Gem Physics

of the Textbook

PHYSICS XI

A Revised Version of the Book

"Important Articles"

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Significant Figures:
Accurately known digits and the first doubtful digit.

A **precise** measurement is the one which has less absolute uncertainty.
An **accurate** measurement is one which has less fractional or percentage uncertainty.

Dimensions of Physical Quantity:
$$[F] = [m][a] = [M][LT^{-2}] = [MLT^{-2}]$$

Vector Addition by Rectangular Components
Consider vectors $\vec{A}$ & $\vec{B}$.
Found resultant $\vec{R}$ by head-to-tail rule.
Did some geometrical work.
From the geometry of the figure,

$$R_x = A_x + B_x \quad \& \; R_y = A_y + B_y$$

Generalizing the equations,

$$R_x = A_x + B_x + C_x + \ldots\ldots$$
$$\& \quad R_y = A_y + B_y + C_y + \ldots\ldots\ldots$$

The magnitude is, $R = \sqrt{R_x^{\,2} + R_y^{\,2}}$

& direction is, $\tan\theta = \dfrac{R_y}{R_x}$

Product of two Vectors
$\vec{A}\cdot\vec{B} = AB\cos\theta$; $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$ & $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

& $\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{AB} = \dfrac{A_xB_x + A_yB_y + A_zB_z}{AB}$

$\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}$; $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$

$\hat{i}\times\hat{j} = \hat{k}$, $\hat{j}\times\hat{k} = \hat{i}$, $\hat{k}\times\hat{i} = \hat{j}$ & $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_yB_z - A_zB_y)\,\hat{i} + (A_zB_x - A_xB_z)\,\hat{j} + (A_xB_y - A_yB_x)\,\hat{k}$$

Law of conservation of linear momentum

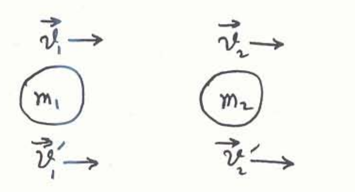
Statement: *The total linear momentum of an isolated system remains constant.*

Alternate statement: *If there is no external force applied to a system the total linear momentum of that system remains constant in time.*

Proof:

For an isolated system,
Let

m_1 = mass of first ball
m_2 = mass of second ball
$\vec{v}_1$ = velocity of first ball before collision
$\vec{v}_2$ = velocity of second ball before collision
$\vec{v}_1'$ = velocity of first ball after collision
$\vec{v}_2'$ = velocity of second ball after collision
$\vec{F}$ = action force of first ball
$\vec{F}'$ = reaction force of second ball



From the definition of impulse we have

$$\text{Impulse} = \vec{F} \times t = m\,\vec{v}_f - m\,\vec{v}_i$$

The impulse (or change in momentum) of first ball is;

$$\vec{F} \times t = m_1\,\vec{v}_1' - m_1\,\vec{v}_1 \quad \ldots\ldots (1)$$

& the impulse (or change in momentum) of second ball will be;

$$\vec{F}' \times t = m_2\,\vec{v}_2' - m_2\,\vec{v}_2 \quad \ldots\ldots (2)$$

adding equations (1) & (2), we have

$$\vec{F} \times t + \vec{F}' \times t = (m_1\,\vec{v}_1' - m_1\,\vec{v}_1) + (m_2\,\vec{v}_2' - m_2\,\vec{v}_2)$$

or $$(\vec{F} + \vec{F}')\,t = (m_1\,\vec{v}_1' - m_1\,\vec{v}_1) + (m_2\,\vec{v}_2' - m_2\,\vec{v}_2) \quad \ldots\ldots (3)$$

since action force $\vec{F}$ is equal and opposite to the reaction force $\vec{F}'$, so we have

$$\vec{F}' = -\vec{F} \quad \ldots\ldots (4)$$

from equations (3) & (4) we have

$$[\vec{F} + (-\vec{F})]\,t = 0 = (m_1\,\vec{v}_1' - m_1\,\vec{v}_1) + (m_2\,\vec{v}_2' - m_2\,\vec{v}_2)$$

or $$(m_1\,\vec{v}_1' - m_1\,\vec{v}_1) + (m_2\,\vec{v}_2' - m_2\,\vec{v}_2) = 0 \quad \ldots\ldots (5)$$

or $$m_1\,\vec{v}_1 + m_2\,\vec{v}_2 = m_1\,\vec{v}_1' + m_2\,\vec{v}_2' \quad \ldots\ldots (6)$$

from equation (5) we conclude,

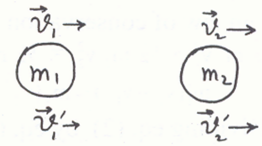
The total linear momentum of an isolated system remains constant.

& from equation (6) we have

Total initial momentum of the system before collision is equal to the total final momentum of the system after collision.

Elastic Collision in One Dimension

Considering two smooth, non-rotating balls of masses m_1 & m_2 with initial velocities v_1 & v_2 and their velocities after collision as v_1' & v_2'.



Applying law of conservation of momentum, we have

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' \quad \ldots.. (1)$$

Also applying law of conservation of K.E. we have

$$\tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2 = \tfrac{1}{2} m_1 v_1'^2 + \tfrac{1}{2} m_2 v_2'^2 \ldots.. (2)$$

From the above two equations, finding final velocities v_1' & v_2' in terms of known quantities of m_1, m_2, v_1 & v_2.

Solving these equations to get following two equations.

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 + \frac{2m_2}{m_1 + m_2} v_2 \quad \ldots.. (3)$$

$$v_2' = \frac{2m_1}{m_1 + m_2} v_1 + \frac{m_2 - m_1}{m_1 + m_2} v_2 \quad \ldots.. (4)$$

Some cases of special interest:

<u>Case 1</u> When $m_1 = m_2$

From equations (3) & (4) we get

$$v_1' = v_2 \;\; \& \;\; v_2' = v_1$$

We conclude : When two particles of equal masses collide elastically, they exchange their velocities.

<u>Case 2</u> When $m_1 = m_2$ & $v_2 = 0$

From equations (3) & (4) we get

$$v_1' = 0 \;\; \& \;\; v_2' = v_1$$

We conclude : The incident particle which was moving with v_1, comes to rest while the target particle that was at rest begins to move with velocity v_1.

<u>Case 3</u> When $m_2 \gg m_1$ & $v_2 = 0$

From equations (3) & (4) we get

$$v_1' \simeq - v_1 \;\; \& \;\; v_2' \simeq 0$$

We conclude: The small incident particle just bounces off in the opposite direction while the heavy target remains almost motionless.

<u>Case 4</u> When $m_1 \gg m_2$ & $v_2 = 0$

From equations (3) & (4) we get

$$v_1' \simeq v_1 \;\; \& \;\; v_2' \simeq 2 v_1$$

We conclude: The incident particle keeps on moving without loosing much energy, while the target particle moves with the double velocity.

For elastic collision, consider two smooth, non-rotating balls, law of conservation of momentum gives

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

or $\quad m_1(v_1 - v_1') = m_2(v_2' - v_2) \qquad \ldots\ldots (1)$

from law of conservation of KE

$$\tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2 = \tfrac{1}{2} m_1 v_1'^2 + \tfrac{1}{2} m_2 v_2'^2$$

$$m_1(v_1^2 - v_1'^2) = m_2(v_2'^2 - v_2^2) \qquad \ldots\ldots (2)$$

Dividing eq. (2) by eq. (1) gives

$$(v_1 + v_1') = (v_2' + v_2) \text{ or } (v_1 - v_2) = (v_2' - v_1')$$

or $\quad (v_1 - v_2) = -(v_1' - v_2')$

The above equation shows that the magnitude of relative velocity of approach is equal to the magnitude of relative velocity of separation

Projectile Motion

Definitions:

"**Projectile motion** is two dimensional motion under constant acceleration due to gravity"

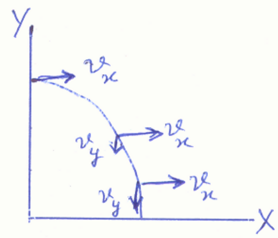
Projectile: "An object launched in an arbitrary direction in space with the initial velocity having no mechanism of propulsion is called a **projectile**"

Consider motion of a ball **thrown horizontally**;

For horizontal motion:

$$x = v_x t$$

$\begin{cases} S = v\,t \\ x = \text{horizontal distance} \end{cases}$



For vertical motion:

We have

$$S = v_i t + \tfrac{1}{2} a t^2$$

$\begin{cases} S = y = \text{vertical distance} \\ v_i = 0 \\ a = g \end{cases}$

or

$$y = \tfrac{1}{2} g t^2$$

Now consider a **projectile fired in a direction** θ with horizontal.

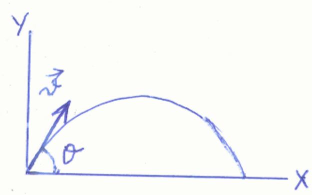
Horizontal component remains constant.

Using $v_f = v_i + a\,t$

or

$$v_{fx} = v_{ix} = v_i \cos\theta$$

$\begin{cases} v_f = v_{fx} \\ v_i = v_{ix} \\ a = a_x = 0 \end{cases}$



for Vertical component:

using $v_f = v_i + a\,t$

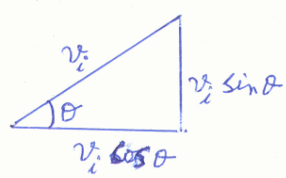
or

$$v_{fy} = v_i \sin\theta - gt$$

$\begin{cases} v_f = v_{fy} \\ v_i = v_i \sin\theta \\ a = -g \end{cases}$

so **Magnitude** of velocity at any instant:

$$v = \sqrt{v_{fx}^2 + v_{fy}^2}$$



and **Angle ϕ:**

$$\tan\phi = \frac{v_{fy}}{v_{fx}}$$

Maximum height h:

[for its definition see the "Definitions" book]

Using equation,

$$2aS = v_f^2 - v_i^2$$

or $$2(-g)h = (0)^2 - (v_i \sin\theta)^2$$

or $$-2gh = -v_i^2 \sin^2\theta$$

or $$h = \frac{v_i^2 \sin^2\theta}{2g}$$

$\begin{cases} a = -g \\ S = \text{height} = h \\ v_f = v_{fy} = 0 \\ v_i = v_{iy} = v_i \sin\theta \end{cases}$

Time of flight t:

[for its definition see the "Definitions" book]

Using equation $S = v_i t + \frac{1}{2} a t^2$

or $0 = v_i \sin\theta\, t + \frac{1}{2}(-g) t^2$

or $0 = v_i \sin\theta\, t - \frac{1}{2} g t^2$

$S = h = 0$, $v_i = v_{iy} = v_i \sin\theta$, $a = -g$

$$\Rightarrow \boxed{t = \frac{2v_i \sin\theta}{g}}$$

Time to reach at max. height:

Using $v_f = v_i + a t$

or $0 = v_i \sin\theta\, t' - g t'$

$v_f = 0$, $v_i = v_i \sin\theta$, $a = -g$

or $$\boxed{t' = \frac{v_i \sin\theta}{g}}$$

Range R:

[for its definition see the "Definitions" book]

Using $S = v t$

$S = R$, $v = v_{ix} = v_i \cos\theta$, $t = \frac{2v_i \sin\theta}{g}$

or $R = v_i \cos\theta \frac{2v_i \sin\theta}{g}$

or $R = \frac{v_i^2}{g} 2 \sin\theta \cos\theta$

$\sin(\theta + \phi) = \sin\theta \cos\phi + \cos\theta \sin\phi$

or $\sin(\theta + \theta) = \sin\theta \cos\theta + \cos\theta \sin\theta$

or $\sin 2\theta = 2 \sin\theta \cos\theta$

or $$\boxed{R = \frac{v_i^2}{g} \sin 2\theta}$$

Maximum Range R_{max}:

We know that maximum value of $\sin\theta$ is $\sin 90^\circ = 1$.

So from the above equation, for maximum value,

$\sin 2\theta = 1$ or $2\theta = 90^\circ$ or $\theta = 45^\circ$

$$\Rightarrow \boxed{R_{max} = \frac{v_i^2}{g}}$$

Applications:

1. **Ballistic Missiles:** [see the "Definitions" book]
   The ballistic missiles are useful only for short ranges. For long ranges powered and remote control **guided** missiles are used.
2. **Bazooka:** A weapon consisting of a launching tube for military rockets, used by infantry personnel against armored vehicles such as tanks. Now developed an explosive projectile that, using a shaped charge, and are able to penetrate armor.
3. **Cannon:** The first cannon used gunpowder charges to fire stones or metal balls. Modern cannon demonstrate a laser-guided artillery shell, or "cannon-launched guided projectile," that can be fired with great accuracy.
4. **Rifle:** Any firearm having the interior of its barrel rifled, that is, engraved with spiral grooves so as to give spin to a projectile as it is fired.
5. **Explosives:** They are used as propellants for projectiles and rockets and as bursting charges for demolition purposes and for projectiles, bombs, and mines.

Work Done by Gravitational Field

Definitions:

Gravitational field : "The space around the earth within which it exerts a force of attraction on other bodies."

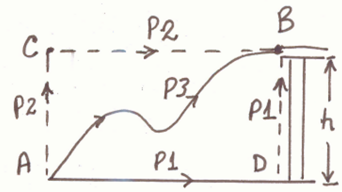
Conservative field : "In which the work done between two points in the field is independent of the path followed between the two points."

To prove:

1. Work done in the Earth's gravitational field is independent of the path followed.
2. Total work done along a closed path in a gravitational field is equal to zero.

Consider

An object of mass m being displaced with constant velocity from point A to B along various paths under gravitational force.



i) To calculate work done along the path ADB

$$W_{ADB} = W_{A\to D} + W_{D\to B}$$
$$= mgd\cos 90^o + mgh\cos 180^o$$
$$= 0 + mgh(-1)$$

or $\boxed{W_{ADB} = -mgh}$ ..... (1)

$\vec{W} = \vec{F}\cdot\vec{d}$
or $W = Fd\cos\theta$ $F = mg$
or $W = mgh\cos\theta$ $d = h$

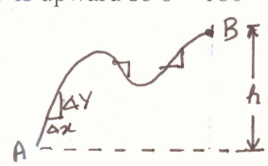
$\cos 90^o = 0$
$\cos 180^o = -1$

ii) To calculate work done along the path ACB

$$W_{ACB} = W_{A\to C} + W_{C\to B}$$
$$= mgh\cos 180^o + mgd\cos 90^o$$
$$= mgh(-1) + 0$$

or $\boxed{W_{ADB} = -mgh}$ ..... (2)

since directions of mg is downward & h is upward so $\theta = 180^o$



iii) To calculate work done along the curved path 3

$$W_{AB} = \Sigma\, mgd\cos\theta$$
$$= mg\left(\sum_{i=1}^{n}\Delta x_i + \sum_{i=1}^{n}\Delta y_i\right)$$
$$= mg(x_1\cos 90^o + x_2\cos 90^o + \ldots\ldots + y_1\cos 180^o + y_2\cos 180^o + \ldots\ldots)$$
$$= mg\left(\sum_{i=1}^{n}\Delta y_i(-1)\right)$$

or $\boxed{W_{AB} = -mgh}$ ..... (3)

directions of mg & x are ⊥ so $\theta = 90^o$ & directions of mg & y are opposite so $\theta = 180^o$

$\sum_{i=1}^{n}\Delta y_i = h$

iii) To calculate work done along the path B to A

Selecting any one of the paths. Lets take path 1.

