Self Study Series

Important Articles

of the Textbook

PHYSICS XII

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PREFACE

The vital portion of your Text book of Physics for class 12 is included in this book of 'Important Articles'.

Here no attempt is made to write extra information or high knowledge for impressing students. Articles are written in brief, no details, but to the point, hoping you will not miss the main points in your exam papers.

Foot notes and side notes are not for reproducing in the exams. They are written just for understanding the related article.

Text and some figures are made in such a way so that you can reproduce easily in the exams.

If you stuck! Just prepare this book, to go through your exams.

Best of luck.

February 22, 2005   Ross Nazir Ullah

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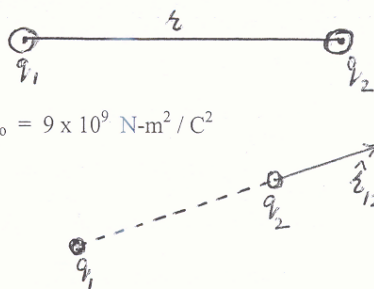
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Coulomb's Law

Statement:
"The magnitude of the force between two point charges is directly proportional to the product of the magnitudes of the charges and inversely proportional to the square of the distance between them". Mathematically

$$F_e \propto q_1 q_2 / r^2$$

or $\boxed{F_e = k \dfrac{q_1 q_2}{r^2}}$ where $k = 1/4\pi\epsilon_0 = 9 \times 10^9$ N-m^2/C^2

in vector form

$$\boxed{\vec{F}_e = k \frac{q_1 q_2 \, \hat{r}_{12}}{r^2_{12}}}$$

Coulomb's Torsion balance:
As shown in the figure, the torsion balance consists of a horizontal insulated rod R carrying two metallic spheres a & a' at the ends of the rod. The rod is suspended from middle with a fibre supported from F. A small mirror M is attached to the fibre and a beam of light reflected from this mirror falls on the scale. Another insulated rod P, carrying a small sphere b, equal in size a, can be brought near the small sphere a.
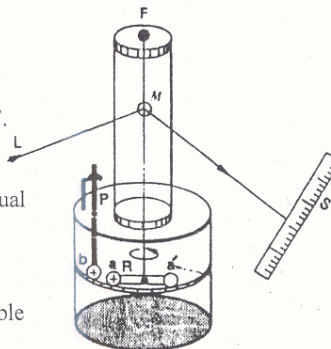
Experimental details:
The sphere b is charged by rubbing it with some suitable material. When sphere b is touched with sphere a, it shares its charge so that both have equal charges,

$$q_1 = q_2 = q$$

The charge on b can be further divided into q/2, q/4, so on, by touching it with the spheres of equal sizes.
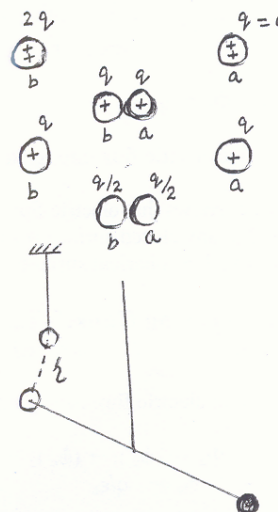By keeping the distance a and b fixed, we find

$$F \propto q_1 q_2$$

On varying the distance between two spheres with constant charge, it found that

$$F \propto 1/r^2$$
Combining the above two relations, we get

$$\boxed{F \propto q_1 q_2 / r^2}$$

which is Coulomb's Law.

Gauss's Law

Statement:

"The flux through any closed surface is $1/\epsilon_0$ times the total charge enclosed in it".

First to prove $\Phi = q/\epsilon_0$ for a closed surface:

To show electric flux through a closed surface
is equal to q/ϵ_0.
Consider
a sphere of radius r having a charge $+q$
placed at its centre.
We divide the closed surface into very small
elements of area $\vec{\Delta A_1}$, $\vec{\Delta A_2}$, ..., $\vec{\Delta A_n}$.
The magnitude of E will be same at each point.
Let
$\vec{E_1}$ be the intensity at the centre of $\vec{\Delta A_1}$.
The directions of both $\vec{E_1}$ and $\vec{\Delta A_1}$ will be same.
So electric flux $(\Delta\phi_e)$ will be

$$(\Delta\phi_e)_1 = \vec{E_1}.\vec{\Delta A_1} = E_1.\Delta A_1 \cos 0 = E.\Delta A_1 \qquad ..... (1)$$

similarly

$$(\Delta\phi_e)_2 = \vec{E}.\vec{\Delta A_2}, \quad (\Delta\phi_e)_3 = \vec{E}.\vec{\Delta A_3} \quad .... \quad (\Delta\phi_e)_n = \vec{E}.\vec{\Delta A_n}$$

