vec{j} = -ne\vec{v_D}}$ — (4)

where the $-ve$ sign shows that the electrons move in opposite direction to the vector $\vec{i}$

Ohm's Law - A Microscopic View

"The current (i) in a conductor is proportional to the potential difference (V) between its ends. This leads to $V = iR$, where R is the conductor's resistance".

Ohm's law is not a fundamental law of electromagnetism because it depends on the properties of the conducting medium.

In microscopic form, the Ohm's law can be written,

from the equation $\vec{E} = \rho \vec{j}$

$$\text{or } \rho = E/j \qquad \text{——— (1)}$$

We define:

Conduction electrons: In a metal the valence electrons which are not attached to individual atoms but are free to move about within the lattice.

Free-electron Model:

A theory of electrical conduction in metals, in which the conduction electrons are assumed to move freely throughout the conducting material, somewhat like molecules of gas in a container.

Electron gas:

The assembly of conduction electrons moving in a conductor.

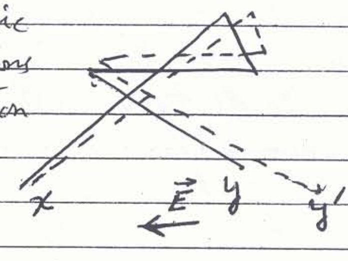
The classical Maxwellian velocity distribution for the electron gas would suggest that the conduction electrons have a broad distribution of velocities from zero to infinity, with well-defined average. This average speed depends strongly on temperature. The effective speed obtained from the quantum distribution is nearly independent of temperature.

We have $\rho = \frac{RA}{L}$ or $R = \rho L/A$ & $j = i/A$ or $i = jA$

& $E = \frac{dV}{ds} = \frac{V}{L}$ or $V = EL$ taking $\boxed{V = iR}$ putting ~~values~~ values $EL = iA \times \rho L/A$

or $\boxed{\rho = E/j}$

When we apply an electric field to a metal, the electrons modify their random motion in such a way that they drift slowly, in the opposite direction to that of the field, with an average drift speed v_d. The electrons drifts steadily, ending at y' rather than at y.



To calculate drift speed v_d and hence resistivity ρ

we have $F = ma$

$$\text{or } a = \frac{F}{m} = \frac{qE}{m} = \frac{eE}{m} \quad \text{—— (2)}$$

$$\& \quad v_d = a\tau = \frac{eE\tau}{m} \qquad \left[a = \frac{v}{t}\right]$$

$$v_d = \frac{j}{ne} = \frac{eE\tau}{m} \qquad \left[\begin{array}{l} v_d = j/ne \\ \text{eq (3) in last section} \end{array}\right.$$

$$\text{or } j = \frac{ne^2E\tau}{m} \quad \text{—— (3)}$$

From eqs (1) & (3)

$$\rho = \frac{E}{ne^2E\tau/m}$$

$$\text{or } \boxed{\rho = \frac{m}{ne^2\tau}}$$

where m, n & e are constants

The metals will obey Ohm's law if τ is constant. i.e. we must show that mean free time τ does not depend on the applied electric field E. In many metals under certain temperature range ρ is independent of E, and the materials obey Ohm's Law.

RC Circuits

Suppose we charge the capacitor by throwing switch S to position a. Applying Kirchhoff's Voltage rule to the fig.

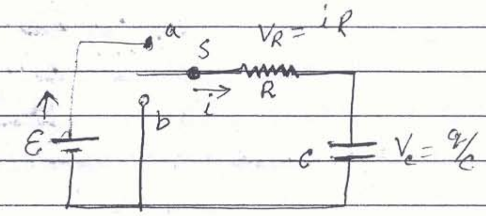


Fig-1

$$\mathcal{E} - iR - q/C = 0$$

or $\quad \mathcal{E} = iR + q/C$ ——— (1)

or $\quad \mathcal{E} = R\frac{dq}{dt} + q/C \qquad [i = dq/dt$

or $\quad \frac{dq}{q - \mathcal{E}C} = -\frac{dt}{RC}$ ——— (2)

The tentative solution of the above equation is

$$q = C\mathcal{E}(1 - e^{-t/RC}) \quad \text{——— (3)}$$

Differentiating eq (3) w.r.t. t,

$$i = \frac{dq}{dt} = \frac{\mathcal{E}}{R} e^{-t/RC} \quad \text{——— (4)}$$

We see that eq (2) will be satisfied after putting the values of q & dq/dt from eqs (3) & (4). So eq (3) is the solution of eq (2).

We can measure i & q by measuring V_R across resistor and V_c across capacitor by CRO. The resulting plots are shown in the fig. Please note

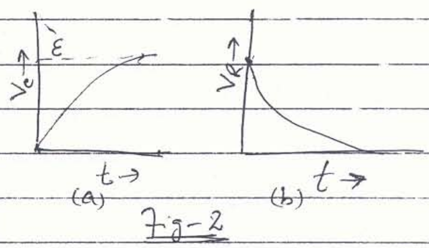


Fig-2

1- At $t = 0$, $V_R = \mathcal{E}$ & $V_c = 0$

2- As $t \to \infty$, $V_R = 0$ & $V_c \to \mathcal{E}$

3- At all times $V_R + V_c = \mathcal{E}$ as eq (1) requires

We define: Capacitive time constant (τ_c)

"The time at which the charge on the capacitor has increased to 63% of its final value ($C\mathcal{E}$)".

As putting $t = \tau_c = RC$ in eq (3) gives

$$q = C\mathcal{E}(1 - e^{-1}) = 0.63\, C\mathcal{E}$$

Fig 2(c) shows that if a resistance is included in a circuit with a charging capacitor, the increase of the charge of the capacitor toward its limiting value is delayed by a time characterized by the time constant RC.

When switch S is closed on 'a', the charge on the capacitor is initially zero, so the potential difference across the capacitor is initially zero. Because of this current, charge flows to the capacitor and the potential difference across the capacitor increases with time.

Any increase in the potential difference across the capacitor must be balanced by a corresponding decrease in the potential difference across the resistor, with a similar decrease in the current.

The entire potential difference of the emf now appears across the capacitor, which is fully charged ($q = C\mathcal{E}$). Unless changes are made in the circuit, there is no further flow of charge.

Discharging a Capacitor

Now the switch S is closed on 'b' in the fig 1. the capacitor discharges through the resistor. There is no emf in the circuit, putting $\varepsilon = 0$ in eq ①

$$iR + q/C = 0 \quad \text{——— ⑤}$$

$$\text{or} \quad R\,dq/dt + q/C = 0 \qquad [\,i = dq/dt$$

$$\text{or} \quad dq/q = -dt/RC \qquad [\,\int e^{-x}dx = -e^{-x}$$

The solution of the above equation after integration will be

$$q = q_0\, e^{-t/RC} \quad \text{——— ⑥}$$

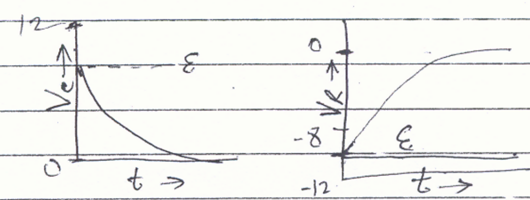
where q_0 is the initial charge of the capacitor.