$W_{BA} = W_{B\rightarrow d} + W_{D\rightarrow A}$

$= mgh \cos 0^o + mgd \cos 90^o$

$= mgh + 0$

or $\boxed{W_{BA} = mgh}$ ..... (4)

Conclusion

Equations (1) , (2) & (3) show that
Work done in the Earth's gravitational field is independent of the path followed.

Adding equations (3) & (4) , we have

$W_{A\rightarrow B\rightarrow A} = W_{AB} + W_{BA} = -mgh + mgh = 0$

So Work done along a closed path in a gravitational field is **zero**

Examples of Conservative field:

i) Gravitational field
ii) Electric field
iii) Magnetic field

While frictional forces and air resistance made non-conservative forces and corresponding field, e.g. rough surfaces and air, will be non-conservative field.

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Some information:

Work, in physics, product of a force applied to a body and the displacement of the body in the direction of the applied force. While work is done on a body, there is a transfer of energy to the body, and so work can be said to be energy in transit.

Gravitation may also be described in a completely different way. A massive object, such as the earth, may be thought of as producing a condition in space around it called a gravitational field. This field causes objects in space to experience a force. The gravitational field around the earth, for instance, produces a downward force on objects near the earth surface. The field viewpoint is an alternative to the viewpoint that objects can affect each other across distance. This way of thinking about interactions has proved to be very important in the development of modern physics.

Absolute Potential Energy

Definition:

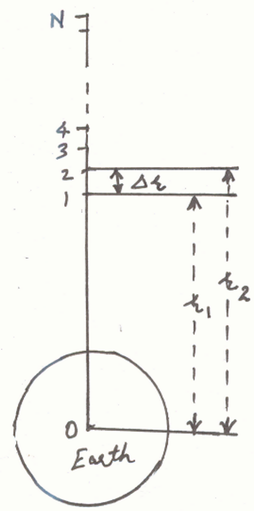
Absolute potential energy : "Energy required to move a mass from the earth up to an infinite distance".

To calculate the value of absolute gravitational potential energy,

Consider

A body of mass m which moves from point 1 to far off point N with constant velocity in the gravitational field.

As gravitational force changes with distance, so divide the distance between 1 to N into small steps, each of length Δr.



We have

$$\text{Mean distance} = r = \frac{r_1 + r_2}{2} \qquad \text{.....} \quad (1)$$

$$\& \quad r_2 - r_1 = \Delta r \qquad \text{.....} \quad (2)$$

$$\text{or} \quad r_2 = r_1 + \Delta r \qquad \text{.....} \quad (3)$$

putting the value of r_2 from eq. (3) in eq. (1), we get

$$r = \frac{r_1 + r_1 + \Delta r}{2}$$

$$\text{or} \quad r^2 = \left(\frac{r_1 + r_1 + \Delta r}{2}\right)^2$$

$$\text{or} \quad r^2 = \left(\frac{2r_1}{2} + \frac{\Delta r}{2}\right)^2$$

$$\text{or} \quad r^2 = r_1^2 + \frac{(\Delta r)^2}{4} + r_1 \Delta r$$

Neglecting $\frac{(\Delta r)^2}{4}$ as $(\Delta r)^2 \ll r_1^2$

$$\text{So} \quad r^2 = r_1^2 + r_1(r_2 - r_1) \qquad \text{[from eq. (2)]}$$

$$\text{or} \quad r^2 = r_1^2 + r_1 r_2 - r_1^2$$

$$\text{or} \quad r^2 = r_1 r_2 \qquad \text{.....} \quad (4)$$

Now, if M is the mass of the earth, the gravitational force at the center of the small step is

$$F = G\frac{Mm}{r^2} \qquad \text{.....} \quad (5)$$

from equations (4) and (5) we get

$$F = G\frac{Mm}{r_1 r_2} \qquad \text{.....} \quad (6)$$

As this force is assumed to be constant during the interval Δr, so the work done is

$$W_{1\to 2} = \vec{F} \cdot \Delta\vec{r} = F\Delta r \cos 180^\circ = -F\Delta r$$

$$\text{or} \quad W_{1\to 2} = -G\frac{Mm}{r_1 r_2}(r_2 - r_1) \qquad [\ \Delta r = r_2 - r_1$$

$$\text{or} \quad W_{1\to 2} = -GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$W_{1\to 2} = -GMm\frac{r_2 - r_1}{r_1 r_2}$$

$$\text{or} \quad = -GMm\left(\frac{r_2}{r_1 r_2} - \frac{r_1}{r_1 r_2}\right)$$

$$\text{or} \quad W_{1\to 2} = -GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\text{similarly} \quad W_{2\to 3} = -GMm\left(\frac{1}{r_2} - \frac{1}{r_3}\right)$$

$$W_{3\to 4} = -GMm\left(\frac{1}{r_3} - \frac{1}{r_4}\right)$$

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$$W_{N-1\to N} = -GMm\left(\frac{1}{r_{N-1}} - \frac{1}{r_N}\right)$$

So the total work done is

$$W_{total} = W_{1\to 2} + W_{2\to 3} + W_{3\to 4} + \ldots\ldots + W_{N-1\to N}$$

$$= -GMm\left[\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \left(\frac{1}{r_2} - \frac{1}{r_3}\right) + \left(\frac{1}{r_3} - \frac{1}{r_4}\right) + \ldots\ldots + \left(\frac{1}{r_{N-1}} - \frac{1}{r_N}\right)\right]$$

$$= -GMm\left[\frac{1}{r_1} - \frac{1}{r_2} + \frac{1}{r_2} - \frac{1}{r_3} + \ldots\ldots + \frac{1}{r_{N-1}} - \frac{1}{r_N}\right]$$

$$\text{or} \quad W_{total} = -GMm\left(\frac{1}{r_1} - \frac{1}{r_N}\right)$$

If the point N is at an infinite distance from the Earth, then

$$\frac{1}{r_\infty} = 0 \qquad [\ \frac{1}{\infty} = 0\ ]$$

$$\text{Hence} \quad W_{total} = \frac{-GMm}{r_1}$$

So the general expression will be

$$\boxed{U = \frac{-GMm}{r}} \quad \ldots\ldots (7)$$

[U is Gravitational Potential Energy]

Negative sign shows that when r increases U becomes less negative, i.e. as we raise a body upward PE increases.

Taking $r_1 = R$, the radius of Earth, then this total work done would be equivalent to the absolute gravitational potential energy,

$$\text{So} \qquad \boxed{U_g = -\frac{GMm}{R}} \quad \ldots\ldots (8)$$

Escape Velocity

Definitions:

Escape velocity : "The initial velocity, which a projectile must have at the earth's surface in order to go out of earth's gravitational field."

(Initial) $KE = \frac{1}{2} m v_{esc}^2$ [$KE = \frac{1}{2} mv^2 \Rightarrow KE \propto v$

Absolute potential energy : "Energy required to move a mass from the earth up to an infinite distance."

$$U = \left| -\frac{GMm}{R} \right| = \frac{GMm}{R}$$

The Energy [Initial KE / Increase in PE] needed to go free from 'g' [earth's gravitational field / infinite distance] implies,

$$KE_{initial} = PE_{absolute}$$

or $$\frac{1}{2} m v_{esc}^2 = \frac{GMm}{R}$$

or $$v_{esc}^2 = \frac{2GM \times R}{R \times R} = \frac{2GMR}{R^2}$$

or $$v_{esc}^2 = 2R \times \frac{GM}{R^2} = 2Rg$$

or $$\boxed{v_{esc} = \sqrt{2gR}}$$

We have $F = mg$ & $F = \frac{GMm}{R^2}$

$\Rightarrow mg = \frac{GMm}{R^2}$

or $g = \frac{GM}{R^2}$

Interconversion of PE & KE

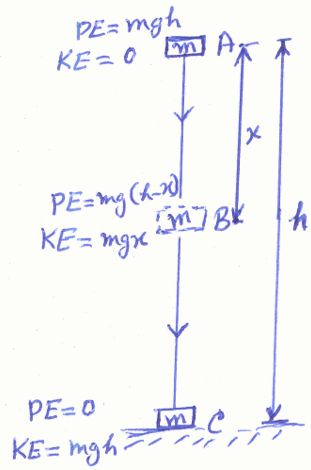
Definitions:

Kinetic Energy : "The energy possessed by a body due to its motion."
Potential energy : "It is the energy possessed by a body due to its position."

To show that the total Energy at each point (A, B & C) remains same.

Consider

A body of mass m at rest, at a height h above the surface of the Earth.



At point A

Total Energy = T = PE + KE

or T = mgh + 0 = $\boxed{mgh}$

At point B

First calculating v_B , by using the equation

$$v_f^2 = v_i^2 + 2aS$$

putting the values

$$v_B^2 = 0 + 2gx = 2gx$$

Now T = PE + KE

or T = mg(h-x) + ½ m x 2gx

or T = mgh – mgx + mgx

or T = $\boxed{mgh}$

At point C

First calculating v_C , by using the equation

$$v_f^2 = v_i^2 + 2aS$$

putting the values

$$v_C^2 = 0 + 2gh = 2gh$$

Now T = PE + KE

or T = 0 + ½ m x 2gh

or T = $\boxed{mgh}$

It shows as the body falls, its velocity increases that results increase in KE and its height decreases that results decrease in PE. So we conclude

$$\boxed{\text{Loss of PE} = \text{Gain in KE}} \quad \ldots\ldots (\alpha)$$

$$\text{or} \quad mg(h_1 - h_2) = \frac{1}{2} m(v_2^2 - v_1^2)$$

From equation (α), we have

PE = W. done = KE

or $F d \cos\theta = \frac{1}{2} m v^2$

Assuming a frictional force f is present during the downward motion, then Total downward force will be (F - f) = (mg -f) , so

$(mg - f) h \cos 0^{\circ} = \frac{1}{2} m v^2$

$mgh - fh = \frac{1}{2} m v^2$

or $\boxed{mgh = \frac{1}{2} m v^2 + fh}$

or Loss of PE = Gain in KE + W. done against friction

Centripetal Force

Definitions:

Centripetal Force: "The force needed to bend the normally straight path of the particle into a circular path."

Or "A force that causes a body to move in a circular path."

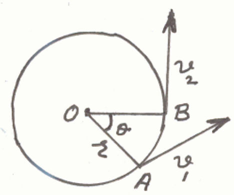
Centripetal Acceleration: "The instantaneous acceleration of an object traveling with uniform speed in a circle and is directed towards the center of the circle."

Or "An acceleration directed towards the center of a circle."

To calculate the magnitude of Centripetal Acceleration

Consider

m = mass of the particle
ω = angular speed of the particle
r = radius of the circle
v = its linear velocity along the tangent
v_1 = velocity at point A
v_2 = velocity at point B



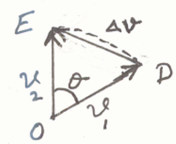
since two velocities at points A and B are same,

so $\quad v_1 = v_2 = v \qquad \ldots\ldots (1)$

from fig. (b), we have

$$\vec{v}_1 + \Delta\vec{v} = \vec{v}_2$$

or $\quad \Delta\vec{v} = \vec{v}_2 - \vec{v}_1 \qquad \ldots\ldots (2)$



now angle AOB = angle DOE = $\Delta\theta$

for small angle chord Δv = arc DE $\quad \ldots\ldots (3)$

and $\sin\theta = \theta \quad \ldots\ldots (4)$

We have

$$\sin\theta = \frac{\Delta v}{v_2}$$

putting the values from equations (1), (3) & (4), we get

$$\theta = \frac{\Delta v}{v} \qquad [\text{for small change; } \theta = \Delta\theta\ ]$$

or $\quad v\,\Delta\theta = \Delta v \qquad \ldots\ldots (5)$

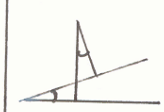
multiplying and dividing by Δt to L.H.S. , we get

$$v\frac{\Delta\theta}{\Delta t}\Delta t = \Delta v$$

or $\quad \Delta v = v\,\omega\, t \qquad \ldots\ldots (6) \qquad [\omega = \theta / t\,]$

or $\quad \dfrac{\Delta v}{\Delta t} = \omega v \qquad \ldots\ldots (7)$

A Geometry Theorem: Angle between the perpendiculars of the sides of an angle is equal to that angle.



Now we define

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} \qquad \ldots\ldots (8)$$

Taylor's series expansion for:

$$\sin\theta = \theta - \frac{\theta^3}{3\cdot 2\cdot 1} + \frac{\theta^5}{5!} - \ldots\ldots$$

so $\quad a = \omega\, v \qquad \ldots\ldots (9)$

$v = r\,\omega$
or $\omega = v / r$

or
$$\boxed{a = \omega^2 r = \frac{v^2}{r}} \qquad \ldots\ldots (10)$$

In vector form,

$$\vec{a} = -\omega^2\vec{r} = -\frac{v^2}{r^2}\vec{r} \qquad \text{..... (11)}$$

where negative sign indicates that the acceleration is towards the center. [It is indicated by angle ϕ in fig. (a)]

To calculate Centripetal Force

We have

$$\vec{F} = m\vec{a} \qquad \text{..... (12)}$$

from equations (11) & (12), we get

$$\vec{F} = -m\omega^2\vec{r} = -\frac{mv^2}{r^2}\vec{r} \qquad \text{..... (13)}$$

$$\boxed{F_c = \frac{mv^2}{r}}$$

or in angular measure, we have

$$\boxed{F_c = mr\omega^2}$$

EXAMPLES

1. A stone is whirled in a horizontal circle by means of a string.
2. Planets move around the sun.
3. When a racing car moves round a circular track the friction at the wheels provide the centripetal force.