therefore total electric flux Φ_e through closed surface

$$\Phi_e = (\Delta\phi_e)_1 + (\Delta\phi_e)_2 + ... + (\Delta\phi_e)_n$$

$$= \vec{E}.\vec{\Delta A_1} + \vec{E}.\vec{\Delta A_2} + ... + \vec{E}.\vec{\Delta A_n}$$

$$= E \sum \Delta A_i = E \times (\text{total area of closed surface})$$

therefore $\Phi_e = \dfrac{1}{4\pi\epsilon_0} \dfrac{q}{r^2}(4\pi r^2) = \dfrac{q}{\epsilon_0}$ $\qquad$ [area of sphere $= 4\pi r^2$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ [$E = (1/4\pi\epsilon_0)(q/r^2)$

or $\quad \Phi_e = q/\epsilon_0$

which is electric flux through a sphere.

Figure shows that electric flux
through any closed surface is same
as through a spherical surface.
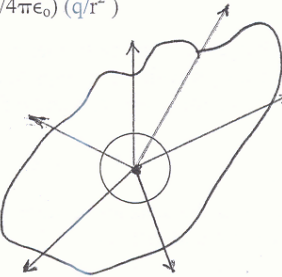
Now proving Gauss's Law:

Consider a closed surface enclosing points q_1, q_2, q_3, ... , q_n as shown in the figure.

The total electric flux through the closed surface will be

$$\Phi_e = (\phi_e)_1 + (\phi_e)_2 + (\phi_e)_3 + ... + (\phi_e)_n$$

as $\quad \phi_e = q/\epsilon_0$

so

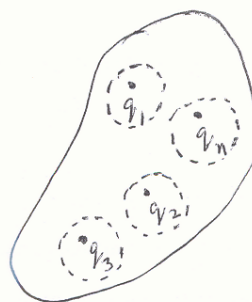$$\Phi_e = q_1/\epsilon_0 + q_2/\epsilon_0 + q_3/\epsilon_0 + \ldots + q_n/\epsilon_0$$

or

$$\Phi_e = (1/\epsilon_0)\{q_1 + q_2 + q_3 + \ldots + q_n\}$$

$$= 1/\epsilon_0 \sum q_i$$

or $\quad \Phi_e = 1/\epsilon_0 \times Q \qquad \ldots (1)$

where Q is the total charge enclosed by the closed surface.

Eq. (1) states that
the flux through any closed surface is $1/\epsilon_0$ times the total charge enclosed in it,
which is Gauss's Law.

Applications of Gauss's Law

1. Electric Intensity due to a sheet of charge:

To calculate the electric intensity $\vec{E}$ at a point P close to the sheet,
Consider a plane sheet of
infinite extent on which
positive charges are
uniformly distributed, with
surface charge density $= \sigma$

Imagine
a closed surface in the form of a cylinder
passing through sheet, whose
one face contains point P.
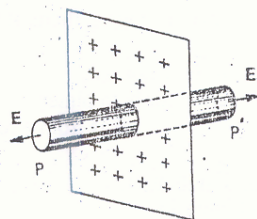
The direction of $\vec{E}$ will be
normal to the sheet,
i.e. parallel to curved surface and
normal to the end faces.
[taking sheet of infinite extent]

The flux through the cylinder will be

$$\Phi_e = (\phi_e)_{curved\ surface} + (\phi_e)_{two\ end\ faces} \qquad [\,(\phi_e) = \vec{E}\cdot\vec{A} = EA\cos 90^\circ = 0$$
$$= 0 + 2EA, \quad \text{where A is area of end face}$$
or $\Phi_e = 2EA \qquad \ldots (1)$

Applying Guass's Law, we have

$$\Phi_e = 1/\epsilon_0 (\text{charge enclosed})$$

or $\quad \Phi_e = 1/\epsilon_0 \times \sigma A \qquad \ldots (2)$

from equations (1) and (2), we have

$$2EA = 1/\epsilon_0 \cdot \sigma A$$

or $\quad E = \sigma/2\epsilon_0$

or $\quad \vec{E} = \sigma/2\epsilon_0 \cdot \hat{r}$

where $\hat{r}$ is the unit vector normal to the sheet and directed outward.

In case the sheet is negatively charged

$$\vec{E} = -\sigma/2\epsilon_0 \cdot \hat{r}$$

And the intensity is directed towards the sheet.