Differentiating eq ⑥,

$$i = dq/dt = -\frac{q_0}{RC}\, e^{-t/RC}$$

$$\text{or} \quad i = -\frac{\varepsilon}{R}\, e^{-t/RC} \quad \text{——— ⑦} \qquad \left[\begin{array}{l} q_0 = C\varepsilon \\ \tau_c = RC \end{array}\right.$$

Fig-3

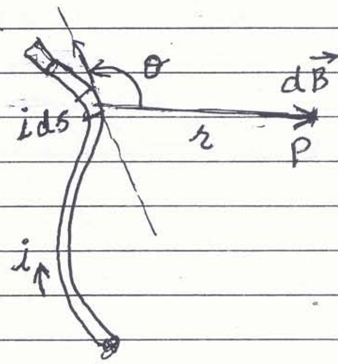


We see that $t = \tau_c = RC$, the capacitor charge is reduced to $q_0 e^{-1}$, which is about 37% of initial charge q_0. The negative sign in eq ⑦ shows that the current is now reversed.

Again we can measure voltages across R and C. We note that V_C falls exponentially from maximum. V_R is negative and rises exponentially to zero & $V_C + V_R = 0$.

The Biot-Savart Law

The elemental field strength dB at a point distant r from a current element $i\,ds$ in free space is given by:



$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2}$$

where θ is the angle between $d\vec{s}$ and $\vec{r}$.

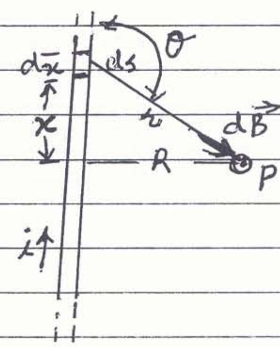
or in vector form:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s}\times\vec{r}}{r^3}$$

Applications

1- A long straight wire

To find $\vec{B}$ due to a current i in a long straight wire at point P.



The fig. shows a typical current element $i\,d\vec{s}$.

Applying Biot-Savart law

$$dB = \frac{\mu_0 i\,ds\sin\theta}{4\pi\,r^2}$$

For all current elements

$$B = \int dB = \frac{\mu_0 i}{4\pi}\int_{-\infty}^{+\infty}\frac{\sin\theta\,dx}{r^2}$$

$$\left\{\begin{aligned} r &= \sqrt{x^2+R^2} \\ \sin\theta &= \sin(\pi-\theta) \\ &= \frac{R}{\sqrt{x^2+R^2}} \end{aligned}\right.$$

or

$$B = \frac{\mu_0 i}{4\pi}\int_{-\infty}^{+\infty}\frac{R}{(x^2+R^2)^{3/2}}\,dx$$

from symmetry of the problem, writing $-\infty$ to $+\infty$ as twice the integral from 0 to ∞,

$$B = \frac{\mu_0 i}{4\pi} \times 2 \int_0^\infty \frac{R}{(x^2+R^2)^{3/2}}\, dx$$

$$= \frac{\mu_0 i}{4\pi} 2R \int_0^\infty \frac{1}{(x^2+R^2)^{3/2}}\, dx$$

$$= \frac{\mu_0 i\, 2R}{4\pi} \left[\frac{x}{R^2 (x^2+R^2)^{1/2}} \right]_0^\infty$$

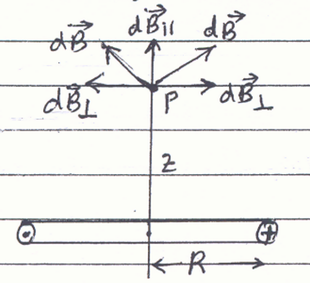
$$= \frac{\mu_0 i}{2\pi R} \left[\frac{x}{x\sqrt{1+R^2/x^2}} \right]_0^\infty$$

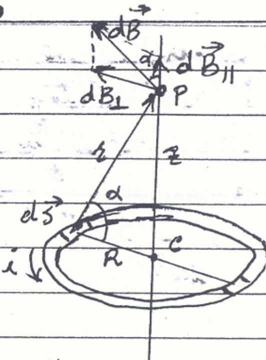
$$= \frac{\mu_0 i}{2\pi R} \left| \frac{1}{\sqrt{1+R^2/x^2}} \right]_0^\infty$$

$$= \frac{\mu_0 i}{2\pi R} \left[\lim_{x\to\infty} \frac{1}{\sqrt{1+R^2/x^2}} - 0 \right] = \frac{\mu_0 i}{2\pi R} (1)$$

or $\boxed{B = \frac{\mu_0 i}{2\pi R}}$

A Circular Current loop





To calculate $\vec{B}$ at a point P on the axis a distance Z from the centre of the loop.

Applying Biot-Savart law

$$dB = \frac{\mu_0 i}{4\pi} \frac{ds \sin\theta}{r^2}$$

[Angle θ is $\perp$ to the planes $i d\vec{s}$ & $\vec{r}$ So $\sin 90° = 1$

The components $d\vec{B}_{\perp}$ point in different directions perpendicular to the axis, and sum of all $d\vec{B}_{\perp}$ for complete loop is zero. Only $d\vec{B}_{\parallel}$ contributes to the total magnetic field $\vec{B}$ at point P.

$$dB_{\parallel} = dB\cos\alpha = \frac{\mu_0 i}{4\pi}\frac{ds\cos\alpha}{r^2}$$

$$B = \int dB_{\parallel} = \int \frac{\mu_0 i}{4\pi}\frac{R}{(R^2+z^2)^{3/2}}\, ds$$

[$r = \sqrt{R^2+z^2}$

$\cos\alpha = R/r = \frac{R}{\sqrt{R^2+z^2}}$

$$= \frac{\mu_0 i R}{4\pi (R^2+z^2)^{3/2}} \int ds$$

[$\int ds = 2\pi r$

$$= \frac{\mu_0 i R}{4\pi (R^2+z^2)^{3/2}} \times 2\pi R$$

or $\boxed{B = \frac{\mu_0 i R^2}{2(R^2+z^2)^{3/2}}}$ $\Rightarrow$ magnitude of magnetic field on the axis of circular current loop

& at the plane of the loop ($Z = 0$)

$$\boxed{B = \frac{\mu_0 i}{2R}}$$

For $Z >> R$, $\boxed{B = \frac{\mu_0 i R^2}{2 Z^3}}$

For a tightly wound N loops

$$B = \frac{\mu_0 i R^2 N}{2 z^3}$$

[$A = \pi R^2$ or $A/\pi = R^2$

$$B = \frac{\mu_0 N i A}{} = \frac{\mu_0 \mu}{2\pi z^3}$$ where $\mu = NiA$

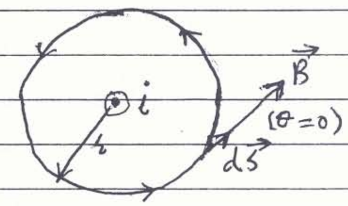
Ampere's Law

"The sum or integral of the magnetic flux density B times the path length along a closed path around a current-carrying conductor is proportional to the current i." Mathematically,

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i$$

Applications

1. Find the magnetic field at a distance r from a long straight wire.



In the fig. choose the Amperian path a circle of radius r.
From the symmetry of the problem, B can depend only on r.
Applying Ampere's law

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i \quad \text{——} \; (1)$$

or $\oint \vec{B}\cdot d\vec{s} = \oint B\,ds\cos\theta$ $\quad [\vec{A}\cdot\vec{B} = AB\cos\theta$

or $\oint B\,ds = B\oint ds$ $\quad [\theta = 0,\ \cos 0° = 1$

or $\oint \vec{B}\cdot d\vec{s} = B\,2\pi r$ —— (2) $\quad$ [B is constant by symmetry

From eqs (1) & (2)

$$B\,2\pi r = \mu_0 i$$

or $$\boxed{B = \frac{\mu_0 i}{2\pi r}}$$

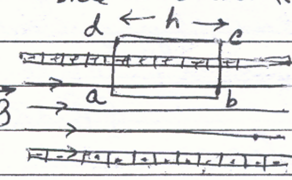
It is the same result, as we obtained using Biot-Savart law.