Artificial Gravity

We define

Artificial gravity: "The gravity like effect produced in orbiting space ship to overcome weightlessness."

We have

$$a = r\,\omega^2$$

$$\omega = \theta / t = 2\pi / T, \quad r = R, \quad f = 1/T \text{ or } T = 1/f$$

or $$a_c = R\left(\frac{2\pi}{T}\right)^2$$

or $$a_c = R\left(\frac{2\pi}{1/f}\right)^2$$

or $$a_c = R4\pi^2 f^2 \quad \text{or} \quad f^2 = \frac{a_c}{R4\pi^2}$$

or $$f = \frac{1}{2\pi}\sqrt{\frac{a_c}{R}}$$

to increase f so that a_c equals g , and from the above equation, we have

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{g}{R}}}$$

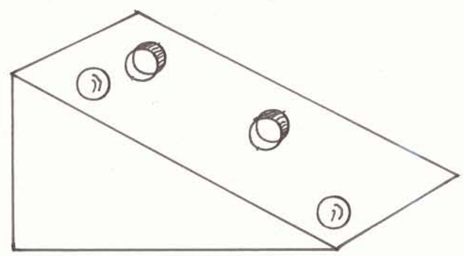
When the space ship rotates with this frequency, the artificial gravity like Earth is provided to the inhabitants of the space ship.

Rotational KE of a Disc & a Hoop

Definitions:

Disc: "A flat circular plate or anything resembling it."

Hoop: "A circular band such like a ring; anything curved like a ring."



To prove $v_{disc} > v_{hoop}$

For Disc,

PE at the top = total KE at the bottom

$PE_{top} = KE_{trans} + KE_{rot}$

i.e. $mgh = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$

or $mgh = \frac{1}{2} mv + \frac{1}{2} (\frac{1}{2} mr^2) (v^2 / r^2)$

or $mgh = \frac{1}{2} m v^2 + \frac{1}{4} m v^2$

or $gh = \frac{3}{4} v^2$

or $v^2 = \frac{4gh}{3}$ or $\boxed{v_{disc} = \sqrt{\frac{4gh}{3}}}$ ..... (1)

for a disc:
$I = \frac{1}{2} mr^2$
$\omega = v / r$

For Hoop,

PE at the top = total KE at the bottom

$PE_{top} = KE_{trans} + KE_{rot}$

i.e. $mgh = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$

or $mgh = \frac{1}{2} mv + \frac{1}{2} (mr^2) (v^2 / r^2)$

or $mgh = \frac{1}{2} m v^2 + \frac{1}{2} m v^2$

or $gh = v^2$

or $v^2 = gh$ or $\boxed{v_{hoop} = \sqrt{gh}}$ ..... (2)

for an hoop:
$I = mr^2$
$\omega = v / r$

from equations (1) & (2), we conclude

$$\boxed{v_{disc} > v_{hoop}}$$

when rolls down an inclined plane of height h.

Terminal velocity

Definitions:

Terminal velocity:" At the end, the extreme, or maximum velocity reached by certain object".

Density (ρ): "The ratio of the mass of a substance to its volume".

Stokes' law: In fluid resistance, the drag force F of a sphere of radius r moving with a velocity v through a fluid of infinite extent is

$$F = 6 \pi \eta r v, \quad \text{where } \eta \text{ is the viscosity.}$$

Terminal velocity of water droplet moving in air:

The dragging force experienced by a tiny water droplet (of fog or mist) falling freely will be given by Stokes Law as:

$$F_D = 6\pi\eta rv \qquad \text{.....(1)}$$

The weight of the droplet is given by

$$w = mg \qquad \text{.....(2)}$$

so the net downward force will be

$$F = w - F_D$$

$$\text{or} \quad F = mg - 6\pi\eta rv \qquad \text{.....(3)}$$

From 2nd Law of motion,

$$F = ma \qquad \text{.....(4)}$$

From eqs. (3) & (4), we have

$$ma = mg - 6\pi\eta rv$$

$$\text{or} \quad a = \frac{mg - 6\pi\eta rv}{m}$$

$$\text{or} \quad a = g - \frac{6\pi\eta rv}{m}$$

At terminal velocity $v = v_T$ & $a = 0$, so

$$0 = g - \frac{6\pi\eta rv_T}{m}$$

$$\text{or} \quad 6\pi\eta rv_T = mg$$

$$\text{or} \quad v_T = mg / 6\pi\eta r \qquad \text{.....(5)}$$

$$\text{or} \quad v_T = \frac{(4/3\pi r^3)\rho g}{6\pi\eta r} \qquad \left[\rho = \frac{\text{mass}}{\text{vol.}} = \frac{m}{4/3\pi r^3} \right.$$

$$\left[\text{or } m = (4/3)\pi r^3) \rho \right.$$

$$\text{or} \quad v_T = \frac{(4/3\pi r^3)\rho g}{6\pi\eta r}$$

$$\text{or} \quad \boxed{v_T = \frac{2\rho g r^2}{9\eta}} \qquad \text{.....(6)}$$

$$\text{since} \quad \frac{2\rho g}{9\eta} = \text{constant},$$

$$\text{so} \quad \boxed{v_T \propto r^2} \qquad \text{.....(7)}$$

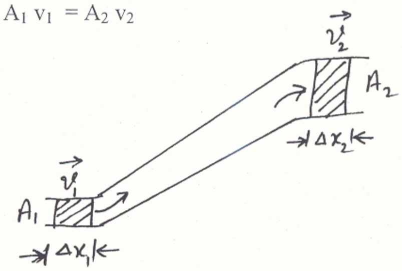
i.e. The terminal velocity of a sphere of given material varies directly with the square of the radius.

Equation of Continuity

STATEMENT:

"The product of cross-sectional area of the pipe and the fluid speed at any point along the pipe is a constant". Mathematically, $A_1 v_1 = A_2 v_2$

PROOF:



Consider

A fluid flowing through a pipe of non-uniform size.

And the flow of the liquid is streamline and incompressible.

Let

As shown in the figure

At left side:

Velocity of the fluid = v_1

Move through distance = Δx_1

Area of cross-section = A_1

So volume = $V_1 = \Delta x_1 . A_1$

& mass passing during Δt

$\Delta m_1 = \rho_1 V_1 = \rho_1 \Delta x_1 . A_1$

or $\Delta m_1 = \rho_1 A_1 v_1 . \Delta t$ ..... (1)

[Density = mass / volume
$\rho = m / V$
or $m = \rho V$]

[$S = v t$
or $\Delta x = v_1 t$]

At right side:

Velocity of the fluid = v_2

Move through distance = Δx_2

Area of cross-section = A_2

So volume = $V_1 = \Delta x_2 . A_2$

& mass passing during Δt

$\Delta m_2 = \rho_2 V_2 = \rho_2 \Delta x_2 . A_2$

or $\Delta m_2 = \rho_2 A_2 v_2 . \Delta t$ ..... (2)

As the streamline flow is incompressible, so

$\Delta m_1 = \Delta m_2$ .....(3)

from equations (1), (2) & (3) we have

$\rho_1 A_1 v_1 . \Delta t = \rho_2 A_2 v_2 . \Delta t$

since density is constant, i.e., $\rho_1 = \rho_2 = \rho$, so

$\rho A_1 v_1 = \rho A_2 v_2$

or $A_1 v_1 = A_2 v_2$

Which is Equation of Continuity.

Core: $\rho = \frac{m}{V}$ or $m = \rho V = \rho \times \Delta x \cdot A = \rho A v \Delta t$ [S = vt]

as $\Delta m_1 = \Delta m_2$

so $\rho A_1 v_1 \Delta t = \rho A_2 v_2 \Delta t$ or $A_1 v_1 = A_2 v_2$

Bernoulli's Equation

STATEMENT:

In a steady frictionless motion of a fluid acted on by external forces which possess a gravitational potential ρgh, then

$$P + \frac{1}{2}\rho v + \rho gh = \text{constant}$$

where P & ρ are the pressure and density of the fluid, v is the velocity of the fluid along a streamline.

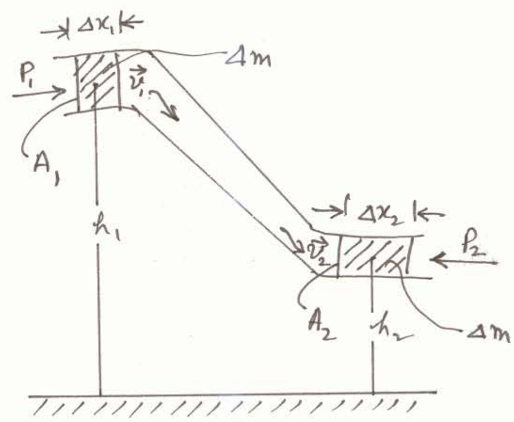
PROOF:

Consider

A fluid is flowing , (in the figure)

And assume;

1) The fluid is incompressible,
2) Non-viscous,
3) Moving with streamline flow



Let (shown in the figure)

A liquid of mass (Δm), flowing through a pipe during time (t),

At left side:

Pressure = P_1
Velocity of the fluid = v_1
Move through distance = Δx_1
Area of cross-section = A_1
Height from the bottom = h_1

At right side:

(for the same mass Δm)
Pressure = P_2
Velocity of the fluid = v_2
Move through distance = Δx_2
Area of cross-section = A_2
Height from the bottom = h_2

We have

Pressure = P = Force / Area = F / A or F = P A ..... (1)

Work done = W = force x displacement = F x Δx = PA Δx ..... (2)

Also S = Δx = v t .....(3)

& $\rho = m / V$ or $V = m / \rho$

as volume = area x length

so $A \cdot \Delta x = A v t = V = m / \rho$ .....(4)

for the same mass flowing during time t, through both ends, the volume will be

$A_1 v_1 t = A_2 v_2 t = A v t$ .....(5)

Now from equations (2) & (4) we have

W = P A v t

or W = P m / ρ .....(6)

Now we have

Kinetic energy = KE = $\frac{1}{2} m v^2$ .....(7)

& Potential energy = PE = m g h .....(8)

Taking mass (Δm) of the fluid flowing from upper end to lower end as same.
Applying the Law of conservation of energy to this volume ((Δm) of fluid:

Net Work done = change in KE + change in PE

or $W_{upper\ end} + W_{lower\ end} = \{KE_{upper} - KE_{lower}\} + \{PE_{upper} - PE_{lower}\}$ .....(9)

From equations (6) to (9) we have

$P_1 m/\rho + \{(-P_2) m/\rho\} = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 + m g h_2 - m g h_1$

or $m/\rho (P_1 - P_2) = m(\frac{1}{2} v_2^2 - \frac{1}{2} v_1^2 + g h_2 - g h_1)$

or $P_1 - P_2 = \rho(\frac{1}{2} v_2^2 - \frac{1}{2} v_1^2 + g h_2 - g h_1)$

or $P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 + \rho g h_2 - \rho g h_1$

or $P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$

or $P + \frac{1}{2}\rho v^2 + \rho g h =$ constant

Which is Bernoulli's Equation.

Core: Net $W = \Delta KE + \Delta PE$ or $W_U + W_L = KE_U - KE_L + PE_U - PE_L$

$$W = Fd = PAd = PV = P\frac{m}{\rho} \qquad [P = F/A \ \& \ \rho = m/V]$$

$$P_1\frac{m}{\rho} + (-P_2)\frac{m}{\rho} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 + mgh_2 - mgh_1$$

or $$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 + \rho g h_2 - \rho g h_1$$

or $$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2 \quad \text{or} \quad P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

Applications:

Torricelli's Theorem

The speed of efflux is equal to the velocity gained by the fluid in falling through the distance ($h_1 - h_2$) under the action of gravity.

Suppose we have a large tank with two small orifices, one at the top and the other at the bottom. Applying Bernoulli's equation,

or $$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2 \quad [v_2 \gg v_1, \text{ so ignoring top velocity } v_1]$$

or $$P_1 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2 \quad \text{or} \quad \Rightarrow v_2 = \sqrt{2g(h_1 - h_2)} \qquad [P_1 = P_2]$$

Venturi Relation

Applying Bernoulli's equation to Venturi meter,

or $$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2 \quad [A_2 < A_1 \Rightarrow v_1 < v_2 \ \& \ h_1 = h_2]$$

or $P_1 - P_2 = \frac{1}{2}\rho v_2^2$, called Venturi relation.

Relation between Speed & Pressure

Suppose water is flowing through a pipe having different area of cross sections at two points A & B. then applying Bernoulli's equation

or $$P_A + \frac{1}{2}\rho v_A^2 + \rho g h_A = P_B + \frac{1}{2}\rho v_B^2 + \rho g h_B \quad [\rho g h_A = \rho g h_B]$$

$$\Rightarrow P_A + \frac{1}{2}\rho v_A^2 = P_B + \frac{1}{2}\rho v_B^2$$

It shows that, *where the speed is high, the pressure will be low.*

SHM & Uniform Circular Motion

Consider, a point P moving along circular trajectory around the center O, with angular speed ω.

The radius of the circle is x_o , speed of the moving point P is

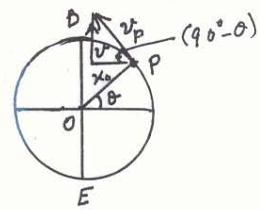
$$v = x_o \omega \quad \ldots\ldots (1) \qquad [v = r\omega]$$

To calculate **time period T** of P, we have

$$\omega = \frac{\theta}{t} \quad \text{or} \quad t = \frac{\theta}{\omega}, \qquad \theta = 2\pi \text{ rad}$$

so

$$T = \frac{2\pi}{\omega}$$



From the figure, the **displacement x** is

$$x = x_o \sin\theta \qquad [\theta = \omega t]$$

or

$$x = x_o \sin\omega t$$

From the figure, the **speed v** of the point P is

$$v = v_P \sin(90^0 - \theta) = v_P \cos\theta = x_o\omega\cos\theta \quad [v_P = x_o\omega]$$

or $$v = x_o\omega\sqrt{1-\sin^2\theta} = x_o\omega\sqrt{1-\frac{x^2}{x_o^2}} = x_o\omega\sqrt{\frac{x_o^2 - x^2}{x_o^2}}$$

or $$v = x_o\omega\sqrt{\frac{1}{x_o^2}(x_o^2 - x^2)} = \frac{x_o\omega}{x_o}\sqrt{x_o^2 - x^2} = \omega\sqrt{x_o^2 - x^2}$$

or

$$v = \omega\sqrt{x_o^2 - x^2}$$

$\sin^2\theta + \cos^2\theta = 1$

or $\cos^2\theta = 1 - \sin^2\theta$

or $\cos\theta = \sqrt{1-\sin^2\theta}$

& also we have

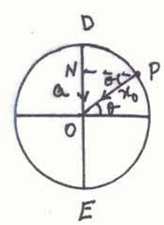
$$\sin\theta = \frac{x}{x_o}$$

Now to calculate **acceleration a** of the point P,

As P describes a constant angular speed ω , N oscillates to and fro along the diameter DE. As N move towards O, it speeds up, i.e., its acceleration is directed towards O.