2. Electric Intensity between two oppositely charged plates:

Consider two parallel plates of
some conducting material,
placed close to each other

The plates have charges,
$+\sigma$, $-\sigma$ [opposite charges

these plates are of infinite extent
        [to avoid fringing fields at their ends

the direction of $\vec{E}$ is normal,
and same direction for both plates
        [$\vec{E}$ is from +ve to –ve

<u>To calculate $\vec{E}$ at point P</u>
The value of intensity
for both plates,

$$\vec{E} = \vec{E_1} + \vec{E_2}$$

From previous knowledge,
E for single sheet of charge is

$$\vec{E} = \sigma/2\epsilon_0 \,\hat{r}$$

so
$$\vec{E} = (\sigma/2\epsilon_0 \, r)\,\hat{r} \text{ from +ve to -ve} + (\sigma/2\epsilon_0 \, r)\,\hat{r} \text{ from +ve to -ve}$$
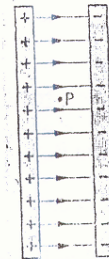
or
$$\vec{E} = 2\,\sigma/2\epsilon_0 \,\hat{r}$$

or
$$\vec{E} = \sigma/\epsilon_0 \,\hat{r} \text{ , } \hat{r} \text{ is from +ve to –ve plate}$$

The direction of electric field is normal to the plate directed from positively charge plate to negatively charged plate.

Electric Potential

Definition:

"Work done on a unit positive charge in moving it from infnity to that point against the electrical forces".

Calculating Electric Potential:

To calculate electric potential at a point due to a point charge,
Consider a point charge q,
Take two points A & B close to each other, so that E remains constant between them, then from the figure

$$r_B = r_A + \Delta r \qquad\qquad ..... (1)$$
$$\Delta r = r_B - r_A \qquad\qquad ..... (2)$$

the centre of this displacement from the point charge is

$$r = \frac{r_B + r_A}{2}$$

first calculating r^2

$$(r)^2 = \left\{ \frac{r_B + r_A}{2} \right\}^2 \qquad\qquad ..... (3)$$

from equations (1) and (3), we have

$$r^2 = \left\{ \frac{r_A + \Delta r + r_A}{2} \right\}^2$$

$$= (2r_A/2 + \Delta r/2) = (r_A + \Delta r/2)^2$$
$$= r^2_A + 2 r_A \Delta r/2 + \Delta r^2/4$$

neglecting $\Delta r^2/4$, Δr being very small, we get

$$r^2 = r^2_A + r_A \Delta r \qquad\qquad ..... (4)$$

from equations (2) and (4), we have

$$r^2 = r^2_A + r_A (r_B - r_A)$$
$$\text{or} \quad r^2 = r^2_A + r_A r_B - r^2_A$$
$$\text{or} \quad r^2 = r_A r_B \qquad\qquad ..... (5)$$

Now calculating work done

If a unit positive charge move from point B to A $[\Delta r = r_A - r_B]$,
which is equal to potential difference,

$$W_{B \to A} = \Delta V = V_A - V_B$$
$$\text{or} \quad V_A - V_B = - E \cdot \Delta r$$
$$= - (1/4\pi\epsilon_0)(q/r^2)(r_A - r_B)$$
$$= \frac{+1}{4\pi\epsilon_0} \frac{q (r_B - r_A)}{r_A r_B}$$

$$\begin{cases} E = - \Delta V / \Delta r \\ \text{or } \Delta V = - E \cdot \Delta r \\ \& \ E = (1/4\pi\epsilon_0)(q/r^2) \\ \text{since } \Delta r = (r_A - r_B) \end{cases}$$
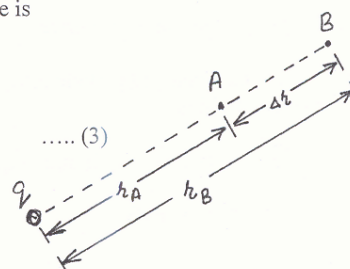
$$\text{or} \quad V_A - V_B = (q/4\pi\epsilon_0)(1/r_A - 1/r_B)$$

put $V_B = 0$ at $r_B = \infty$, it gives

$$V_A = \frac{1}{4\pi\epsilon_0} \frac{q}{r_A}$$

The general expression of absolute potential V_r or electric potential at a distance r from charge q is

$$V_r = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

Force on a current-carrying conductor

Consider a conductor moving in a uniform magnetic field.
Take direction of flow of charge through the
conducting wire at right angle to B.

Let

No. of free electrons per unit volume = n
Length of the wire = l
Area of cross section = A
Then , charge of magnitude = nAl

So, charge for electrons = q = nAle

We have force on a charged particle moving
in the magnetic field is

$$\vec{F} = q\,(\vec{v} \times \vec{B}) = qvB \sin\theta = qvB\sin 90°$$

or   F = qvB

or F = nAle x $\dfrac{I}{nAe}$ x B

or F = I l B

or $\boxed{\vec{F} = I\,\vec{l} \times \vec{B}}$
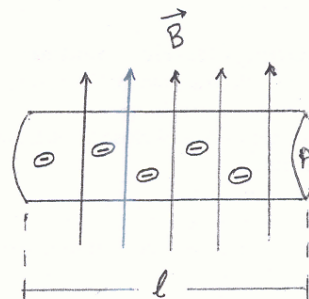
$$\begin{cases} q = nAle \\ t = S/v = l/v \\ I = \dfrac{q}{t} = \dfrac{nAle \times v}{l} \\ \text{or}\quad v = \dfrac{I}{nAe} \end{cases}$$

Ampere's Law

It is also called Ampere's Circuital Law.