Solenoids

"A long helically wound coil of insulated wire"

To find the magnetic field $\vec{B}$ inside a Solenoid.

In the fig. for an ideal solenoid, applying Ampere's law to the rectangle abcd



$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i \quad \text{—— (1)}$$

writing integral $\oint \vec{B}\cdot d\vec{s}$ as the sum of four integrals

$$\oint \vec{B}\cdot d\vec{s} = \int_a^b \vec{B}\cdot d\vec{s} + \int_b^c \vec{B}\cdot d\vec{s} + \int_c^d \vec{B}\cdot d\vec{s} + \int_d^a \vec{B}\cdot d\vec{s} \quad \text{—— (2)}$$

As $\int_b^c \vec{B}\cdot d\vec{s} = \int_d^a \vec{B}\cdot d\vec{s} = 0$ [$\vec{B}$ is $\perp$ to $d\vec{s}$

& $\int_c^d \vec{B}\cdot d\vec{s} = 0$ [$\vec{B}$ is taken as zero for external points

$\int_a^b \vec{B}\cdot d\vec{s} = \oint Bh = Bh$ [$ds = h$ = length of the path

so from eq (2)

$$\oint \vec{B}\cdot d\vec{s} = Bh + 0 + 0 + 0$$

$$\text{or} \quad \oint \vec{B}\cdot d\vec{s} = Bh \quad \text{—— (3)}$$

Now net current i in the Amperian loop abcd is

$$i = i_0 n h \quad \text{—— (4)}$$

[i_0 = current in solenoid
n = number of turns/unit length

from eqs (1), (3) & (4) we get

$$Bh = \mu_0 i_0 n h$$

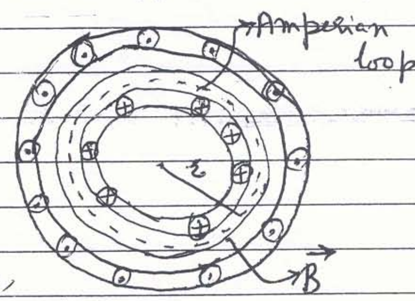
$$\text{or} \quad \boxed{B = \mu_0 i_0 n} \quad \text{—— (5)}$$

which shows the magnetic field inside a Solenoid does not depend upon diameter or length of Solenoid but it depends only on current i_0 and number of turns per unit length.

Toroids

"A solenoid wound on a circular support instead of a straight one."

To find the magnetic field at the interior points in a toroid.



Applying Amperes law

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i \quad \text{——} ①$$

From symmetry of the problem, the lines of $\vec{B}$ form concentric circles.

Lets choose a concentric circle of radius r as an Amperian loop and traverse in clockwise direction

$$\oint \vec{B} \cdot d\vec{s} = \oint B\, ds \cos 0°$$

$$\text{or} \quad \oint \vec{B} \cdot d\vec{s} = \oint B\, ds = B \oint ds = B(2\pi r) \quad \text{——} ②$$

& current i in the circle is

$$i = i_0 N \quad \text{——} ③ \qquad \left\{ \begin{array}{l} i_0 = \text{current in toroid (will same for the circle)} \\ N = \text{total number of turns} \end{array} \right.$$

from eqs ①, ② & ③ we get

$$B\, 2\pi r = \mu_0 i_0 N$$

$$\text{or} \quad \boxed{B = \frac{\mu_0 i_0 N}{2\pi r}} \quad \text{——} ④$$

In contrast to the solenoid, B is not constant over the cross-section of a toroid.

Toroids form the central feature of the tokamak, a device for fusion power reactor.

1 - Show that B is zero for points outside the toroid.

2 - Prove the statement that "a toroid is a solenoid bent into the shape of a doughnut".

Faraday's Law of Induction

"The induced emf in a circuit is equal to the negative of the rate at which the magnetic flux through the circuit is changing with time." Mathematically

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \quad \text{——— ①}$$

For a closed loop we can write as a line integral of $\vec{E}$ around the circle.

$$\mathcal{E} = \oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt} \quad \text{——— ②}$$

From ① & ②

$$\boxed{\oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}}$$

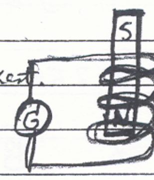
$\left[V = \frac{W}{q} = \frac{F \times d}{q} = E \times d\right.$

or $V = \mathcal{E} = \vec{E}\cdot\vec{d}$ $\left[E = \frac{F}{q}\right.$

which is one of the four basic Maxwell equations of electromagnetism.

The negative sign in the equation tells us the direction of the induced emf, explained further in Lenz's law.

We see that the relative motion between the loop and the magnet causes the current.



There are many ways of changing the magnetic flux through a loop.

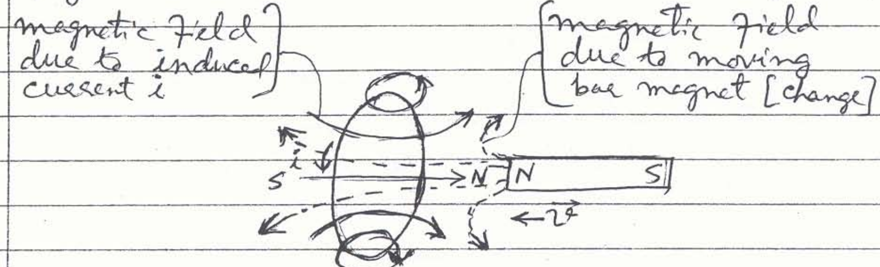
1- By moving a magnet relative to the loop [Fig-1]
2- By changing the current in a nearby circuit [transformers]
3- By moving the loop in a non-uniform field.
4- By rotating the loop in a fixed magnetic field [generator]
5- By changing the size or shape of the loop.
6- By starting, stopping or changing the current in a neighboring circuit [Mutual Induction]
7- By starting, stopping or changing the current in the circuit itself. [Self-induction]

Lenz's Law

"The induced current in a closed conducting loop appears in such a direction that it opposes the change that produced it."

Right Hand Rule

"Grasp the wire in the right hand, with the thumb in the direction of the current, and the fingers curl in the direction of the magnetic field".



The direction of magnetic field due to induced current is determined by applying right hand rule. This field direction <u>opposes</u> the direction of magnetic field due to pushing of bar magnet, which is the <u>change</u>.