The magnitude of acceleration of the point P is

$$a_c = \frac{v_P^2}{x_o} = \frac{x_o^2\omega^2}{x_o} = x_o\omega^2 \qquad \left[a = \frac{v^2}{r} \quad \& \quad v_P = x_o\omega\right]$$



its component along DOE is

$$a = x_o\omega^2 \sin\theta$$

since it is directed towards center and $x = x_o \sin\theta$,

so

$$a = -\omega^2 x$$

Thus the point N has acceleration proportional to displacement and directed towards the center, which is the characteristic of SHM. So the projection of P executes SHM. We can define SHM as "the projection of uniform circular motion upon any diameter of a circle".

A Horizontal Mass Spring System

Consider,
The vibrating mass attached to a spring whose **acceleration** at any instant is given by

$$a = -\frac{k}{m}x$$

$$\left\{ \begin{array}{l} F = -kx \ \& \ F = ma \\ \Rightarrow ma = -kx \\ \text{or } a = -\frac{k}{m}x \end{array} \right.$$

also we have $a = -\omega^2 x$

comparing the above two equations, we get

$$\omega = \sqrt{\frac{k}{m}}$$

so the **time period** of the mass attached to a spring is

$$T = \frac{2\pi}{\omega}$$

$$\text{or} \quad T = \frac{2\pi}{\sqrt{k/m}} \quad \text{or} \quad \boxed{T = 2\pi\sqrt{\frac{m}{k}}} \qquad \ldots.. (1)$$

for **instantaneous displacement** taking the equation

$$x = x_o \sin \omega t$$

$$\text{or} \quad \boxed{x = x_o \sin\sqrt{\frac{k}{m}}\, t} \qquad \ldots.. (2)$$

for **instantaneous velocity** taking the equation

$$\text{or} \quad v = \omega\sqrt{x_o^2 - x^2}$$

$$\text{or} \quad v = \sqrt{\frac{k}{m}}\sqrt{x_o^2 - x^2} = \sqrt{\frac{k}{m}(x_o^2 - x^2)} = \sqrt{x_o^2\left\{\frac{k}{m}\left(\frac{x_o^2 - x^2}{x_o^2}\right)\right\}}$$

$$\text{or} \quad \boxed{v = x_o\sqrt{\frac{k}{m}\left(1 - \frac{x^2}{x_o^2}\right)}} \qquad \ldots.. (3)$$

putting $x = 0$ in the above equation to get **maximum velocity** v_o, we have

$$\boxed{v_o = x_o\sqrt{\frac{k}{m}}} \qquad \ldots.. (4)$$

from equations (3) & (4), instantaneous velocity in terms of maximum velocity is

$$\boxed{v = v_o\sqrt{1 - \frac{x^2}{x_o^2}}} \qquad \ldots.. (5)$$

Energy Conservation in SHM

To calculate **potential energy**, we have

$$F = kx$$

$$P.E. = W.\ done = F_{av} \cdot x = \frac{1}{2}kx \cdot x = \frac{1}{2}kx^2$$

$F = kx$
F_{min} at $x = 0$ is $F = k(0) = 0$
& F_{max} at $x = x$ is $F = kx$
so $F_{av} = \dfrac{0 + kx}{2} = \dfrac{1}{2}kx$

or $$\boxed{P.E. = \frac{1}{2}kx^2} \quad \text{..... (6)}$$

From the above equation we see that **maximum PE** will be at $x = x_o$, i.e.

or $$\boxed{P.E._{max} = \frac{1}{2}kx_o^2} \quad \text{..... (7)}$$

& the **minimum PE** will be at $x = 0$,

$$P.E._{min} = \frac{1}{2}k(0)^2 \quad \text{or} \quad \boxed{P.E._{min} = 0} \quad \text{..... (8)}$$

And **kinetic energy** will be

$$K.E. = \frac{1}{2}mv^2 = \frac{1}{2}m\left(x_o\sqrt{\frac{k}{m}\left(1 - \frac{x^2}{x_o^2}\right)}\right)^2 = \frac{1}{2}mx_o^2\frac{k}{m}\left(1 - \frac{x^2}{x_o^2}\right)$$

or $$\boxed{K.E. = \frac{1}{2}kx_o^2\left(1 - \frac{x^2}{x_o^2}\right)} \quad \text{..... (9)}$$

The above equation shows that the **maximum kinetic energy** will be at $x = 0$,

So $$\boxed{K.E._{max} = \frac{1}{2}kx_o^2} \quad \text{..... (10)}$$

& **minimum kinetic energy** will be at $x = x_o$, so

$$K.E._{min} = \frac{1}{2}kx_o^2\left(1 - \frac{x_o^2}{x_o^2}\right) = \frac{1}{2}kx_o^2(1-1) = \frac{1}{2}kx_o^2 \times 0$$

or $$\boxed{K.E._{min} = 0} \quad \text{..... (11)}$$

Now calculating **total energy** at any displacement x will be

$$E_{total} = P.E. + K.E.$$

$$= \frac{1}{2}kx^2 + \frac{1}{2}kx_o^2\left(1 - \frac{x^2}{x_o^2}\right) = \frac{1}{2}kx^2 + \left(\frac{1}{2}kx_o^2 - \frac{1}{2}kx^2\right)$$

$$= \frac{1}{2}kx^2 + \frac{1}{2}kx_o^2 - \frac{1}{2}kx^2 \quad \text{or} \quad \boxed{E_{total} = \frac{1}{2}kx_o^2} \quad \text{..... (12)}$$

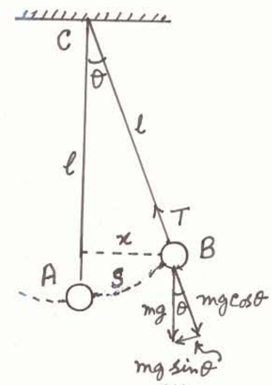
From equations (7), (10) & (12), we see
The energy oscillates back and forth between kinetic energy and potential energy but total energy of the mass remains constant everywhere.

Simple Pendulum

Definition:

"A **simple pendulum** consists of a single isolated particle suspended from a frictionless support by a light, inextensible string".

When a simple pendulum is disturbed from its mean position, it performs a vibratory motion.



To show

The motion of the bob is simple harmonic.

Let the bob is at position B during its vibratory motion.

Two forces are acting on the bob.

i) Weight mg of the bob in vertically downward direction.

ii) Tension T acting along the string.

mg is resolved into two components

Component of mg along the string = mg cos θ ..... (1)

Since there is no motion along the string, so

$$T = mg\cos\theta$$

Component of mg perpendicular to the string = mg sin θ ..... (2)

We have

$$F = m\,a \qquad \ldots\ldots (3)$$

The component (mg sin θ) is responsible for the motion, directed towards the mean position, so from equations (2) & (3), we get

$$ma = -mg\sin\theta$$

$$\text{or} \quad a = -g\sin\theta$$

We suppose that angle θ is very small,

so $\sin\theta = \theta$

$$\text{and} \quad a = -g\theta = -g\frac{x}{\ell}$$

$$\text{or} \quad a = -\frac{g}{\ell}x \qquad \ldots\ldots (4)$$

$$\text{or} \quad a \propto -x \qquad \ldots\ldots (5)$$

$S = r\theta$

or $x = \ell\theta$

or $\theta = x/\ell$

Taylor's series expansion for,

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \ldots\ldots\ldots\ldots$$

Equation (5) show that the acceleration is proportional to the displacement and directed towards the mean position, so the motion of the bob of simple pendulum execute Simple Harmonic Motion.

To calculate **time period** of simple pendulum,
We have from general expression of SHM,

$$a = -\omega^2 x \qquad \ldots\ldots \quad (6)$$

comparing equations (5) & (6), we get

$$\omega^2 = \frac{g}{\ell}$$

$$\text{or} \quad \omega = \sqrt{\frac{g}{\ell}} \qquad \ldots\ldots \quad (7)$$

We have time period from general expression of SHM

$$T = \frac{2\pi}{\omega} \qquad \ldots\ldots \quad (8)$$

From equations (7) & (8), we get

$$T = \frac{2\pi}{\sqrt{g/\ell}}$$

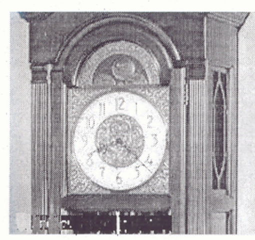
$$\text{or} \quad T = 2\pi\sqrt{\frac{\ell}{g}} \qquad \ldots\ldots \quad (9)$$

equation (9) shows that the time period T of simple pendulum,

i) is independent of the mass
ii) depends upon the length ℓ
iii) depends on the value of g

By determining T and ℓ we can accurately measure the value of g at certain place.

The most basic type of pendulum is the *simple pendulum.* It oscillates back and forth in a single plane, all the mass of the device can be considered to reside entirely in the suspended object. The motion of pendulums such as those in clocks closely approximates the motion of a simple pendulum. A *spherical pendulum* is not confined to a single plane, and as a result its motion can be much more complicated than the motion of a simple pendulum.



The principle of the pendulum was discovered by Italian physicist and astronomer Galileo, who established that the period for the back-and-forth oscillation of a pendulum of a given length remains the same, no matter how large its arc, or amplitude. (If the amplitude is too large, however, the period of the pendulum is dependent on the amplitude.) This phenomenon is called *isochronism.* Because of the role played by gravity, however, the period of a pendulum is related to geographical location. For example, the period will be greater on a mountain than at sea level. Thus, the pendulum can be used to determine accurately the local acceleration of gravity.

Speed of Sound in Air

Sound: "The series of disturbance in matter to which the human ear is sensitive".

The speed of sound waves depends upon the density, ρ of the medium.
Also it depends upon the elasticity, E of the medium.
Following is
Newton's formula for the velocity of sound in air.
"Speed of sound is directly proportional to the square root of the elasticity and inversely proportional to the square root of the density of the medium".

Mathematically, $$v = \sqrt{\frac{E}{\rho}} \quad \ldots\ldots (1)$$

Newton assumed that sound waves travel through gases in such a condition that there is no change in temperature (isothermal).

To prove:

Elasticity of volume E is equal to pressure P.
Consider the volume V of the air at a pressure P

Isothermal Process:
The process in which the temperature of the system remains constant.

For constant temperature, if we increase pressure from P to P + ΔP, the volume will decrease from V to V - ΔV, we have from Boyle's Law,

$$P_1 V_1 = P_2 V_2$$

or $$PV = (P + \Delta P)(V - \Delta V)$$

$$PV = PV - P\Delta V + V\Delta P - \Delta P\Delta V$$

Neglecting $\Delta P\Delta V$ as $\Delta P\Delta V << P \;\&\; V$,

we get $P\Delta V = V\Delta P$ or

or $$P = \Delta P\frac{V}{\Delta V} = \frac{\Delta P}{\Delta V/V} = \frac{\text{stress}}{\text{strain}} = E$$

or $$P = E \quad \ldots\ldots (2)$$

from equations (1) & (2), we have

$$v = \sqrt{\frac{P}{\rho}} \quad \ldots\ldots (3)$$

Boyle's Law:
The volume of a given mass of a gas is inversely proportional to the pressure, if the temperature is kept constant.

$$P \propto \frac{1}{V} \quad \text{or} \quad PV = \text{constant}$$

Elasticity (E) :
The property of a material body to regain its original condition, on the removal of deforming forces.

Stress:
The distorting force per unit area set up inside the body.

Strain:
The change produced in the dimensions of a body under a system of forces.

There is difference of 16 % in the theoretical value of velocity of sound in air determined from the above formula and the experimental value.

Laplace's Correction

To account for the difference, Laplace pointed out that the compressions and rarefactions occur so rapidly that heat of compressions remains confined to the region where it is generated and does not have time to flow to the neighbouring cooler regions, which have undergone an expansion. So temperature of the medium does not remain constant. In such case Boyles's law takes the form

$$PV^{\gamma} = \text{Constant} \quad \ldots\ldots (4)$$

If we increase pressure from P to P + ΔP, the volume will decrease from V to V - ΔV, so we have

$$PV^{\gamma} = (P+\Delta P)(V-\Delta V)^{\gamma}$$

$$\text{or} \quad PV^{\gamma} = (P+\Delta P)\left\{\frac{(V-\Delta V)V}{V}\right\}^{\gamma}$$

$$\text{or} \quad PV^{\gamma} = (P+\Delta P)\left\{1-\frac{\Delta V}{V}\right\}^{\gamma} V^{\gamma}$$

$$\text{or} \quad P = (P+\Delta P)\left\{1-\frac{\Delta V}{V}\right\}^{\gamma}$$

From Binomial theorem we get

$$P = (P+\Delta P)\left(1-\gamma\frac{\Delta V}{V}-\frac{\gamma(\gamma-1)}{1\cdot 2}\frac{(\Delta V)^2}{V^2}-\ldots\ldots\right)$$

Neglecting squares and higher powers of (ΔV / V) as ΔV <<V, we get

$$P = (P+\Delta P)\left(1-\gamma\frac{\Delta V}{V}\right)$$

$$\text{or} \quad P = P - P\gamma\frac{\Delta V}{V} + \Delta P - \Delta P\gamma\frac{\Delta V}{V}$$

Neglecting $\Delta P\gamma\frac{\Delta V}{V}$ as ΔPΔV << P & V,

$$\Delta P = P\gamma\frac{\Delta V}{V}$$

$$\text{or} \quad \gamma P = \frac{\Delta P}{\Delta V/V} = \frac{\text{stress}}{\text{strain}} = E \quad \ldots\ldots (5)$$

From equations (1) & (5) we get

$$v = \sqrt{\frac{\gamma P}{\rho}} \quad \ldots\ldots (6)$$

which is Laplace's modified expression for the speed of sound.

Adiabatic process:
Process in which no heat flows into or out of the system.

Specific heat at constant pressure, C_P
It is the amount of heat energy required to raise the temperature of one mole of a gas through 1^0 K at constant pressure.

Specific heat at constant volume, C_V
It is the amount of heat energy required to raise the temperature of one mole of a gas through 1^0 K at constant volume.