Statement:

"The product of total path elements around a conductor and magnetic flux density in a complete loop equals μ_o times the total current enclosed by the loop". Mathematically

$$\sum_{i=1}^{n} (\vec{B} \cdot \vec{\Delta l})_r = \mu_o I$$

Illustration:

We have in the figure,
a long straight wire carrying a current I,
consider
a closed curve consisting of a circle of
radius r and with centre at the wire.
According to Ampere, magnitude of
flux density B is directly proportional to
the strength of current and inversely
proportional to the distance of the point
from the conductor, i.e.

$$B \propto I$$

And

$$B \propto 1/r$$

Combining the above relations

$$B \propto I/r$$

or $B = \dfrac{\mu_o I}{2\pi r}$
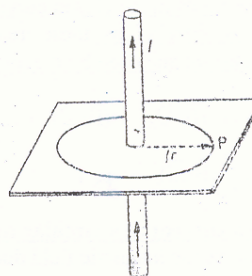
where $\mu_o / 2\pi$ is constant of proportionality.

or $2\pi r B = \mu_o I$

but $2\pi r$ is the length of the path around the wire.

Over this path, the value of B is same at every point,
So we can write,

$$\sum_{i=1}^{n} (\vec{B} \cdot \vec{\Delta l})_r = \mu_o I$$

This is mathematical form of Ampere's Circuital Law.

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Applications

1. A closed curved path around the current carrying conductor

In the figure, this closed curved path is divided
into a large number of elements each of
length Δl. Taking product

$$\vec{B} \cdot \vec{\Delta l}$$

for each element, then taking
sum of all the elements,

$$(\vec{B} \cdot \vec{\Delta l})_1 + (\vec{B} \cdot \vec{\Delta l})_2 + (\vec{B} \cdot \vec{\Delta l})_3 + \ldots + (\vec{B} \cdot \vec{\Delta l})_n$$

since $\vec{B} \cdot \vec{\Delta l} = B \cdot \Delta l \cos \theta$
where θ is the angle between $\vec{B}$ and $\vec{\Delta l}$.
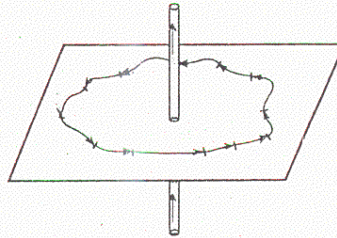We can consider to take component of $\vec{B}$ parallel to $\vec{\Delta l}$.
The above expression is the <u>sum of quantities $\vec{B} \cdot \vec{\Delta l}$</u>
<u>for all path elements in the closed curved.</u>
According to <u>Ampere's Law,</u> these are equal to
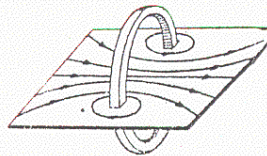μ_0 times the current enclosed by the loop, i.e.

$$(\vec{B} \cdot \vec{\Delta l})_1 + (\vec{B} \cdot \vec{\Delta l})_2 + (\vec{B} \cdot \vec{\Delta l})_3 + \ldots + (\vec{B} \cdot \vec{\Delta l})_n = \mu_0 I$$

$$\text{or} \quad \sum_{t=1}^{n} (\vec{B} \cdot \vec{\Delta l})_r = \mu_0 I$$

2. Field due to a current in a circular coil

The figure shows the magnetic field due to a current in a circular coil.

We can <u>trace the field</u> by <u>dusting iron fillings</u> on a card board.
Also by <u>plotting with a compass needle</u>.
In this case the value of B will be

$$B = \frac{2\pi n I}{10 r}$$

where   n is number of turns of the coil
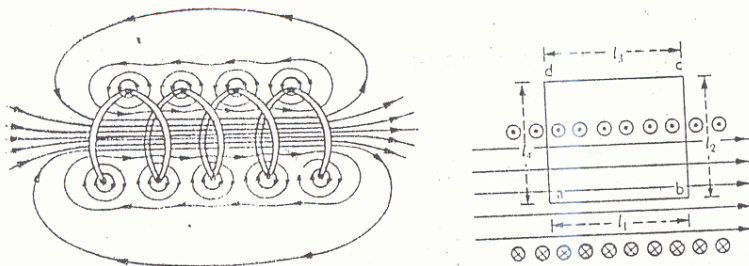I is current passing through it
r is the radius of the coil.