When the north pole of a magnet is brought near a coil the right face of the coil should acquire a north polarity due to induced current, because it will be able to oppose the movement of the magnet. Similarly when the north pole of the magnet is taken away, the right face should acquire a south polarity so that it again opposes the movement of the magnet by attracting the north pole. As <u>Lenz's law is in accordance with the law of conservation of energy.</u>

If the current were in opposite direction to that shown, we would only need to push the magnet slightly to start the process and then the action would be self-perpetuating.

Practical applications

1- The oscillations of a moving coil galvanometer are damped by short-circuiting its terminals. As the coil moves in a strong magnetic field, a current is induced in the coil. By Lenz's law, the direction of this induced current is such that it tends to oppose the motion of the coil, so the oscillations are quickly damped.

2- In induction motor, a magnetic field rotates about an axis. The field cuts the conducting bars of a rotor. The current induced in the rotor is in such a direction that it brings about forces under whose influence the rotor starts rotating in the direction of the rotating field. The rotation of the rotor decreases the relative motion between the rotor and the rotating magnetic field, as it is this relative motion which is the change of the induced current in the rotor.

Lenz's Law & Faraday's Law

Lenz's law is a verbalization of the sign convention and the negative sign of Faraday's law. If only the direction of the induced emf is desired, often it is easier to apply Lenz's law than Faraday's Law. However, Lenz's law does not predict the magnitude of the induced emf. If both magnitude and direction are desired, Faraday's law will yield both at once.

Eddy Currents

"Closed loops of induced current circulating in plates perpendicular to the magnetic flux."

These eddy currents in a solid mass of metal are a source of energy dissipation in alternating current machinery.

When a magnet is suddenly moved towards a metal sheet, currents in it are set up as shown in the Fig-1

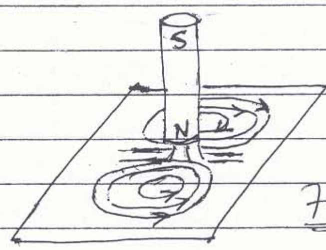
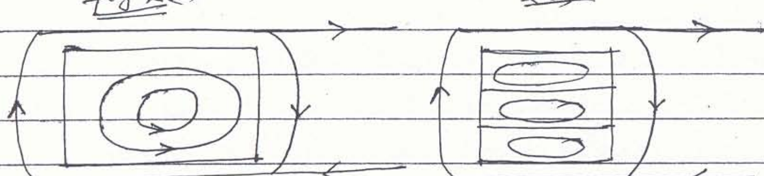


Fig-1

Fig 2~~(a)~~ ~~(b)~~



(a) Large eddy currents in one big mass of metal

(b) small eddy currents in small parts insulated from each other.

The eddy currents set up in metallic masses heat up the latter, so there is a waste of energy. To reduce these currents, the metal is made in parts laminated and so insulated from one another. When the eddy currents tend to flow in circular paths, the insulation between the parts blocks their flow.

Practical applications:

1- Induction furnace

In it a sample of material can be heated

using a rapidly changing magnetic field.
In such furnaces melting takes place rapidly and are used in cases in which it is not possible to make thermal contact with the material to be heated, such as when it is enclosed in a vacuum chamber, so that no oxidation is brought about.

2- Induction motor: When a metallic cylinder is placed in a rotating magnetic field, and is capable of rotation about its axis, eddy currents are set up in it. Then the cylinder begins to rotate in the direction of rotation of the field.

3- Magnetic braking: If in the previous case a stationary magnetic field is suddenly applied to a rotating cylinder or disc the eddy currents set up in them exert a couple which tends to stop the motion of the disc. It is used in stopping an electric train.

4- Inductothermy: Heating effect of eddy currents is used for localized heating of body tissues. A thick coil of a few turns of wire is wound over the affected part but separated by a small distance apart from the body, and an alternating current of high frequency is passed through this coil.

5- Damping: In moving coil galvanometers, the eddy currents set up in the metallic frame on which the coil is wound and in the soft iron core when the coil oscillates tend to stop the oscillations and make it dead beat.

6- Laminated Cores: Cores of dynamos, motors and transformers are laminated, to reduce the heating effect due to eddy currents. Thin sheets insulated from one another by a varnish coating. These are placed at right angles to the direction of the current.

Gauss' Law for Magnetism

"The net flux of the magnetic field through any closed surface is zero". Mathematically

$$\Phi_B = \oint \vec{B}\cdot d\vec{A} = 0$$

We need Gauss' law for magnetism. As Ampere's law is not a complete description of magnetostatics.

We see that the magnetic effects of materials can be described in terms of equivalent currents. As they behave like real currents as far as magnetic effects are concerned, they introduce no change in Gauss' law for magnetism from its form with only real currents.

We have Gauss' law for electric field

$$\oint \vec{E}\cdot d\vec{A} = q/\epsilon_0 \qquad \left[\Phi_E = \oint \vec{E}\cdot d\vec{A}\right.$$

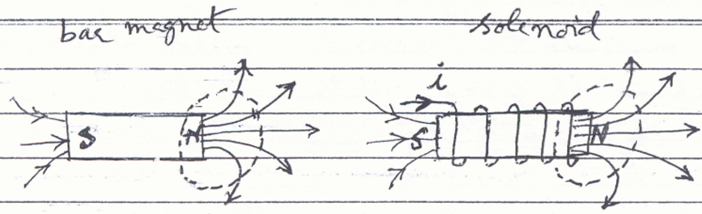
Considering magnetic flux Φ_B, instead electric flux Φ_E.

And if the Gaussian surface contains no net 'magnetic charge' then the flux Φ_B will be zero. So magnetic form of Gauss' law will be:

$$\Phi_B = \oint \vec{B}\cdot d\vec{A} = 0$$

This equation is one of the four Maxwell's equations of electromagnetism.

Looking lines of $\vec{B}$ for a bar magnet and a short solenoid. Both of which can be considered as magnetic dipoles.



Constructing Gaussian surfaces in both figures for the magnetic field. Note that the total inward flux equals the total outward flux, and the net flux Φ_B for the closed surface is zero.

Many searches have been made, no individual magnetic charge has ever been found, from which the lines of $\vec{B}$ originate or to which they converge. There is no isolated magnetic charge or monopole.

Few Definitions

Dipole:
A system of two equal and opposite charges placed at a very short distance apart.

Dipole Moment:
The product of either of the charges and the distance between them; $p = q\,d$

Magnetic dipole: A small loop carrying a current I behaves as a magnetic dipole.

Magnetic dipole moment:
$\mu = IA$, where A is the area of the loop.

Para Magnetism: The property of a substance by which it is feebly attracted by a strong magnet.

Dia Magnetism: The property of a substance whereby it is feebly repelled by a strong magnet.

Ferro magnetism: The property of a substance by which it is strongly attracted by a magnet.

Magnetic Properties of Solids

Magnetism:

"A property that all materials posses as a result of the motion of their electrons".

Origin:

Magnetism is due to the spin and orbital motion of the electrons surrounding the nucleus and is a property of all substances. Each electron orbiting the nucleus behaves like an atomic sized loop of current that generates a small magnetic field. Also each electron possesses a spin that also gives rise to a magnetic field. An atom in which there is a resultant magnetic field, behaves like a tiny magnet and is called a magnetic dipole.