We define: $\gamma = \frac{C_P}{C_V}$

To prove: PV^{γ} = constant

If we have one mole of a gas, then for adiabatic process, we have

$Q_P = n C_V \Delta T + P \Delta V \quad \ldots\ldots (1)$

For small change per unit volume,

$dQ = C_V dT + P dV$

for adiabatic change, we have

$dQ = 0 = C_V dT + P dV$

or $C_V dT + P dV = 0 \quad \ldots\ldots (2)$

Now we have for one mole,

$PV = RT$

Differentiating it, we get

$P dV + V dP = R dT$

$$\text{or} \quad dT = \frac{PdV + VdP}{R} = \frac{PdV + VdP}{C_P - C_V} \quad ..(3)$$

From equations (2) & (3), we have

$$C_V\left(\frac{PdV + VdP}{C_P - C_V}\right) + PdV = 0$$

$$C_V PdV + C_V VdP + C_P PdV - C_V PdV = 0$$

$$\text{or} \quad C_V VdP + C_P PdV = 0$$

$$\text{or} \quad VdP + \frac{C_P}{C_V} PdV = 0$$

put $C_P/C_V = \gamma$, so $VdP + \gamma PdV = 0$

Dividing throughout by PV,

$$\frac{dP}{P} + \gamma\frac{dV}{V} = 0$$

Integrating, $\log P + \gamma \log V$ = constant

or $\log(PV)$ = constant

Taking antilog, PV^{γ} = another constant

So PV^{γ} = constant

If we put the values in the above formula, the theoretical value agrees with the experimental value. So Laplace's correction is correct.

Effect of Pressure

As density is proportional to the pressure, so the speed of sound is not affected by a variation in the pressure of a gas.

Effect of Density

At the same temperature, pressure & γ, from equation (6) we have

$v \propto \frac{1}{\sqrt{\rho}}$ e.g. $\rho_{oxygen} = 16 \times \rho_{hydrogen}$, so v_{oxygen} is $4 \times v_{hydrogen}$

Effect of Temperature

We have $v = \sqrt{\frac{\gamma P}{\rho}} \Rightarrow v_o = \sqrt{\frac{\gamma P}{\rho_o}}$ & $v_t = \sqrt{\frac{\gamma P}{\rho_t}}$

or $$\frac{v_t}{v_o} = \frac{\sqrt{\gamma P / \rho_t}}{\sqrt{\gamma P / \rho_o}} = \sqrt{\frac{\rho_o}{\rho_t}} \quad \ldots\ldots (1)$$

We have

$$V_t = V_o(1+\beta t)$$

$\beta = \frac{\Delta V}{V_o \Delta T} = \frac{V_t - V_o}{V_o(t-0)}$

or $\beta = \frac{V_t - V_o}{V_o t}$ or $\beta V_o t = V_t - V_o$

or $V_t = V_o + \beta V_o t$ or $V_t = V_o(1+\beta t)$

As coefficient of volume expansion,

$\beta = 1/273$ for all gases,

$$V_t = V_o\left(1+\frac{t}{273}\right) \quad \left[\rho = \frac{m}{V} \text{ or } V = \frac{m}{\rho}\right]$$

or $\frac{m}{\rho_t} = \frac{m}{\rho_o}\left(1+\frac{t}{273}\right)$ or $\rho_o = \rho_t\left(1+\frac{t}{273}\right)$

or $$\frac{\rho_o}{\rho_t} = 1+\frac{t}{273} \quad \ldots\ldots (2)$$

from equations (1) & (2) we have

$$\frac{v_t}{v_o} = \sqrt{1+\frac{t}{273}} \quad \ldots\ldots (3)$$

or $$\frac{v_t}{v_o} = \sqrt{\frac{273+t}{273}} = \sqrt{\frac{T}{T_o}} \Rightarrow \boxed{v = \sqrt{T}} \quad \ldots\ldots (4)$$

i.e. speed of sound varies directly as the square root of absolute temperature.

From equation (3) we have

$$\frac{v_t}{v_o} = \left(1+\frac{t}{273}\right)^{1/2}$$

Binomial Theorem:

$$(1+x)^n = 1+\frac{n}{1}\cdot x + \frac{n(n-1)}{1\cdot 2}\cdot x^2 + \frac{n(n-1)(n-2)}{1\cdot 2\cdot 3}\cdot x^3 + \ldots\ldots$$

Expanding R.H.S. by applying Binomial theorem, we get

$$\frac{v_t}{v_o} = 1+\frac{1}{2}\times\frac{t}{273} + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{1\cdot 2}\left(\frac{t}{273}\right)^2 + \ldots.$$

Neglecting higher powers, we get

$$\frac{v_t}{v_o} = 1+\frac{t}{546} \quad \text{or} \quad v_t = v_o\left(1+\frac{t}{546}\right) = v_o + \frac{v_o t}{546} \quad \left[v_o = 332 \text{ m s}^{-1}\right]$$

or $$v_t = v_o + \frac{332\times t}{546} \quad \text{or} \quad \boxed{v_t = v_o + 0.61\times t}$$

Stationary Waves in a Stretched String

To make

General formulas for wavelength λ and frequency f of transverse stationary waves. Consider a string of length ℓ which is kept stretched at two ends so that tension in string is T.

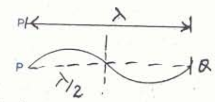


Fig.(a)

In fig. (b), string is plucked at the middle, the string vibrates in one loop, with a frequency, say f_1 , so

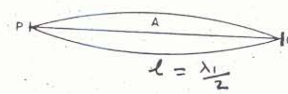


Fig.(b)

$\ell = \frac{\lambda_1}{2}$ and $v = f_1\lambda_1$

or $\lambda_1 = 2\ell$, $v = f_1 \times 2\ell$

or $\lambda_1 = \frac{2\ell}{1}$.....(1) , $f_1 = \frac{v}{2\ell}$......(2)

In fig. (c), string is plucked from one quarter, then it vibrates in two loops, with a frequency, say f_2 , so

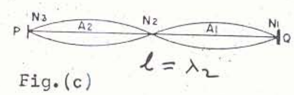


Fig.(c)

$\ell = \lambda_2$ and $v = f_2\lambda_2 = f_2\ell$

or $\lambda_2 = \ell$, $f_2 = \frac{v}{\ell} = 2\frac{v}{2\ell}$

$\lambda_2 = \frac{2\ell}{2}$.....(3) , $f_2 = 2f_1$......(4)

In fig. (d), string is plucked in such a way that it vibrates in three loops, with a frequency, say f_3 , so

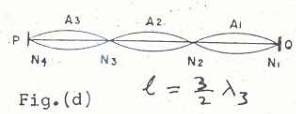


Fig.(d)

$\ell = \frac{3}{2}\lambda_3$ and $v = f_3\lambda_3 = f_3\frac{2\ell}{3}$

or $\lambda_3 = \frac{2\ell}{3}$.......(5) , $f_3 = 3\frac{v}{2\ell} = 3f_1$.....(6)

Generalizing equations (1), (3), (5) and (2), (4), (6) we get

or $\lambda_n = \frac{2}{n}\ell$ .....(7) and $f_n = nf_1$.......(8)

Now if m' is the total mass of the string, tension F and length ℓ, then speed v is

$$v = \sqrt{\frac{F \times \ell}{m'}} = \sqrt{\frac{F}{m'/\ell}}$$

when m is mass per unit length $= \frac{m'}{\ell}$

so $v = \sqrt{\frac{F}{m}}$ ..... (9)

from equations (2) & (9) we get

$$f_1 = \frac{1}{2\ell}\sqrt{\frac{F}{m}} \quad ..... (10)$$

$v^2 = v \times v$

or $v^2 = v \times \frac{\ell}{t}$

$= \frac{v}{t} \times \ell$

or $v^2 = a \times \ell$

or $v^2 = \frac{F \times \ell}{m}$

or $v = \sqrt{\frac{F \times \ell}{m}}$

S=vt

$v = \frac{S}{t} = \frac{\ell}{t}$

$a = \frac{v}{t}$

$F = ma$

or $a = \frac{F}{m}$

From equation (8) we see that the stationary waves have a discrete set of frequencies f_1 , $2f_1$, $3f_1$, …..nf_1 which is known as harmonic series. The fundamental frequency f_1 corresponds to the first harmonic, f_2 corresponds to second harmonic and so on. The stationary waves can be set up on the string with the frequencies of harmonic series determined by the tension, length and mass per unit length of the string.

Stationary Waves

1) Two waves of equal frequency traveling in opposite direction..
2) The resultant of two waves of the same wavelength, frequency and amplitude traveling in opposite directions through the same medium.
3) Waves apparently standing still resulting from two similar wave trains traveling in opposite directions.

Node: A point of no disturbance of a stationary wave.

Antinode: A point which oscillate with the maximum amplitude in stationary waves.

Production of Nodes and Antinodes:

The points on the cord which do not oscillate at all are called nodes, and the point which oscillate with the maximum amplitude are called antinodes. The distance between two nodes or antinodes is always equal to $\frac{1}{2} \lambda$.

Reason of production:

Stationary waves are set up as a result of super-position of two exactly similar waves moving along the same line but in opposite directions.

Condition for production:

The phenomenon of stationary waves takes place in any medium wherever its particles are simultaneously agitated by two similar waves moving along the same line in opposite directions.

Detection:

The presence of stationary waves in a medium can be easily detected by the fact that the particles at the nodal points will be at rest and the particles at the antinodes will be vibrating quite strongly.

Quantization of frequencies:

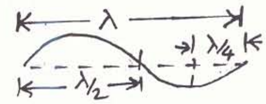
In any medium stationary waves of all frequencies cannot be set up. The waves having a discrete set of frequencies only, can be set up in the medium.

Energy:

Energy in a wave moves because of the motion of the particles of the medium. At nodes PE & KE alternates. At antinodes PE is stored for extreme positions and KE for their equilibrium (mean) positions.

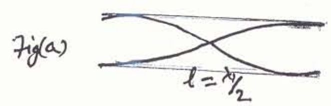
Stationary Waves in Air Columns

I—A pipe open at both ends



From the fig. (a) we have

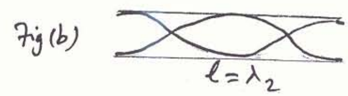
$\ell = \frac{\lambda_1}{2}$ and $v = f_1\lambda_1$



or $\lambda_1 = 2\ell$, $v = f_1 \times 2\ell$

or $\lambda_1 = \frac{2\ell}{1}$.....(1) , or $f_1 = \frac{v}{2\ell}$......(2)

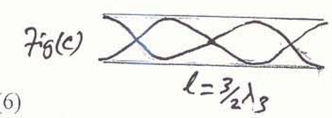
In fig. (b), we have $\ell = \lambda_2$ and $v = f_2\lambda_2$



or $\lambda_2 = \ell$, $f_2 = \frac{v}{\lambda_2} = \frac{v}{\ell}$

$\lambda_2 = \frac{2\ell}{2}$.....(3) , $f_2 = 2\frac{v}{2\ell} = 2f_1$......(4)

In fig. (c), we have $\ell = \frac{3}{2}\lambda_3$ and $v = f_3\lambda_3$



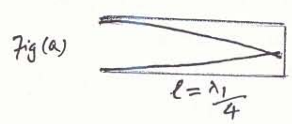
or $\lambda_3 = \frac{2\ell}{3}$.......(5) , $f_3 = \frac{v}{\lambda_3} = 3\frac{v}{2\ell} = 3f_1$.....(6)

Generalizing the equations (1), (3), (5) and (2), (4), (6) give,

or $\lambda_n = \frac{2}{n}\ell$ .....(7) and $f_n = nf_1$.......(8) where n = 1, 2, 3, 4, 5, ………

II—A pipe closed at one end & open at the other

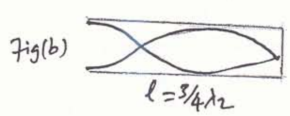
From the fig. (a), $\ell = \frac{\lambda_1}{4}$ and $v = f_1\lambda_1$



or $\lambda_1 = 4\ell$, $v = f_1 \times 4\ell$

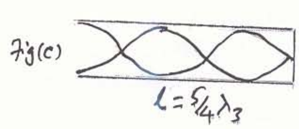
or $\lambda_1 = \frac{4\ell}{1}$.....(1) , or $f_1 = \frac{v}{4\ell}$......(2)

In fig. (b), $\ell = \frac{3}{4}\lambda_2$ and $v = f_2\lambda_2$



or $\lambda_2 = \frac{4\ell}{3}$ ......(3) , $f_2 = \frac{v}{\lambda_2} = \frac{v}{4\ell} \times 3$......(4)

In fig. (c), $\ell = \frac{5}{4}\lambda_3$ and $v = f_3\lambda_3$



or $\lambda_3 = \frac{4\ell}{5}$.......(5) , $f_3 = \frac{v}{\lambda_3} = \frac{v}{4\ell} \times 5$.....(6)

Generalizing the equations (1), (3), (5) and (2), (4), (6) give,

or $\lambda_n = \frac{4\ell}{n}$ .....(7) and $f_n = n\frac{v}{4\ell}$.......(8) where n = 1, 3, 5, 7, ……..

Equation (8) shows that in the pipe closed at one end, only odd harmonics are generated. And equation (8) of Section I show that the pipe, which is open at both ends, is rich in harmonics.

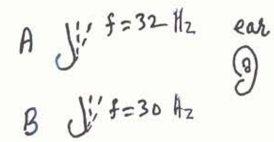
Beats

Definition: 1) The condition whereby two sound waves form an outburst of sound followed by an interval of comparative silence.

2) The periodic alternations of sound between maximum and minimum loudness.

Explanation: If two sources of nearly equal frequencies are sounded at the same time, then only a single note is heard. This note rises and falls in loudness alternately.

Illustration: Consider two tuning forks, having frequencies 30 and 32, be sounded together and placed upon a table. Suppose at a certain instant, at $t = ½$ seconds, fork A completes 16 vibrations and fork B 15 vibrations. The right hand prongs of both the forks just start moving to the right sending out compressions. These compressions arrive at the ear together and a loud sound is heard. During this interval one beat is heard.