The value of B is not uniform in this case. Only for a point on the axis of the coil it is nearly
uniform.

3. Field due to current in a solenoid

Definition

"A long helically wound coil of insulated wire is called solenoid".

Figure (a) shows the magnetic field produced by the solenoid

To calculate B

In figure (b) consider a rectangular path abcda.

It is divided into four path elements l_1, l_2, l_3, l_4.

Applying Ampere's Law to this closed path

$$\Sigma\,((B \cdot \Delta l)_i = \mu_o\, I$$

$$\text{or} \quad \vec{B} \cdot \vec{l_1} + \vec{B} \cdot \vec{l_2} + \vec{B} \cdot \vec{l_3} + \vec{B} \cdot \vec{l_4} = \mu_o \times (\text{current enclosed}) \qquad \ldots\ldots (1)$$

since

$\vec{B} \cdot \vec{l_1} = Bl_1 \cos 0^\circ = Bl_1$ [inside B is parallel to l_1

$\vec{B} \cdot \vec{l_2} = Bl_2 \cos 90^\circ = 0$ [B is perpendicular to l_2.

$\vec{B} \cdot \vec{l_3} = Bl_3 \cos 0^\circ = 0$ [B is zero outside the solenoid

$\vec{B} \cdot \vec{l_4} = Bl_4 \cos 90^\circ = 0$ [B is perpendicular to l_4

From the above relations we have from eq. (1),

$$B\, l_1 + 0 + 0 + 0 = \mu_o \times (\text{current enclosed}) \qquad \ldots\ldots (2)$$

If current I is enclosed in each turn, and

n are the number of turns, then

the current enclosed by the loop abcda will $= n\, l_1\, I$ $\ldots\ldots (3)$

from relations (2) and (3) we have
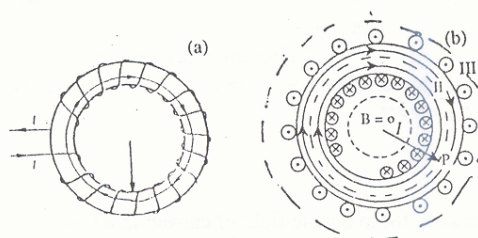
$$B\, l_1 = \mu_o\, n\, l_1\, I$$

$$\text{or} \qquad B = \mu_o\, n\, I$$

where the direction of B is along the axis of the solenoid.

4. Field due to current in a toroid

Definition

"A helically wound coil of insulated wire (solenoid) bent into a circle is called toroid".

A toroid having current I. The dotted lines are three paths for applying Ampere's Law.

1. **Field B inside the toroidal ring**
   Here if there is some field,
   it will be tangent to the path at all points.
   Applying Ampere's Law,
   $$\sum_{i=1}^{n}(B\cdot\Delta l)_i = \mu_o I$$
   or B x circumference = μ_o x current enclosed
   since current enclosed is zero,
   so
   $$B \times 2\pi\, r_1 = \mu_o \times 0 = 0$$
   i.e. the field B is zero inside the toroidal ring.

2. **Filed B outside the toroidal ring**
   Here also if there is some field, it will be tangent to the path at all points.
   Each turn of the winding passes twice through it,
   so net current is zero.
   Applying Ampere's Law
   $$\sum_{i=1}^{n}(B\cdot\Delta l)_i = \mu_o I$$

   $$B\cdot 2\pi\, r_3 = \mu_o I = \mu_o \times 0 = 0$$
Hence the field B outside the toroidal ring is zero.

3. **Field B in the space enclosing the windings**
   In this case, consider a circular path of radius r,
   By symmetry the field is tangent to the path, so

   $$\sum_{i=1}^{n}(B\cdot\Delta l)_i = 2\pi\, r\, B \qquad\qquad \ldots\,(1)$$

each winding passes once through it, so

total current enclosed = N I $\qquad$ ..... (2) $\qquad$ [N = total number of turns
applying Ampere's Law,
$$\sum_{i=1}^{n}(B\cdot\Delta l)_i = \mu_o \times \text{ current enclosed} \qquad \ldots\,(3)$$

from relations (1), (2) and (3) we have
$$2\pi\, r\, B = \mu_o N I$$

$$\text{or } B = \frac{\mu_o\, N I}{2\pi\ \ r} \qquad\qquad \ldots\,(4)$$

[circumference = $2\pi\, r$
taking number of turns per unit length = n = N $/2\pi\, r$ $\qquad$ ..... (5)
from relations (4) and (5), we get

$$B = \mu_o n I$$

Which is the required relation for magnetic field of current in a toroid
for a winding in vacuum.