Types:

1. **Paramagnetic substances**
   The substances in which, the orbits and the spin axes of the electrons in the atom are so oriented that their fields support each other and the atoms behaves like a tiny magnet.
   e.g. transition metals (such as scandium, vanadium, etc.) and rare earth elements (such as lanthanium, neodymium, etc.)

2. **Diamagnetic substances**
   The substances in whose atoms, there is no resultant field as the magnetic fields produced by both orbital and spin motions of the electrons might add up to zero.
   e.g. atoms of water, copper, bismuth and antimony.

3. **Ferromagnetic substances**
   The substances in which, the atoms cooperate with each other in such a way so as to exhibit a strong magnetic effect.
   e.g. iron, cobalt, nickel, chromium dioxide, and Alnico.
   [Alnico: Alloy for making magnets; an alloy of iron, aluminum, and nickel together with one or more of cobalt, copper, and titanium, used for making strong permanent magnets.]

Domains:

"A region inside a ferromagnetic material in which all the atomic magnetic fields point the same way".

Domains exist in ferromagnetic substances. Their size are of the order of millimeter or less but large enough to contain 10^{12} to 10^{16} atoms. Each domain is magnetized to saturation. They behave as a small magnet with its own north and south poles. In unmagnetized iron, the domains are oriented in a disorderly fashion. When the specimen is placed in an external magnetic field, the entire specimen becomes saturated. The combination of a solenoid and a specimen of iron I side it thus makes a powerful magnet and is called an electromagnet.

Soft magnetic material:

Such material in which domains are easily oriented on applying an external field and also readily return to random positions when the field is removed. For example iron. This is desirable in an electromagnet and also in transformers.

Hard magnetic material:

Such material in which domains are not so easily oriented to order. They require very strong external fields. but once oriented, retain the alignment. For example, steel and Alnico V, makes a good permanent magnet.

Effect of temperature:

Curie temperature

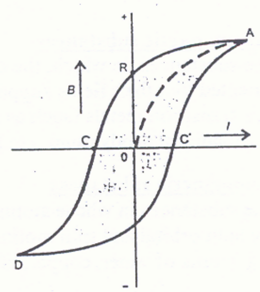
"The temperature above which a ferromagnetic substance loses its ferromagnetism".

Thermal vibrations tend to disturb the orderliness of the domains. Ferromagnetic materials preserve the orderliness at ordinary temperatures. When heated, they begin to loose their orderliness due to increased thermal motion. This process begins at Curie temperature. For iron its value is 750 °C. Above this temperature iron is paramagnet but not ferromagnetic.

Hysteresis & Hysteresis Loop:

Hysteresis

The lagging of magnetization of ferromagnetic material behind the magnetizing force.



(a) Hysteresis loop of steel

Hysteresis loop

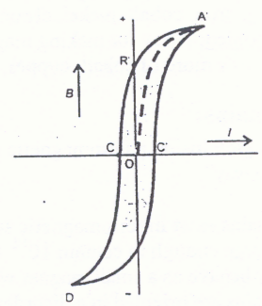
The loop formed by magnetic hysteresis.

Saturation

The magnetic flux density increases from zero and reaches a maximum value. At this stage the material is said to be magnetically saturated.

Remanence (or Retentivity)

i) The residual magnetic flux density in a substance when the magnetizing field strength is returned to zero.
ii) When substances are applied forces for producing magnetization and then force removed then power of retaining their original magnetization is called retentivity.



(b) Hysteresis loop of soft iron

OR = Retentivity
OC = Coercivity

Coercivity

Degree of reversed magnetizing force required to deprive the metal of the whole of its original magnetization.

Hysteresis loss

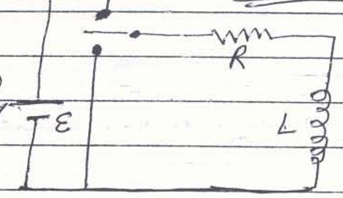
The dissipation of energy that occurs, due to magnetic hysteresis, when the magnetic material is subjected to cyclic changes of magnetization.

Area of the loop

The area of the loop is a measure of the energy needed to magnetize and demagnetize the specimen during each cycle of the magnetized current. This is the energy required to do work against internal friction of the domains. Hard magnetic materials (such as steel) have large loop area and soft magnetic materials (such as iron) have small loop area.

LR Circuits

Fig-(a)

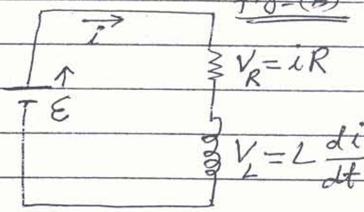


Switching on the battery as shown in Fig(b)
Analyzing the circuit quantitatively
Applying Kirchhoff's Voltage rule,

$$\mathcal{E} = iR + L\frac{di}{dt}$$

or $L\dfrac{di}{dt} + iR = \mathcal{E}$ ——— ①

Fig-(b)



taking its tentative soln.

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right) \text{ ——— ②}$$

differentiating the above eqn.

$$\frac{di}{dt} = \frac{\mathcal{E}}{R}\frac{1}{\tau_L}e^{-t/\tau_L} \text{ ——— ③}$$

eq① will be satisfied if

$\boxed{\tau_L = L/R}$ — called inductive time constant

so complete solution is

$$i(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-tR/L}\right) \text{ ——— ④}$$

Defining time constant τ_L:
Putting $t = \tau_L$ in eq②

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-1}\right) = \frac{\mathcal{E}}{R}(1 - 0.37) = 0.632\frac{\mathcal{E}}{R}$$

or $i = 0.632\, i_0$

Inductive time constant (τ_L) is defined as 'the time in which the current rises from zero to 0.632 or 63% of its final value'.

Properties of V_R & V_L:

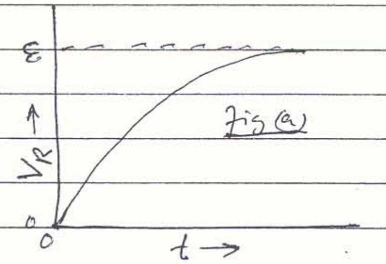
from eq② we have

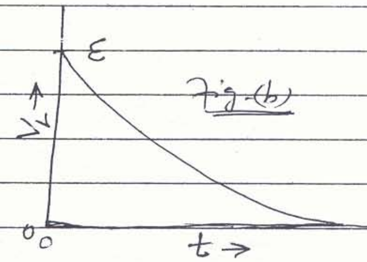
$$V_R = iR = \mathcal{E}(1 - e^{-t/\tau_L})$$

This gives

$$t=0, V_R = 0 \ \& \ t \to \infty, V_R = \mathcal{E}$$

as shown in fig(a)





from eq③ we have

$$\frac{di}{dt} = \frac{\mathcal{E}}{R}\frac{1}{L/R} e^{-t/\tau_L}$$

or $V_L = L\frac{di}{dt} = \mathcal{E} e^{-t/\tau_L}$

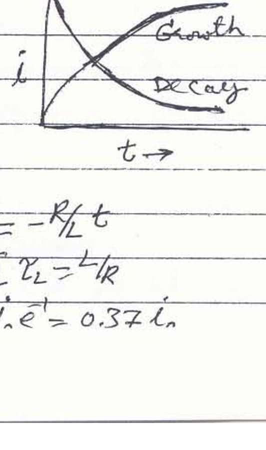
This gives: $t=0, V_L = \mathcal{E}$ & $t \to \infty, V_L = 0$, shown in fig(b).