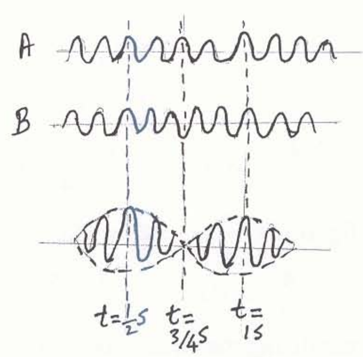


After $t = ¾$ seconds, the fork A completes 24 vibrations and fork B 22 ½ vibrations. Again compression from A and the rarefaction from B cancel each other and no sound is heard. After $t = 1$ second, the fork A completes 32 vibrations and fork B 30 vibrations. Both these forks will be sending compressions and again a loud sound will be heard.
During this interval another beat is heard. So the total number of beats heard is 2, which is also equal to the frequency difference of two forks.

Conclusion: ***The number of beats per second is equal to the difference between the frequencies of the tuning forks.***

Displacement curves:

From the principle of superposition, the resultant displacement of any particle will be the sum of the displacements due to each of the two waves. The resultant wave which is produced from two waves A and B are shown in fig. The variations of amplitude give rise to variations of loudness which is called beats.



Applications of Beats:

1. The phenomenon of beats is used in finding the unknown frequencies.
2. It is used in tuning the musical instruments, e.g. pianos, organs.
3. Presence of dangerous gases in mines is sometimes detected by means of beats.
4. It is also made use in the Heterodyne method of radio reception.

[Heterodyne: Having alternating currents of two different frequencies that are combined to produce two new frequencies the sum and difference of the original frequencies, either of which may be used in radio or TV receivers.]

DOPPLER EFFECT

Statement

The change in the frequency of the waves caused b the relative motion of either the source of waves or the observer.

Explanation

When an observer is standing on a railway platform the pitch (frequency heard) of the whistle of an approaching locomotive is heard to be higher. But when the same vehicle moves away, the pitch of the whistle becomes lower.

Illustration

Consider this effect under the following cases.

Case 0: **Both source and the observer are stationary**

Here the waves received by the observer in one second are

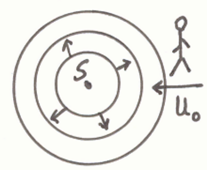
$$f = \frac{v}{\lambda}$$

1. **Observer is moving towards the stationary source**

An observer A moves with velocity = u_o

Then the relative velocity = $v + u_o$

The number of waves received in one second, f_A will be



$$f_A = \frac{v + u_o}{\lambda} \qquad \left[\lambda = \frac{v}{f}\right]$$

or

$$\boxed{f_A = f\left(\frac{v + u_o}{v}\right)}$$

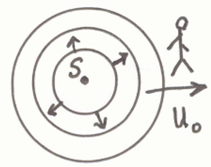
As $f_A > f$, therefore the frequency heard / observed by the observer will ncrease.

2. **Observer is receding from the stationary source**

An observer B moving away with velocity = u_o

Then the relative velocity = $v - u_o$

The number of waves received in one second, f_A will be



$$f_B = \frac{v - u_o}{\lambda} \qquad \left[\lambda = \frac{v}{f}\right]$$

or

$$\boxed{f_B = f\left(\frac{v - u_o}{v}\right)}$$

As $f_B < f$, therefore the frequency heard / observed by the observer will be reduced.

3. **Source is moving towards the stationary observer**

The waves are compressed by an amount $\Delta\lambda$ (Doppler shift).

As they are contained in a shorter space, there will be decrease in the wavelength. So

The wavelength for observer C will be = $\lambda_C = \lambda - \Delta\lambda$

or $\lambda_C = \left(\frac{v}{f} - \frac{u_S}{f}\right) = \left(\frac{v - u_S}{f}\right)$

or $f_C = \frac{v}{\lambda_C} = \frac{v}{\left(v - u_S/f\right)}$ or $f_C = \left(\frac{v}{v - u_S}\right) f$

$f = \frac{v}{\lambda}$ or $\lambda = \frac{v}{f}$

& $\Delta\lambda = \frac{u_S}{f}$

As $f_C > f$, therefore the frequency heard / observed by the observer will ncrease.

3. **Source is moving away from the stationary observer**

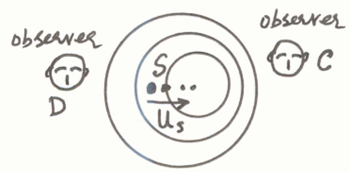
The waves will an increase in the wavelength

for observer D = $\lambda_D = \lambda + \Delta\lambda$

or $\lambda_D = \left(\frac{v}{f} + \frac{u_S}{f}\right) = \left(\frac{v + u_S}{f}\right)$

or $f_D = \frac{v}{\lambda_D} = \frac{v}{\left(v + u_S/f\right)}$

or $f_C = \left(\frac{v}{v - u_S}\right) f$



As $f_D < f$, therefore the frequency heard / observed by the observer will be decreased.

Applications of Doppler's effect

1. Applied to light:
The frequency of light from certain stars is found to be slightly more and from other stars slightly less than the frequency of the same light emitted from the source on earth. Their velocities can be obtained from this frequency difference.
2. Ultrasonic waves from a bat:
A bat determines the location and nature of objects by sending ultrasonic waves.
3. Reflection of radar waves:
The frequency of the reflected radar waves is decreased if the plane is moving away and increased if it is approaching. From the observed frequency difference the speed and direction of the plane can be calculated.
4. Detection of submarines:
When under-water sound waves (sonar) are reflected from a moving submarine, we can detect its location.
5. Velocities of earth satellites:
These velocities are determined from the Doppler shift in the frequency of their transmitted waves.
6. Radar speed trap:
Microwaves are emitted from a transmitter in short bursts. Each burst is reflected off by any car in the path. By measuring Doppler shift, the speed of car is calculated by computer program.

[Doppler shift: Apparent change in frequency due to relative motion of source and observer.]

Young's Double Slit Experiment

Interference:

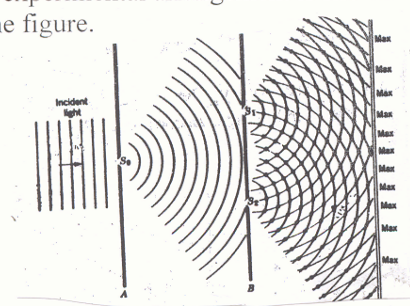
"The phenomenon in which the two waves support each other at some points and cancel at other".

To obtain interference of light waves, the following conditions must be fulfilled.

i) The beams of light must be monochromatic.
ii) The interfering beams must be phase coherent.
iii) Linear superposition should be applicable.

Young's double-slit experiment gives the experimental evidence for Huygen's wave theory of light.

The experimental arrangement is shown in the figure.



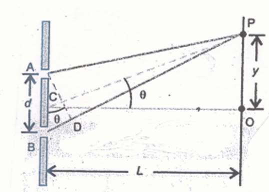
A screen A with slit S_0 is placed in front of a monochromatic source of light.

Phase coherence: Producing of two waves of same wavelength and time period at the same instant.
Phase: The phase of a vibrating particle at any instant is its state or condition as regards its position and direction of motion with respect to the mean position.
Coherent sources: Sources which are emitting light waves continuously of the same wavelength, time period and amplitude. They must maintain a constant phase difference between them.
Monochromatic: Light consisting of only one colour.
Superposition: Combining the displacements of two or more wave motions algebraically to produce a resultant wave motion.
Principle of linear superposition: When two waves act upon a body imultaneously they pass each other without disturbing each other, and act upon the particles of the quite independent of each other, and their resultant displacement is the resultant of all individual waves.

The wave fronts emerge on the other side of screen A. These wave fronts arrive at screen B, which has two slits S_1 and S_2. These two slits behave as coherent sources. These wave fronts produce interference. The resulting interference pattern is obtained on the screen, consisting of alternate bright and dark parallel bands called fringes.

To derive

The equation for maxima and minima, looking in the figure.



Consider a point P on one side of the central point O on the screen. AP and BP are the paths of the rays reaching P.

The path difference is

$$\Delta S = BP - AP = BD \qquad \ldots\ldots (1)$$

The separation between the centers of the two slits is $AB = d$

Distance of the screen from the slits is $CO = L$

AD is drawn perpendicular to BP.
Since $L >> d$, so $AP \cong DP$
For P to be bright fringe, i.e., for constructive interference,
$BD = \Delta S = m\lambda$, $m = 0, 1, 2, \ldots$
since $BD = d\sin\theta$ *

So $d\sin\theta = m\lambda$, ... (2)

And for dark fringes,

$$d\sin\theta = \left(m + \frac{1}{2}\right)\lambda \qquad m = 0, 1, 2, 3, \ldots$$

Now from the figure,

$$\tan\theta = \frac{y}{L} \quad \text{or} \quad y = L\tan\theta$$

or $y = L\sin\theta$

from equation (2), we have

$$\sin\theta = m\frac{\lambda}{d}$$

Constructive interference:
The interference of two waves, so that they reinforce one another. Its condition is
Path difference = $\Delta S = m\lambda$

Destructive interference:
The interference of two waves, so that they cancel one another. Its condition is:

$$\Delta S = \left(m + \frac{1}{2}\right)\lambda$$

*Angle θ between any two lines is equal to the angle between their perpendiculars.

Since $y << L$ so $OC \cong PC$
or $\tan\theta \cong \sin\theta$

so $y = m\frac{\lambda L}{d}$ ..... (3) or $\lambda = \frac{yd}{mL}$ ..... (4)

from equation (4), we can calculate λ.

Equation (3) is: Position of m[th] bright fringe = $y_m = m\frac{\lambda L}{d}$

And for (m + 1)th bright fringe: $y_{m+1} = (m+1)\frac{\lambda L}{d}$

And fringe width = $\Delta y = y_{m+1} - y_m = (m+1)\frac{\lambda L}{d} - m\frac{\lambda L}{d}$

$$\text{or} \quad \Delta y = \frac{\lambda L}{d} \qquad \ldots\ldots (5)$$

From the above equation, the wavelength λ can be determined, if other factors are known.

Also equation (5) shows that fringe spacing increases if red light (long wavelength) is used instead of blue light (short wavelength).

Young summed up his work, in 1807, "the middle of the pattern is always light, and the bright stripes on each side are at such distances that the light coming to them from one of the apertures must have passed through a longer space than that which comes from the other by an interval which is equal to the breadth of one, two, three, or more of the wavelengths."

The trouble with this understanding of light emerged at the beginning of the 20th century, when Max Planck and then Albert Einstein showed that light could be treated—as if it were a stream of little particles.
The way particles pass through two holes in a wall is very different, in the everyday commonsense world, from the way waves behave. If you stood on one side of a wall in which there were two holes, and threw stones (or tennis balls) in the general direction of the wall, some would go through each of the holes. You certainly would not get these balls, or rocks, halfway between the two holes in the wall.

The discovery that light can behave like a wave or like a particle is an example of wave-particle duality.

Michelson's Interferometer

Definition:

Michelson's interferometer is an instrument that can be used to measure distance with extremely high precision.

Explanation:

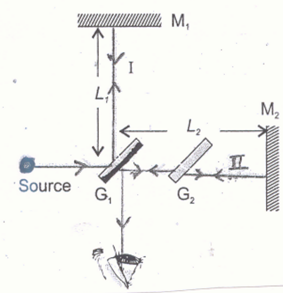
Device includes one half silvered mirror and two plane mirrors, using interference of light waves to measure very small distances.

It splits a light beam into parts and then recombines them to form an interference pattern. It is used for accurate measurement of wavelength.

Experimental arrangement:

In the figure, monochromatic beam of light is split into two rays through half silvered mirror G_1. One ray is reflected towards M_1 and second ray is transmitted through G_1 towards mirror M_2.

After reflecting from mirrors M_1 and M_2, the two rays recombine to produce interference, seen through a telescope by an observer. The glass plate G_2 is placed to compensate the path length.



In **practical interferometer**, the mirror M_1 can be moved along the direction perpendicular to its surface by means of a precision screw. If M_1 is displaced through a distance of $\lambda/4$, the path difference changes by $\lambda/2$. Then destructive interference giving rise a dark fringe. When M_1 is moved further $\lambda/4$, the total distance covered is λ, a bright fringe will appear. Thus we see successive bright and dark fringes, as the mirror M_1 moved a distance $\lambda/4$.

A fringe is shifted, each time the mirror is displaced through $\lambda/2$. Hence counting the number m of the fringes, which are shifted by the displacement L of the mirror, we can write the equation,

$$L = m\frac{\lambda}{2}$$

This interference is used to make very accurate measurements.

Michelson measured the length of standard meter in terms of the wavelength of red cadmium light and showed that the standard meter was equivalent to 1,553,163.5 wavelengths of this light.

Simple Microscope

Definitions

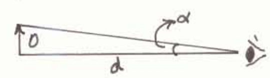
Simple Microscope: *An ordinary convex lens held close to the eye is called simple microscope or magnifying glass.*

Least distance of distinct vision (d): *The minimum distance from the eye (equal to 25 cm) for a normal person at which an object appears to be distinct.*

Magnifying Power (or Angular Magnification) M:
The ratio of the angles subtended by the image as seen through the optical device to that subtended by the object at the unaided eye.

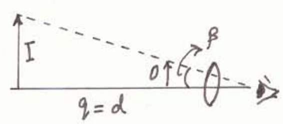
Mathematically, $M = \frac{\beta}{\alpha}$ ..... (1)

From the figures we have,



$$\tan\alpha = \alpha = \frac{\text{Object size}}{\text{Object distance}} = \frac{O}{d} \quad ..... (2)$$

$$\& \quad \tan\beta = \beta = \frac{\text{Image size}}{\text{Image distance}} = \frac{I}{q} = \frac{I}{d} \quad ..... (3)$$



We have from equations (1), (2) & (3)

$$M = \frac{I/d}{O/d} = \frac{I}{O} \quad ..... (4)$$

In figure (b), as the two triangles are similar, so the ratio of the corresponding sides should be equal,

$$\frac{I}{O} = \frac{d}{p} \quad ..... (5)$$

From equations (4) & (5) we have

$$M = \frac{d}{p} \quad ..... (6)$$

Now from the lens formula, [image is virtual]

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{-q} = \frac{1}{p} - \frac{1}{d} \quad [q = d] \ ..... (7)$$

Taylor's series for,

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - ....$$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - ....$$

for small θ,

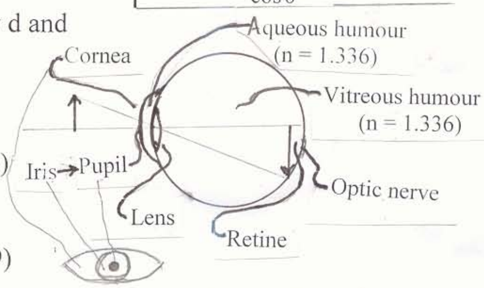
$\sin\theta = \theta$ & $\cos\theta = 1$

so $\tan\theta = \frac{\sin\theta}{\cos\theta} = \theta$

Multiplying both side of eq.(7) by d and simplifying will give

$$\frac{d}{f} = \frac{d}{p} - \frac{d}{d} = \frac{d}{p} - 1$$

$$\text{or} \quad \frac{d}{p} = 1 + \frac{d}{f} \quad ..... (8)$$



From equations (6) & (8) we get

$$M = 1 + \frac{d}{f} \quad ..... (9)$$

The above equation shows that magnifying power of a magnifying glass is inversely proportional to f. Lesser the focal length, greater will be its magnification.