Measurement of e/m

We will determine the ratio of the charge e of the electron to its mass m.

A charged particle describes a circular motion when it enters perpendicularly in a uniform magnetic field.

The magnetic force provides the necessary centripetal force.

Equating the magnetic and centripetal forces on a single electron, we get

$$F_m = F_c$$
$$\text{or } Bev = m v^2 / r$$
$$\text{or } \frac{e}{m} = \frac{v^2}{r} \times \frac{1}{Bv}$$

$$\left\{ \begin{array}{l} F_m = Bev \sin\theta \\ \quad = Bev \sin 90^\circ \\ \text{or } F_m = Bev \\ \& \ F = mv^2 / r \end{array} \right.$$

$$\text{or } e/m = v / Br \quad \quad \dots\dots (1)$$

If v and r are known and by measuring B we can determine e/m.

Measurement of radius r:

The radius r is measured by making the beam visible. This is done by filling the tube with a gas at low pressure. The electrons will suffer collisions with gas and it will result emission of light. The beam will become visible as a circular ring of light. The diameter of the ring can easily be determined.

Measurement of velocity v:
There are **two methods** of measuring v.

1. Potential difference method
Here we should know the potential difference through which the electrons were accelerated before entering the magnetic field.
We define potential difference as work done per unit charge,

$$V = W / q$$
$$\text{or } W = qV$$
$$\text{or } W = eV$$
$$\text{or } W = \text{energy gained} = E = Ve$$
$$\text{also } \text{kinetic energy} = E = \tfrac{1}{2} mv^2$$
from equations (2) and (3) we have

$$\left\{ \begin{array}{l} q = e = \text{charge on electron} \\ V = \text{voltage} \end{array} \right.$$
$$\dots\dots (2)$$
$$\dots\dots (3)$$

$$V e = \tfrac{1}{2} mv^2$$

$$\text{or } v^2 = 2V e / m \quad \quad \text{or } v = \sqrt{2V e / m} \quad \quad \dots\dots (4)$$

from equations (1) and (4) we get $\dfrac{e}{m} = \dfrac{\sqrt{2V\,e/m}}{Br}$

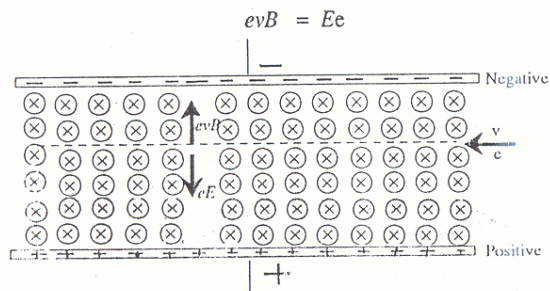squaring both sides

$$\dfrac{e^2}{m^2} = \dfrac{2V\,e}{mB^2\,r^2}$$

$$\dfrac{e}{m} = \dfrac{2V}{B^2\,r^2} \qquad\qquad \dots\dots (5)$$

2. Particle velocity selector

In the figure,

two oppositely charged parallel plates produce electric field E which exerts force Ee downward on an electron.

The magnetic field B exerts an upward force evB on the same electron.

$$evB = Ee$$

If the voltage across the plates is adjusted so that both forces are equal, then the particle will not deflect, so

$$F_m = F_e$$

$$evB = Ee \qquad \dots\dots (6)$$

or $\quad v = E/B \qquad \dots\dots (7)$

$$\begin{cases} E = F/q \\ \text{or } F_e = qE = Ee \\ \text{also } F_m = Bev\,\sin 90^\circ \\ \text{or } F_m = Bev \\ \text{also } E = -\Delta V/\Delta r \\ \text{or } E = V/d \end{cases}$$

We can determine E by adjusting
plate voltage V and measuring distance d between plates.

From equations (1) and (7) we have

$$\dfrac{e}{m} = \dfrac{E}{B^2\,R}$$
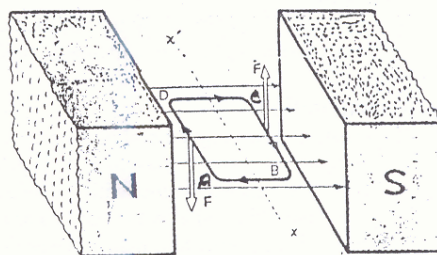
Torque on a current-carrying coil

Consider a current-carrying rectangular
coil ABCD in a uniform magnetic field.

The coil is capable of rotation about an axis.