Growth and Decay of Current

Taking eq④ $i = i_0(1 - e^{-tR/L})$ $\quad [\frac{\mathcal{E}}{R} = i_0$

This gives:

$t=0, i=0$ & $t = L/R$, $i = i_0(1-e^{-1}) = 0.63 i_0$ shown in fig.



Now short circuiting the circuit by putting $\mathcal{E}=0$ in eq①, we have

$$L\frac{di}{dt} + iR = 0$$

or $di/i = -R/L \, dt$

Integration gives: $\log i = -\frac{R}{L}t + A$

when $t=0$, $i = i_0$ & $A = \log i_0$

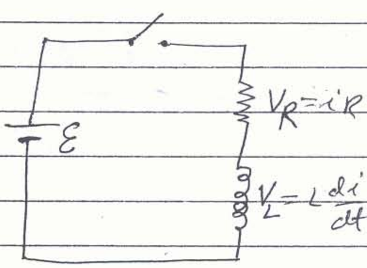
so $\log i - \log i_0 = -\frac{R}{L}t$ or $\log i/i_0 = -\frac{R}{L}t$

or $i/i_0 = e^{-\frac{R}{L}t}$ or $i = i_0 e^{-t/\tau_L}$ $\quad [\tau_L = L/R$

This gives: $t=0, i=i_0$ & $t = L/R$, $i = i_0 e^{-1} = 0.37 i_0$

Energy Storage in a Magnetic field

Consider the energy stored in the magnetic field of an inductor.



Applying Kirchhoff's voltage rule to the circuit

$$\mathcal{E} = iR + L\frac{di}{dt} \quad \text{——— (1)}$$

Multiplying both sides by i

$$\mathcal{E}i = i^2R + Li\frac{di}{dt} \quad \text{——— (2)}$$

The above equation has the following interpretation in terms of work and energy.

1- The left side of eq (2) is <u>the rate at which the seat of emf delivers energy to the circuit.</u> $\left[V = \frac{W}{q} \text{ or } W = dq\, V \text{ or } \frac{W}{dt} = V\frac{dq}{dt} \right.$

2- The second term <u>i^2R is the rate at which energy is dissipated in the resistor.</u> $\left[\text{Power} = P = \frac{W}{t} = \frac{qV}{t} = VI = I^2R \right.$

3- Energy delivered to the circuit but not dissipated in the resistor must be stored in the magnetic field of the inductor.

So the last term $Li\,di/dt$ is the <u>rate at which energy is stored in the magnetic field.</u> $\left[\mathcal{E}_L = L\frac{di}{dt} \text{ or } \mathcal{E}_L i = Li\frac{di}{dt} \right.$

Now let U_B is the energy stored in the magnetic field. Then the rate at which the energy is stored is $\frac{d}{dt}U_B$.

from eq ②

$$\frac{d}{dt}U_B = L\,i\,\frac{di}{dt}$$

or $$dU_B = L\,i\,di$$

Integration gives

$$\boxed{U_B = \frac{1}{2}L\,i^2}$$

[Compare: $U_E = \frac{1}{2}\frac{q^2}{C}$

It represents total stored magnetic energy in an inductance L carrying a current i.

If we open a switch in an inductive circuit quickly enough, we can cause a very large potential difference across the largest resistance which is the gap between the switch or even a spark discharge. A variation of this method is used in a car to produce potential difference of many thousands of volts across the gaps of the <u>spark plugs</u> from a battery with an emf of only a few volts.

Alternating Current Circuits

Alternating current in a circuit is given by

$$i = i_m \sin(\omega t - \phi) \quad \text{——— (1)}$$

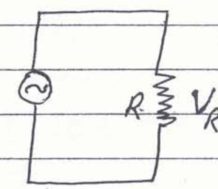
This a.c. current is driven by a source of emf that varies with time,

$$\mathcal{E} = \mathcal{E}_m \sin \omega t \quad \text{——— (2)}$$

Resistive element

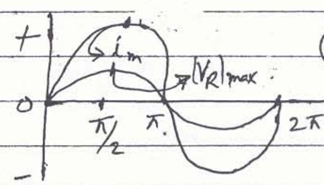
We have

$$V_R = iR = i_m R \sin(\omega t - \phi) \quad \text{——— (3)}$$



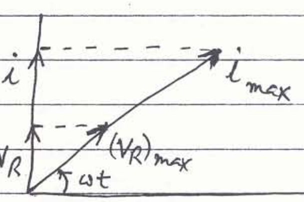
(a)

We see in fig(b) that in an a.c. circuit the time varying quantities i and V_R are <u>in phase.</u>



(b)

In the phasor diagram Fig(c), the projection of a phasor on the vertical axis gives the instantaneous value of the alternating quantity involved.



(c)

<u>Definition</u>

Phasor:

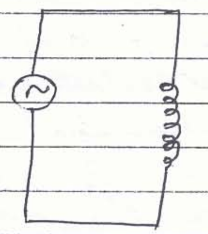
A rotating vector used to represent a sinusoidally varying quantity.

The projection of the vector on a fixed axis represents the amplitude variation with time. A phase angle between quantities (e.g. i & V) is represented by the angle between their phasors.

Inductive element

We have

$$V_L = L\frac{di}{dt} \quad \text{——— (4)}$$



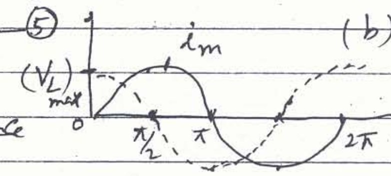
(a)

from eqs (1) & (4)

$$V_L = L\, i_m \omega \cos(\omega t - \phi)$$

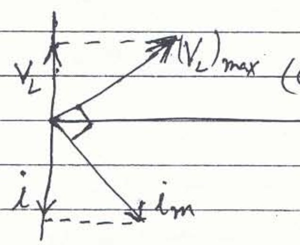
$$\left[\cos\theta = \sin(\theta + \pi/2)\right]$$

or $V_L = L\, i_m \omega \sin(\omega t - \phi + \pi/2)$ ——— (5)



(b)

eqs (1) & (5) show that <u>current lags</u> <u>potential difference</u> by $\pi/2$ (90°), as shown in fig (b) and in phasor diagram fig (c).



(c)

Reactance:

An inductance offers no resistance, but when the current is varying, a back emf is set up in the inductance. This reduces the current and is equivalent to a resistance, called reactance.

We define reactance (X_L) as, <u>the resistance due to inductance in an a.c. circuit.</u>

$$X_L = \omega L = 2\pi f L \quad \text{——— (6)}$$

This reactance is not a constant but is directly proportional to the frequency of the applied emf. From eqs (5) & (6)

$$V_L = i_m X_L \sin(\omega t - \phi + \pi/2) \quad \text{——— (7)}$$

so the maximum value of V_L is

$$(V_L)_{max} = i_m X_L \quad \text{——— (8)}$$

Choke

Sometimes we need to reduce the current in a given circuit for a constant voltage. In d.c. circuits it is done by a rheostat and loss of power is equal to i^2R.