Compound Microscope

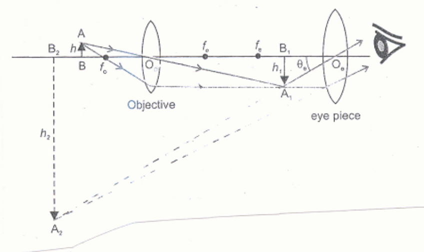
Compound Microscope is a device used to produce a very large magnification of very small objects. It consists of an objective and an eyepiece.

Construction:

It consists of two convex lenses, an object lens of very short focal length and an eyepiece of comparatively longer focal length. The ray diagram of a compound microscope is given in the figure.

Working:

The object AB of height h forms a real, inverted and enlarged image h_1 of the object placed within focal length of eyepiece. The eyepiece is used as a magnifying glass to see the final image h_2 at least distance of distinct vision, d. It is virtual and very much enlarged.



Magnifying Power:

We define:

$$\text{Magnifying power} = \frac{\text{angle formed by final image}}{\text{angle formed by unadided eye}}$$

$$\text{or} \quad M = \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha} = \frac{\tan\theta_e}{\tan\theta} \qquad \text{or} \quad M = \frac{h_2/d}{h/d} = \frac{h_2}{h}$$

multiplying and dividing by h_1, we have

$$M = \frac{h_2}{h} \times \frac{h_1}{h_1} = \frac{h_1}{h} \times \frac{h_2}{h_1} \quad \text{or} \quad M = M_1 M_2 \; ..... (1) \qquad \left[\frac{h_1}{h} = M_1 \quad \& \quad \frac{h_2}{h_1} = M_2\right]$$

Now in the figure, triangles A_1OB_1 and AOB are similar, so

$$\frac{A_1B_1}{AB} = \frac{B_1O}{BO} \quad \text{or} \quad \frac{A_1B_1}{AB} = \frac{h_1}{h} = M_1 = \frac{q}{p} \qquad ..... (2)$$

which is the magnification produced by the objective.

Now magnification produced by the eyepiece,

$$M_2 = \frac{h_2}{h_1} = 1 + \frac{d}{f_e} \quad ..... (3) \qquad \left[M = 1 + \frac{d}{f}\right]$$

so from equations (1), (2) & (3) we have $\quad M = \frac{q}{p}\left(1 + \frac{d}{f_e}\right), \quad f_o < f_e$

usually the object h lies very close to the focal length f_o, so $\quad f_o = p$ and image h_1 lies very close to the eyepiece and image distance q is approximately equal to length L of microscope tube, so $q = L$

Hence, we get $\quad M = \frac{L}{f_o}\left(1 + \frac{d}{f_e}\right), \quad f_o < f_e$

which is required formula for magnification of compound microscope. We see that for high magnification the objective and eyepiece should be of short focal length. However $f_o < f_e$.

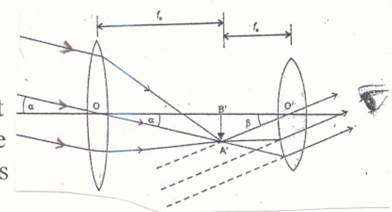
Astronomical Telescope

Definition:

It is a telescope used to see heavenly bodies; it consists of two convex lenses, one for objective and the other as an eyepiece.

Details:

To see distant objects (e.g. distant galaxies) more amount of light is needed. So objective lens used in a telescope is of large focal length with large aperture. It is a convex lens. The eyepiece is also a convex lens. It has short focal length and small aperture. The objective is mounted at one end of a tube and eyepiece is mounted in a small tube to slide inside the bigger tube of the objective.



Working:

The objective form a real, inverted and diminished image at its focus $B^{/}$ of a distant object, in front of eyepiece.

The distance between the eyepiece and this image is adjusted within the focal length so that a magnified and virtual image is formed at the least distance of distinct vision. If the image $A^{/}B^{/}$ is made at the focus of eyepiece then the final image is formed at infinity. It is called the telescope is focused for infinity.

Then, Length of the telescope $= f_o + f_e$

where f_o = focal length of objective & f_e = focal length of eyepiece.

Here the final image is formed inverted, which makes no difference for astronomical purposes.

Magnifying power:

Definition: *It is the ratio of the angle formed by the image at the eye as seen through the telescope to the angle formed by the object with unaided eye, the object and image both lying at infinity.*

Mathematically $M = \frac{\beta}{\alpha}$

Now for small angles,

$$\alpha = \tan\alpha = \frac{A'B'}{B'O}$$

$$\& \quad \beta = \tan\beta = \frac{A'B'}{B'O'}$$

from equations we get

$$M = \frac{A'B'/f_o}{A'B'/f_e} = \frac{A'B'}{f_e} \times \frac{f_o}{A'B'}$$

or $$M = \frac{f_o}{f_e}, \quad f_e < f_o$$

Taylor's series for,

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \ldots\ldots$$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \ldots\ldots$$

for small θ,

$\sin\theta = \theta$ & $\cos\theta = 1$

so $\tan\theta = \frac{\sin\theta}{\cos\theta} = \theta$

Optical Fibres

Definitions

Fibre optics: The use of fine transparent fibres to transmit light. The light passes along the fibres by a series of internal reflections.
Optical fibre: An optical fibre consists of a single flexible rod of high refractive index, less than 1mm in diameter, having polished surfaces coated with transparent material of lower refractive index.
Photo phone: An instrument for talking along a beam of light instead of telegraph wire; telephoning without wires by varying the intensity of a beam of light by the action of voice, and allowing the light to fall upon a piece of crystalline selenium.
Bandwidth: The upper and lower range of frequencies over which a particular characteristic of an electronic device lies within specified limits.
Core: The central part of a wire.
Cladding: A layer of lower refractive index (less intensity) over the central core of high refractive index (high density).
Mode: The method by which light is propagated within the fibre.
Repeater: A device used to amplify or regenerate signals in order to extend the transmission between two stations.

Fibre Optic Principles

Propagation of light in an optical fibre requires that the light should be totally confined within the fibre. This can be done by two phenomena.

1. Total Internal Reflection
The reflection of light at the boundary of two transparent media when the angle of incidence is greater than the critical angle.

Refractive index is a measure of the extent to which a ray of light is bent as it passes from one transparent medium to another.

2. Continuous Refraction
Continuous bending of a wave disturbance at it passes obliquely from one medium into another of different density.

Optical fibres consist of i) glass core, ii) glass cladding, & iii) jacket

Types of Optical Fibres

They are classified on the basis of the mode by which they propagate light. Such as, single- or multi-mode fibre, stepped- or graded index fibre

Step-index fibre: Here indexes of both cladding and core are constant throughout.
Graded index fibre: In it the refractive index of the core decreases radially outwards. Light rays then spiral smoothly around the central axis rather than zig zagging.
Single mode fibre: Here the core is very narrow relative to the cladding and rays travel parallel to the central axis; it may be stepped or graded index.

There are the following three types of optical fibres which are classified on the basis of mode by which they propagate light.

1. **Single (or mono) Mode Index Fibre**

   Definition:

   An optical fibre having a very thin core of about 5 μm diameter and has a relatively larger cladding of glass or plastic.

   Core diameter: 5 μm

   Light source: Laser light

   Capacity: It can carry 14 TV channels or 14000 phone calls.

2. **Multimode Step Index Fibre**

   Definition:

   An optical fibre having a core of relatively larger diameter such as 50 μm is used. The fibre core has a constant refractive index such as 1.52.

   Core diameter: 50 μm

   Light source: White light

   Useful: For short distance only

3. **Multimode Graded Index Fibre**

   Definition:

   An optical fibre in which the central core has high refractive index which gradually decreases towards its periphery.

   There is no noticeable boundary between core and cladding.

   Core diameter: 50 to 1000 μm

   Light source: White light

   Useful: For long distance applications

Applications

1. Transmission to inaccessible place:

   Optical fibres are used to transmit light around corners and could be viewed unobservable places.
2. In medicine:

   Optical fibers are widely used in medical instruments for viewing inside the human body, for laser surgery and in the diagnostic, e.g. in the bore of a dentist's drill.
3. To transmit images:

   Image transmission by optical fibers is used in facsimile systems, in phototypesetting, in computer graphics,
4. In telecommunications:

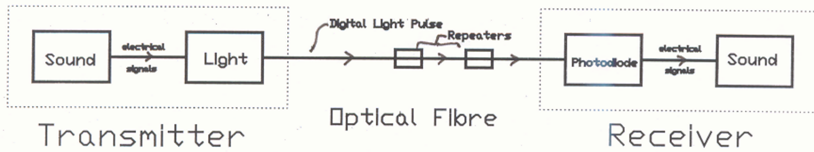
   Optical fibres have ability to transmit thousands of telephone conversations, several television programs and numerous data signals with large bandwidth between stations.
5. In sensing devices:

   It is used in a wide variety of sensing devices, ranging from thermometers to gyroscopes.

   [Gyroscope: A wheel or disc mounted so that it can spin rapidly about an axis which itself can rotate about either of two other axes perpendicular to it and to each other.]

Signal Transmission & Conversion to Sound:

A fibre optic communication system consists of three major components.



i) Transmitter

A device used in a telecommunication system to generate and propagate an electrical signal. This portion convert electrical signals to light signals.

ii) Optical Fibre

An optical fibre consists of a single flexible rod having polished surfaces coated with transparent material.

From the transmitter side , semiconductor laser or LED made digital modulation to move down the fibre.

Repeaters are used to regenerate dim signals nearly with a span of 100 km.

iii) Receiver

At the receiver side a photodiode converts light signals, then amplified and decoded.

Losses of Power:

When a light signal travels along fibres, there is power loss due to the following:

Factors:

i) Scattering

The spreading out of a beam of radiation as it passes through matter, reducing the energy moving in the original direction.

ii) Absorption

In radiation, reduction in the intensity of electromagnetic radiation, or other ionizing radiation, on passage through a medium.

iii) Dispersion

The separation of white light into its component colours.

Results:

Faulty & distorted signals are received.

Remedies:

Use graded index fibre instead step index fibre.

Efficiency:

Time difference is reduced by 1 ns per km (by using graded index fibre), instead of 33 ns per km length of fibre (if used step index fibre).

Kinetic Theory of Gases

The kinetic-molecular theory of gases is based on the following main assumptions first stated by Clausius.

1. A chemically uniform gas consists of very small identical molecules.
2. The molecules are constantly in random motion, moving in all directions with all possible velocities.
3. The molecules behave like smooth elastic spheres.
4. The energy of the gas is all kinetic.
5. The time spent in a collision is negligible as compared with that during which the molecules are moving independently.
6. Between collisions the molecules move in a straight line with uniform velocity.
7. The molecular radii are assumed to be negligibly small as compared with the mean free path.
8. Average kinetic energy of gas molecules is proportional to absolute temperature.

Pressure of Gas

Consider a cubical container of side ℓ

Area of one side = A ,

& Volume = $\ell \cdot A = \ell \times \ell^2 = \ell^3 = V$

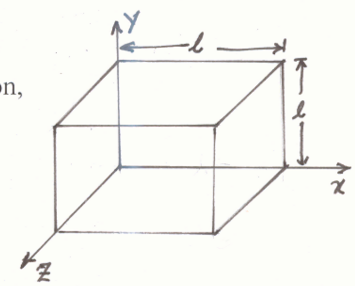
Let a molecule is moving along X-direction,

Its velocity will be = v

Time interval = t

Distance traveled = v t

Distance traveled between two consecutive collisions = 2ℓ



$[S = v\,t$ & $t = S/v]$

Time for one collision = $2\ell / v_{1x}$ ..... (1)

Momentum of the molecule before collision = $m\,v_{1x}$

Momentum of the molecule after collision = $-m\,v_{1x}$

The change in momentum of the molecule = $-m\,v_{1x} - (m\,v_{1x})$

$= -2\,m\,v_{1x}$ ..... (2)

rate of change of momentum = $\dfrac{-2mv_{1x}}{2\ell / mv_{1x}} = \dfrac{-mv_{1x} \times v_{1x}}{\ell} = -\dfrac{mv_{1x}^2}{\ell}$ .....(3)

Now we have

Force = $F_{1x} = m\,a = \dfrac{m(v_f - v_i)}{t}$ = Rate of change of momentum .....(4)

from equations (3) & (4) we get $F_{1x} = \dfrac{-mv_{1x}^2}{\ell}$

Where F_{1x} is the force exerted by the wall on a molecule

So force exerted on the wall by a molecule is

$-F_{1x} = \dfrac{-mv_{1x}^2}{\ell}$ or $F = \dfrac{mv_{1x}^2}{\ell}$ ..... (5)

And total force exerted on right wall by all the molecules, F_x , will be

$$F_x = \frac{mv_{1x}^2}{\ell} + \frac{mv_{2x}^2}{\ell} + \frac{mv_{3x}^2}{\ell} + ... + \frac{mv_{Nx}^2}{\ell}$$

$$\text{or} \quad F = \frac{m}{\ell}\Sigma v_{ix}^2 \qquad ...... (6)$$

So pressure P_x on the wall will be

$$P_x = \frac{F}{A} = \frac{m/_{\ell}\,\Sigma v_{1x}^2}{A(=\ell^2)} = \frac{m}{\ell^3}\Sigma v_{ix}^2 = \frac{\rho}{N}\Sigma v_{ix}^2 \qquad ..... (7)$$

$$\text{or} \quad P_x = \frac{\rho}{N} N < v_x^2 > \; = \rho < v_x^2 > \qquad ..... (9)$$

We have

$$<v^2> = <v_x^2> + <v_y^2> + <v_z^2>$$

as particle is moving in random direction so

$$<v_x^2> = <v_y^2> = <v_z^2> = 1/3 <v^2> \quad ....(10)$$

$$\text{so} \quad P_x = \frac{\rho}{3} < v^2 > ..... (11)$$

From Pascal's Law

$$P_x = P_y = P_z = \frac{\rho}{3} < v^2 >$$

In general form, $P = \frac{1}{3}\rho < v^2 > \; = \frac{mN}{3V} < v^2 >$

$$\text{or} \quad P = \frac{2}{3}\frac{N}{V} < \frac{1}{2}mv^2 > \; = \frac{2}{3}N_o < \frac{1}{2}mv^2 > ...(12)$$

or P = const. <K.E.>

or $P \propto$ <K.E.>

density = $\frac{\text{mass}}{\text{volume}}$

or $\rho = \frac{m}{\ell^3}$

for n molecules

$\rho = \frac{mN}{\ell^3}$ or $\frac{m}{\ell^3} = \frac{\rho}{N}$

We define:

Mean square velocity $< v_x^2 >$

$$< v_x^2 > \; = \frac{v_{1x}^2 + v_{2x}^2 + ... + v_{Nx}^2}{N}$$

$$\text{or} \quad < v_x^2 > \; = \frac{\Sigma v_{1x}^2}{N}$$

$$\text{or} \quad \Sigma v_{1x}^2 = N < v_x^2 > \; ....(8)$$

$$\rho = \frac{mN}{\ell^3} = \frac{mN}{V}$$

$$N_o = \frac{\text{No. of molecules}}{\text{Volume}} = \frac{N}{V}$$

Pascal's law: Pressure applied at any point of a gas/fluid at rest is transmitted without loss to all other parts of the gas/fluid.