The net force on the loop is zero,

The <u>vertical sides</u> of the loop will give rise
to a <u>couple</u>

When seen from the above, there will be a
net anti-clockwise torque τ about xx'-axis,

Taking 'a' as length AB or CD

$$
\begin{aligned}
\text{Total torque} = \vec{\tau} &= \vec{\tau_1} + \vec{\tau_2} \\
&= \vec{F} \times \vec{d_\perp} + \vec{F} \times \vec{d_\perp} \\
\text{or} \quad \vec{\tau} &= 2\vec{F} \times \vec{d_\perp} \qquad\qquad [\ \tau_1 = \tau_2 \\
&= 2F(a/2) \qquad\qquad\quad [\ d_\perp = a/2 \\
\text{or} \quad \tau &= Fa \qquad \dots\dots (1)
\end{aligned}
$$

force due to sides AB and CD is zero   [they are parallel to the field

force due to sides AD and BC will be

$$\vec{F} = I\vec{L} \times \vec{B} = ILB \sin 90^\circ \quad [\ L = \text{length AD or BC}$$
$$\text{or} \quad F = ILB \qquad \dots\dots (2)$$

from equations (1) and (2) we have

$$\tau = ILB \times a = IAB \qquad\qquad [\ A = a \times L$$

if we have n number of turns, then
$$\tau = NIAB$$
if θ is the angle between <u>at right angle to the plane of the coil</u> and the direction of $\vec{B}$, then
                                [area and vector area are at right angle to each other]
$$\tau = NIAB \sin\theta$$

if α is the angle between <u>plane of the coil</u> and the direction of B, then

$$\boxed{\tau = NIAB \cos\alpha} \qquad\qquad [\ \sin(90 + \theta) = \cos\theta$$

this formula is valid even for circular coils.

Galvanometer

Definition:

"An instrument used to measure minute electric currents".

Principle:

The galvanometer is based on interaction between current and a magnet. When a loop carrying current is placed in a magnetic field, it experiences a torque which tends to rotate it.

Construction:

It consists of a rectangular coil of enameled copper wire suspended between the concave shaped poles N and S of U-shaped magnet with the help of a fine metallic suspension wire, which is used as one current lead. The other terminal, serve as the second current lead. A soft iron is placed inside the coil.
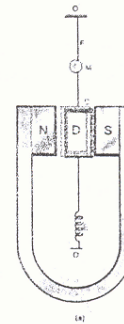
Working:

When a current is passed through the coil, it is acted upon by deflecting couple, which tends to rotate the coil. We have

$$\text{Deflecting couple} = NIBA\cos\alpha \qquad [\cos\alpha = 1$$
$$\text{or deflecting torque} = NIBA \qquad \ldots(1)$$

As the coil turns, it tends to untwist the suspension with a restoring couple.

From Hooke's law

$$\text{Restoring torque} = c\theta \qquad \ldots(2)$$

Under the effect of these two couples, coil comes to rest

$$\text{Deflecting torque} = \text{Restoring torque}$$
$$\text{or } NIBA = c\theta$$
$$\text{or } I = \frac{c}{BAN}\theta$$
$$\text{or } I \propto \theta \ , \ c, B, A \ \& \ N \text{ are constants.}$$

So the current passing through the coil is proportional to angle of deflection.
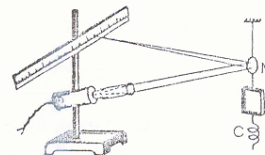
Two methods:

1. Lamp and scale arrangement

The arrangement is shown in the figure.

In sensitive galvanometers, the angle of deflection is observed by a small mirror attached to the coil along with lamp and scale arrangement.
The displacement of the spot of light on translucent scale is proportional to the angle of deflection.

2. Pivoted type galvanometer

Here the coil is pivoted between two jeweled
bearings. The restoring torque is provided by
two hair springs which also serve as current leads.
A light aluminium pointer is attached to the coil
which moves over a scale, which gives angle of
deflection.
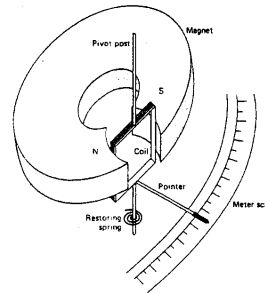
Current Sensitivity:

We define current sensitivity of a galvanometer as "the current in microamperes, required to
produce one millimeter deflection on a scale placed one meter away from the mirror of the
galvanometer".

Dead beat (or Stable) galvanometer:

Such galvanometer in which the coil comes to rest quickly after the current passed through it
or the current is stopped from flowing through it.

Conversion of Galvanometer into Ammeter:

An ammeter is an electrical instrument, which is used to measure current in amperes.
To convert galvanometer into an ammeter, we connect a low value bypass resistor called
shunt. Its value is such that the current I_g for
full scale deflection passes through the
galvanometer and remaining current $(I - I_g)$
passes through the shunt.
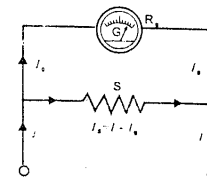
From the figure,

$$I_g R_g = (I - I_g) R$$

or $$R_s = \frac{I_g}{I - I_g} R_g$$

This shunt resistance R_s is usually small and a piece of copper wire serves the purpose.