In a.c. circuit, a choke coil is used for reducing the current. It consists of an inductance which is made by having a large number of turns of an insulated copper wire wound over a closed soft iron laminated core.

Since ohmic resistance of the inductance is small the loss of power across the choke coil is negligible, the only loss bei[ng] due to hysteresis.

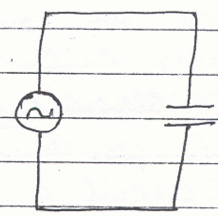
The advantage of using a choke is that, the waste of energy is only due to hysteresis loss which is much less than waste of energy i^2R of an ohmic resistance.

Choke coils are used in flourescent tubes, arc lamps, radio sets and in theatre lights where they may be steadily dimmed by the introduction of an iron cored choke.

Capacitive element

We have

$$V_c = \frac{q}{C} \quad \text{——} (9) \qquad \left[\begin{array}{l} q = cV \\ \text{or } V = q/c \end{array}\right.$$



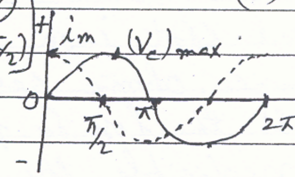
(a)

$$\text{or } V_c = \frac{\int i\,dt}{C} \qquad \left[\begin{array}{l} i = dq/dt \\ \text{or } q = \int dq = \int i\,dt \end{array}\right.$$

$$\& \; i = i_m \sin(\omega t - \phi) \text{ ——— } (1)$$

$$\text{so } V_c = \frac{-i_m \cos(\omega t - \phi)}{\omega C} \qquad \left[\begin{array}{l} \cos\theta = -\sin(\theta - \pi/2) \\ \text{or } -\cos\theta = \sin(\theta - \pi/2) \end{array}\right]$$

(b)

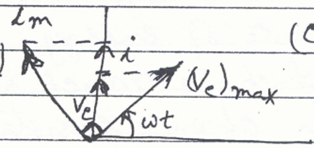


$$\text{or } V_c = \frac{i_m}{\omega C} \sin(\omega t - \phi - \pi/2) \text{ ——— } (10)$$

eqs (1) & (10) show that current leads potential difference by $\pi/2$ (90°) as shown in figs (b) & (c).

We define capacitive reactance (X_c) as

(c)



$$X_c = 1/\omega C = \frac{1}{2\pi f C} \text{ ——— } (11)$$

it is given in ohms if C is in farads.

This capacitive reactance depends on the frequency of alternating current, as the frequency is high the reactance will be small.

An a.c. may be controlled by using a capacitor. It is usually used to separate a.c. from d.c.

From eqs (10) & (11) we have

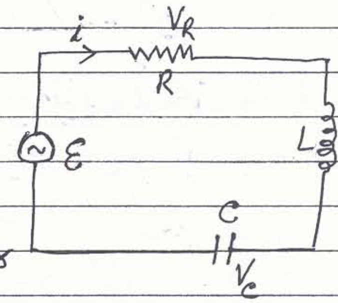
$$V_c = i_m X_c \sin(\omega t - \phi - \pi/2) \text{ ——— } (12)$$

For maximum value

$$(V_c)_{max} = i_m X_c \text{ ——— } (13)$$

RLC Series Circuit

Applying Kirchhoff's voltage rule to the clockwise path around the circuit, we have



$$\sum \text{emf's} = \sum \text{potential drops}$$

or $\mathcal{E} = V_R + V_L + V_C$ —— (14)

eq (14) can be solved by three methods

1- Trigonometric analysis
2- Graphical analysis
3- Differential analysis

Trigonometric Analysis

taking eq (14)

$$\mathcal{E} = V_R + V_L + V_C$$

putting values from eqs (2) (3), (7) & (12), we get

$$\mathcal{E}_m \sin\omega t = i_m R \sin(\omega t - \phi) + i_m X_L \sin(\omega t - \phi + \pi/2) + i_m X_C \sin(\omega t - \phi - \pi/2)$$

or $\mathcal{E}_m \sin\omega t = i_m R \sin(\omega t - \phi) + i_m X_L \cos(\omega t - \phi) - i_m X_C \cos(\omega t - \phi)$ $\left[\begin{array}{l}\sin(\theta + \pi/2) = \cos\theta \\ \sin(\theta - \pi/2) = -\cos\theta\end{array}\right.$

$$= i_m \left[R \sin(\omega t - \phi) + (X_L - X_C)\cos(\omega t - \phi)\right] \text{ —— (15)}$$

put $R = a\cos\phi$ & $X_L - X_C = a\sin\phi$ —— (16)

so $R^2 + (X_L - X_C)^2 = a^2(\cos^2\phi + \sin^2\phi) = a^2$

or $a = \sqrt{R^2 + (X_L - X_C)^2}$ —— (17)

& $\dfrac{\cancel{a}\sin\phi}{\cancel{a}\cos\phi} = \dfrac{X_L - X_C}{R}$ or $\tan\phi = \dfrac{X_L - X_C}{R}$ —— (18)

from eqs (15) & (16) we have

$$\mathcal{E}_m \sin\omega t = i_m \left[a\cos\phi (\sin \omega t - \phi) + a \sin\phi \cos(\omega t - \phi) \right]$$

$$= i_m \left[a \sin(\omega t - \cancel{\phi} + \cancel{\phi}) \right]$$

$$(18) \quad = i_m \left[a \sin\omega t \right] \text{ ——— } (19)$$

$\left\{ \begin{array}{l} \sin(\alpha+\beta) \\ = \sin\alpha\cos\beta \\ \quad + \cos\alpha \sin\beta \end{array} \right.$

from eqs (17)' & (19) we have

$$\mathcal{E}_m \sin\omega t = i_m \sqrt{R^2 + (X_L - X_c)^2} \, \sin\omega t \text{ ——— } (20)$$

$$\text{if } \tan\phi = \frac{X_L - X_c}{R} = \frac{\omega L - 1/\omega c}{R}$$

Now from eq (20)

$$i_m = \frac{\mathcal{E}_m}{R^2 + (X_L - X_c)^2} = \frac{\mathcal{E}_m}{R^2 + (\omega L - 1/\omega c)^2} \text{ ——— } (21)$$

Comparing eq (21) with Ohm's law equation for d.c. circuit, we find that this is the generalized Ohm's law for a.c. circuit, in which $\sqrt{R^2 + (X_L - X_c)^2}$ is total resistance to the flow of a.c. This is called the <u>impedance</u> denoted by Z.

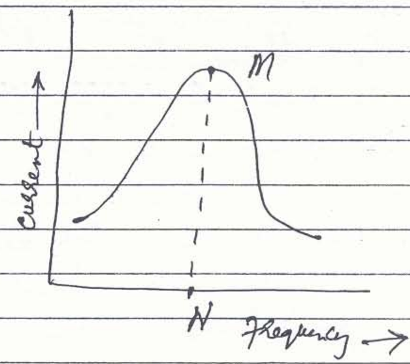
So we define

$$Z = \sqrt{R^2 + (X_L - X_c)^2} \text{ ——— } (22)$$

So $$i_m = \frac{\mathcal{E}_m}{Z} \text{ ——— } (23)$$

Condition for resonance

The variation of current with frequency in RLC circuit is shown in the figure.