Interpretation of Temperature

We have from previous knowledge

$$PV = nRT$$

$$\text{or} \quad P = nRT/V \qquad ...... (13)$$

from equations (12) and (13), we have

$$\frac{nRT}{V} = \frac{2}{3}\frac{N}{V} < \frac{1}{2}mv^2 >$$

$$\text{or} \quad T = \left(\frac{2}{3}\frac{N}{nR}\right) < \frac{1}{2}mv^2 >$$

$$\text{or} \quad T = \text{const.} \times < \frac{1}{2}mv^2 >$$

$$\text{or} \quad T \propto < \frac{1}{2}mv^2 > \quad \text{or} \quad \text{or} \quad T \propto (K.E.)_{av}$$

$V \propto 1/P$ & $V \propto T$

So $V \propto \frac{T}{P}$

or $PV \propto T$

or $PV = RT$

For n moles

$PV = nRT$

Derivation of Gas Laws

We have from previous knowledge,

$$P = \frac{2}{3}\frac{N}{V} < \frac{1}{2}mv^2 > \qquad \ldots(1)$$

If average KE = $<1/2\ mv^2>$ is constant, then

$$P = \text{Const.} \times \frac{2}{3}\frac{N}{V} \text{ or } PV = \text{Const.} \times \frac{2}{3}N = \text{Constant or } P \propto \frac{1}{V} \qquad \ldots\ldots (2)$$

which is **Boyles's Law.**

Now from eq. (1)

$$V = \frac{2}{3}\frac{N}{P} < \frac{1}{2}mv^2 > \qquad \ldots\ldots (3)$$

If pressure P is constant, then

$$V = \text{constant} \times <1/2\ mv^2> \quad \text{or} \quad V \propto (KE)_{av}$$

As $(KE)_{av}$ is measure of Temperature T, so

$$V \propto T \qquad \ldots\ldots (4)$$

which is **Charles' Law.**

Internal energy (ΔU)

i) The sum of all forms of molecular energies (kinetic and potential) of a substance.

ii) Total heat energy retained by the system in the form of potential energy and kinetic energy.

Internal energy retained is in the form of,

i) translational kinetic energy (KE_{trans}), ii) vibrational kinetic energy (KE_{vib}) & iii) rotational kinetic energy (KE_{rot})

Generally internal energy of an ideal gas system is its translational K.E.

Internal energy depends only upon initial and final states,

Internal energy is a function of state (state variable or parameter like P, V & T)

Specific heat: The amount of heat energy required to raise the temperature of unit mass through one degree.

Molar specific heat: The amount of heat energy required to raise the temperature of one mole of a substance through 1 K.

Molar specific heat at constant volume (C_v): The amount of heat energy required to raise the temperature of one mole of a gas through 1 K at constant volume.

Molar specific heat at constant pressure (C_p): The amount of heat energy required to raise the temperature of one mole of a gas through 1 K at constant pressure.

Work & Heat

Heat Q ADDED or heat IN is **+ve**

Heat Q LEAVES or OUT is **-ve**

Work is done BY the system on its environment is **+ve**

Work is done ON the system by the environment is **-ve**

First Law of Thermodynamics

Statement

i) In any thermodynamic process, when heat Q is added to a system, this energy appears as an increase in the internal energy ΔU stored in the system plus the work W done by the system on its surroundings.

ii) The heat energy supplied to a system is equal to the increase in the internal energy of the system from an initial value U_i to the final value U_f plus the work done by the system on its surroundings. Mathematically

$$Q = \Delta U + W \qquad \ldots\ldots (1)$$

Explanation

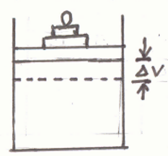
Eq. (1) defines the change in the internal energy of a system. It is equal to the energy flowing in as heat energy minus the energy flowing out as work.
The first law of thermodynamics indicates that there exists a useful state variable of every thermodynamic system called the internal energy.

Applications:

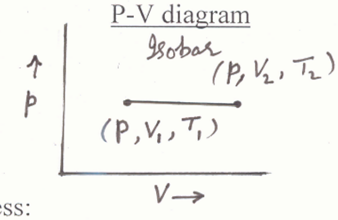
1. Isobaric Process:

"The process in which the pressure of the system remains constant".

Gas-cylinder system



P-V diagram



Applying the equation in this isobaric process:

$$Q = \Delta U + W$$

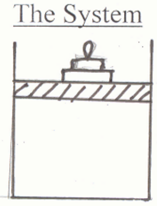
or $$Q = \Delta U + PV$$

Work = force x displacement
$W = F \times d$ [$P = F/A$
or $W = PAd$ [or $F = PA$
or $W = PV$ [$A \times d = V$

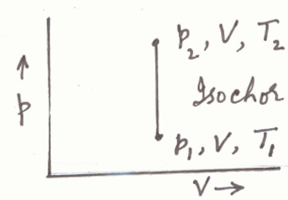
2. Isochoric Process:

"The process in which the volume of the system remains constant".

The System



P-V diagram



$W = 0$

Applying the equation,

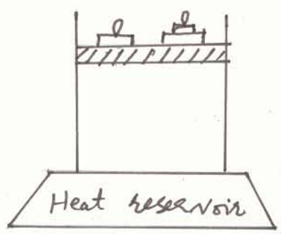
$$Q = \Delta U + W$$

or $$Q = \Delta U$$

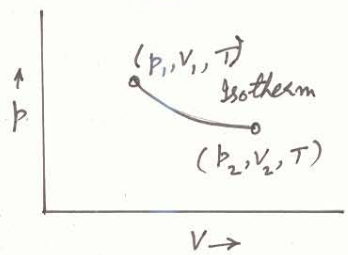
3. Isothermal Process:

"The process in which the temperature of the system remains constant".

The System



P-V diagram



We have for constant temperature,

$P_1V_1 = P_2V_2$ & $\Delta U = 0$

Applying the equation,

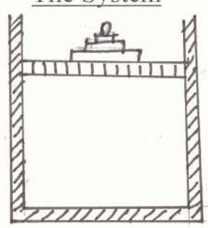
$$Q = \Delta U + W \qquad | \Delta U = 0$$

$$\text{or} \quad Q = W$$

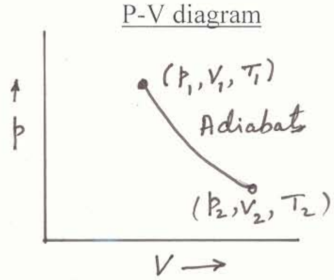
4. Adiabatic Process:

"The process in which no heat enters or leaves the system".

The System



P-V diagram



Applying the equation,

$$Q = \Delta U + W \qquad | Q = 0$$

$$0 = \Delta U + W$$

$$\text{or} \quad W = -\Delta U$$

Also in adiabatic changes the following relation is found to be true,

$$P V^{\gamma} = \text{constant} \qquad \text{where} \qquad \gamma = C_p / C_V$$

[Proof of above equation is in the foot note of the article "Speed of Sound in Air"]

Examples:

1) Rapid air escape when tyre bursts
2) Rapid expansion & compression of sound waves
3) Cloud formation in atmosphere

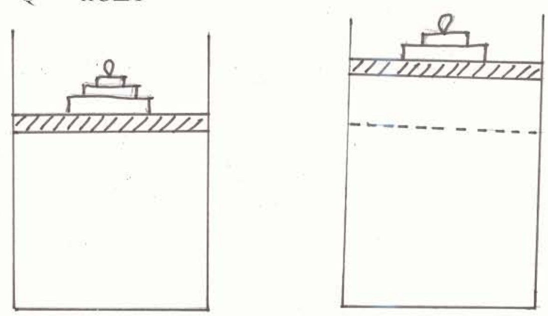
5. Molar Specific Heats of a Gas

We have from previous knowledge

$$Q = mc\Delta T$$

for molecular specific heat

$$Q = nC\Delta T$$



At constant volume:

$$Q = nC_V\Delta T = \begin{Bmatrix}\text{Heat energy used in raising} \\ \text{the temperature through } \Delta T\end{Bmatrix} \quad \ldots\ldots (1)$$

$$\& \quad \begin{Bmatrix}\text{Heat energy used in} \\ \text{doing the external work}\end{Bmatrix} = \Delta W = PAd = P\Delta V = nR\Delta T \quad \ldots\ldots (2)$$

| $PV = nRT$

At constant pressure:

$$Q = nC_p\Delta T_1 = \begin{Bmatrix}\text{Heat energy used in raising} \\ \text{the temperature through } \Delta T\end{Bmatrix} + \begin{Bmatrix}\text{Heat energy used in} \\ \text{doing the external work}\end{Bmatrix} \ldots\ldots (3)$$

from equations (1), (2) & (3), we get

$$nC_p\,\Delta T = nC_V\,\Delta T + nR\Delta T$$

or $\quad \cancel{n}C_p\cancel{\Delta T} = \cancel{n}C_V\cancel{\Delta T} + \cancel{n}R\cancel{\Delta T}$

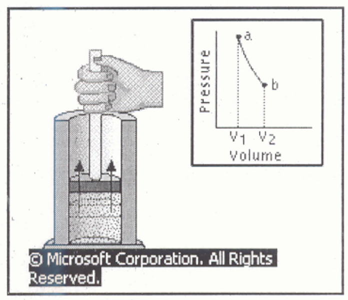
or $\quad C_p = C_V + R$

or $\quad C_p - C_V = R$

It implies $\quad C_p > C_V$

Second Law of Thermodynamics

Definitions:
Reversible Cycle: A succession of events which bring the system back to its initial condition and all the changes are reversible.
Reversible process: If the process can be reversed in such a way that the system and its surroundings are both brought back to their original states, then the process is said to be reversible.
Irreversible process: If a process can not be retraced in the backward direction by reversing the controlling factors, it is an irreversible process.
Heat engine: A device which converts heat energy into mechanical work.
Carnot engine: An ideal heat engine, free from all the imperfections of actual engines, and hence never realized in practice.
Carnot cycle: A cycle in which reversible process occurs.
Heat reservoir : It is supposed to be so big that its temperature remains constant even if some heat enters or leaves the reservoir.
Hot reservoir (or Source):
A hot body, which can supply heat at a high temp. to a cold body.
Cold reservoir (or Sink):
A cold body, which can receive heat at a low temperature from a hot body.



A Heat Engine must have:

i) A source, which can supply heat.
ii) Sink to reject heat.
iii) Working substance.

Lord Kelvin's Statement

"It is impossible to devise a process which may convert heat, extracted from a single reservoir, entirely into work without leaving any change in the working system".

As a consequence of this law, two bodies at different temperatures are essential for the conversion of heat into work. A single heat reservoir, no matter how much energy it contains, can not be made to perform any work.

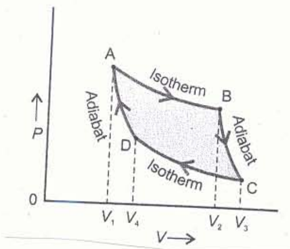
Clausius Statement

"It is impossible to cause heat to flow from a cold body to a hot body without the expenditure of energy".

Carnot Engine

Sadi Carnot described an ideal engine using only isothermal and adiabatic processes. He showed that this is most efficient reversible ideal engine. A Carnot cycle using an ideal gas is shown on PV diagram. It consists of four steps.

1. The gas is allowed to expand isothermally at temperature T_1, absorbing heat Q_1 from hot reservoir.
2. The gas is then allowed to expand adiabatically until its temperature drops to T_2.
3. The gas is compressed isothermally at temp. T_2, rejecting heat Q_2 to cold reservoir.
4. Finally the gas is compressed adiabatically to restore its initial state at temperature T_1.



The net work done during one cycle equals to the area enclosed by the path ABCDA of the PV diagram.

Applying 1st law of thermodynamics,

$$Q = \Delta U + W$$

$$\text{or} \quad Q_1 - Q_2 = 0 + W$$

$$\text{or} \quad W = Q_1 - Q_2$$

Since working substance returns to initial state,
so $\Delta U = 0$

Net heat absorbed during one cycle is, $Q = Q_1 - Q_2$

Now efficiency η will be

$$\eta = \frac{\text{Output}}{\text{Input}} = \frac{Q_1 - Q_2}{Q_1} = \frac{Q_1}{Q_1} - \frac{Q_2}{Q_1}$$

$$\text{or} \quad \eta = 1 - \frac{Q_2}{Q_1}$$

Output = Work = W = $Q_1 - Q_2$
Input = Energy = Q_1

since $Q \propto T$, so $\eta = 1 + \frac{T_2}{T_1}$

& Percentage efficiency, $\%\text{age } \eta = \left(1 + \frac{T_2}{T_1}\right) 100$

The above equation shows that the efficiency of Carnot engine depends on the temperature of hot and cold reservoirs. It is independent of the nature of working substance. It can never be 100 % unless cold reservoir is at absolute zero temperature. Such reservoirs are not available and hence the maximum efficiency even for Carnot engine is less than one or 100 %. And all real heat engines are less efficient than Carnot engine due to friction and other heat losses.

Carnot's Theorem:

No heat engine can be more efficient than a Carnot engine operating between the same two temperatures.

Extension of Carnot's Theorem:

All Carnot's engines operating between the same two temperatures have the same efficiency, irrespective of the nature of working substance.