When frequency of the applied emf equals natural frequency of the circuit, the current reaches a maximum value given by NM, and this gives the condition for resonance.

In eq (21) for maximum current

$$X_L - X_c = 0$$

or $X_L = X_c$

or $\omega L = 1/\omega C$

or $\boxed{\omega = \frac{1}{\sqrt{LC}}}$

Graphical Analysis

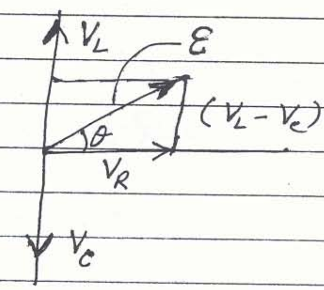
In RLC circuit the applied emf, $\mathcal{E}$ has to do three things.

1- To overcome $i_m R$ drop in voltage in phase with the current.

2- To overcome counter voltage V_L due to inductance L, which leads i by $\pi/2$.

3- To overcome back emf due to capacitor equal to $i_m X_c$ which lags current by $\pi/2$.

The three components of the applied maximum emf are shown in the figure.



From the figure

$$\mathcal{E}_m = \sqrt{V_R^2 + (V_L - V_c)^2}$$

$$= \sqrt{(i_m R)^2 + (i_m X_L - i_m X_c)^2}$$

or $\mathcal{E}_m = i_m \sqrt{R^2 + (X_L - X_c)^2}$

& $\tan\phi = \dfrac{V_L - V_c}{V_R} = \dfrac{i_m X_L - i_m X_c}{i_m R} = \dfrac{X_L - X_c}{R}$

which is identical with eq 21.

Appendix A

Electromagnetic Spectrum

Rays	Frequency	Wave-length	Photon Energy
Cosmic rays	10^{23} Hz	10^{-11} cm	below 10^{8} to10^{20} eV
γ- rays	6×10^{20} to 10^{18} Hz	10^{-10} to 10^{-8} cm	10^{5} to 10^{7} eV
X- rays	6×10^{19} to 6×10^{15} Hz	10^{-9} to 10^{-5} cm	10 to 10^{5} eV
Ultra-violet	2×10^{16} to 8×10^{14} Hz	1.4×10^{-6} to 4×10^{-5} cm	1 to 10^{2} eV
Visible	8×10^{14} to 4×10^{14} Hz	4×10^{-5} to 8×10^{-5} cm	1 eV
Infra-red (heat radiation)	4×10^{14} to 3×10^{11} Hz	8×10^{-5} to 0.04 cm	10^{-3} to 1 eV
Microwaves	10^{13} to 10^{9} Hz	10^{-6} to 10^{-4} cm	10^{-2} to 10^{-3} eV
Electrical radio waves	10^{13} to 10^{3} Hz	0.01 cm to 100 km	10^{-10} to 10^{-1} eV
TV. Radar	7×10^{9} to 2×10^{6} Hz	4×10^{1} to 3.5×10^{-4} m	10^{-3} to 10^{-10} eV
Micropulsation	----------	1×10^{8} to 5×10^{10} m	10^{-14} to 10^{-17} eV

Spectrum of visible portion

Colour of light	Wavelength $\times 10^{-7}$ m	Frequency $\times 10^{14}$ Hz
Red	6.470 to 7.000	4.634 to 4.284
Orange	5.850 to 6.470	5.125 to 4.634
Yellow	5.750 to 5.850	5.215 to 5.125
Green	4.912 to 5.750	6.104 to 5.215
Blue	4.240 to 4.912	7.115 to 6.104
Violet	4.000 to 4.240	7.495 to 7.115

Radio and TV waves

	Range		Frequency	Wavelength
T V	UHF		2.11×10^{8} to 6.9×10^{9}	0.1 to 1 m
T V	VHF		5×10^{7} to 1.2×10^{8}	1 to 10 m
T V	HF (or VL)		2×10^{6} to 4×10^{7}	10 to 100 m
Radio	FM (frequency modulation)		8.8×10^{7} to 1.08×10^{8}	1 to 10 m
Radio	MW	M F	5.3×10^{5} to 1.605×10^{6}	200 to 550 m
Radio	SW 1	M F	2.3×10^{6} to 7.0×10^{6}	49 to 120 m
Radio	SW 2	M F	7.0×10^{6} to 22.0×10^{6}	13 to 41 m
	LF		3×10^{4} to 3×10^{5}	1000 to 10,000 m
	VLF		3×10^{3} to 3×10^{4}	10,000 to 100,000 m
	ELF (extremely)		------- to 3×10^{3}	100,000 to ------- m

Appendix B
Elementary Particles

Particle Name	Symbol	Mass (GeV)	Mass (electron masses)	Electric Charge	Mean life (Sec)
BARYONS					
Proton	p	0.93826	1,836	+1	Stable ($>2.5\times10^{38}$)
Neutron	n	0.93955	1,837	0	1.01×10^{3}
Lambda Hyperon	Λ^{o}	1.1156	2,180	0	2.5×10^{-10}
Sigma Hyperon	Σ^{+}	1.1974	2,300	-1	0.8×10^{-10}
Sigma Hyperon	Σ^{o}	1.1926	2,290	0	less than 10^{-14}
Sigma Hyperon	Σ^{-}	1.1974	2.300	-1	1.65×10^{-10}
Xi Hyperon	Ξ^{o}	1.315	2,590	0	3×10^{-10}
Xi Hyperon	Ξ^{-1}	1.321	2,600	-1	1.7×10^{-10}
MESONS					
Pi meson (Pion)	$\pi^{\pm}$	0.13958	273	+1	2.61×10^{-8}
Pi meson	π^{o}	0.13497	264	0	0.9×10^{-16}
Kappa meson (Kion)	$K^{\pm}$	0.4938	920-960	±1	1.23×10^{-8}
Kion short lived	K^{o}_{s}	0.4978	974	0	0.87×10^{-10}
Kion long lived	K^{o}_{L}	0.4978	974	0	5.2×10^{-8}
Eta meson	η	0.5486	1070	0	---
Theta meson	θ^{o}		965	0	1.5×10^{-10}
Theta meson	$\theta^{\pm}$		955	±1	10^{-9}
LEPTONS		Mass at rest (MeV/c^2)			
Electron neutrino	ν	About 0		0	Stable
Electron	e or e^{-}	0.511		-1	Stable
Muon neutrino	ν_{μ}	About 0		0	Stable
Muon	μ or μ^{-}	106.6		-1	2.2×10^{-6}
Tau neutrino	ν_{τ}	Less than 164		0	Stable
Tau	τ or τ^{-}	1,784		-1	
QUARKS		Mass(GeV)	Mass (emasses)		
Up	u	0.35	310	+2/3	
Down	d	0.35	310	-1/3	
Charm	c	1.5	1,500	+2/3	
Strange	s	0.5	505	-1/3	
Top / Truth	t	30-40	>22,500 Hypothetical particle	+2/3	
Bottom / Beauty	b	4.7	About 5,000	-1/3	