Conversion of Galvanometer into Voltmeter:

A voltmeter is an electrical device which measures the potential difference in volts between
two points.
The voltmeter is made by modifying galvanometer.
We connect a very high resistance R_h placed in series with the meter-movement. Its value
should be such that full scale deflection will be obtained when connected across V volts.

Applying Ohm's law in the figure,

$$V = I_g (R_g + R_h)$$

or $$R_h = \frac{V}{I_g} - R_g$$
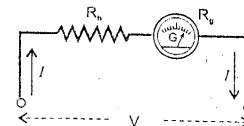
By properly arranging the resistance R_h any voltage can be measured.

Current Generator

Definitions:

"A current generator is a device that converts mechanical energy into electrical energy".

"An alternating current generator is a device that converts mechanical energy into electrical energy and generates alternating emf and current".

Principle:

It is based on Faraday's law of electromagnetic induction. When a coil is rotated in a magnetic field by some mechanical means, magnetic flux through the coil changes, and consequently an emf is induced in the coil.

Construction:

An A.C. generator consists of:
1. A rectangular coil of many turns wound around an iron core, called armature.
2. The magnetic field is usually provided by an electromagnet.
3. Both ends of the coil attached with two metal rings called slip rings. They are concentric with the axis of the loop and rotate with it.
4. The two slip rings slide against stationary carbon brushes to which external circuit is connected.

Working:

The armature is rotated in the magnetic field whose ends are attached with slip rings to slide against carbon brushes, which are connected to an external circuit. An electric current is the output of the generator.
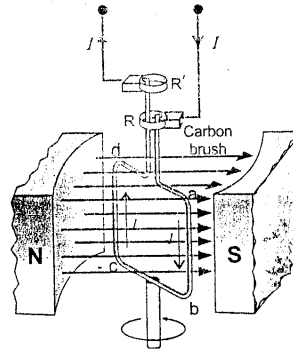
Calculating induced emf:

From Faraday's law,
$$\varepsilon = vBL\sin\theta$$
In the above figure,
$$\varepsilon = \varepsilon_{ab} + \varepsilon_{cd} + \varepsilon_{bc} + \varepsilon_{da}$$

$$\varepsilon_{bc} = \varepsilon_{da} = 0 - v \ \& \ B \text{ have same plane}$$
$$\varepsilon_{ab} = \varepsilon_{cd} = vBL\sin\theta$$

or $\varepsilon = vBL\sin\theta + vBL\sin\theta + 0 + 0$
$$\varepsilon = 2vBL\sin\theta$$
for N turns of the coil
$$\varepsilon = 2NvBL\sin\theta$$
$$= 2N(r\omega)BL\sin\theta \qquad [v = r\omega$$
or $\varepsilon = N\omega(2rL)B\sin\theta \qquad [A = 2rL = \text{area of the coil}$
or $\varepsilon = N\omega AB\sin\omega t \qquad .....(1) \qquad [\theta = \omega t$

or $\varepsilon_0 = N\omega AB \qquad .....(2) \qquad [\text{for max. value: } \sin\omega t = 1$

from equations (1) & (2), we have
$$\varepsilon = \varepsilon_0 \sin\omega t$$
or $\varepsilon = \varepsilon_0 \sin 2\pi ft \qquad .....(3) \qquad [\omega = 2\pi f$

from Ohm's law,

$$I = \varepsilon / R \qquad [R = \text{resistance of the coil}$$

$$\text{or} \quad I = \frac{\varepsilon_0 \sin 2\pi ft}{R}$$

$$[\varepsilon_0 / R = I_0$$

$$\text{or} \quad I = I_0 \sin 2\pi ft \qquad .....(4)$$

Graph for current variation verses θ:

From the above eq. (4) , we have

$$I = I_0 \sin \theta \qquad [\theta = 2\pi ft$$

This equation indicates the variation of current as a function of θ.

The figure shows the graph for the current corresponding to different positions of one loop of the coil.

1. For angle between v & B is $\theta = 0$,
   the plane of the loop is perpendicular to B and current is zero.
2. As θ increases, for $\theta = 90^\circ = 2\pi$ rad,
   the current is maximum.
3. For $\theta = 180^\circ$, the current is zero.
4. For $180^\circ < \theta < 270^\circ$, the current increases but reverses its direction.
5. At $\theta = 270^\circ = 3\pi / 2$ rad, the current is maximum in reverse direction.
6. At $\theta = 360^\circ = 2\pi$ rad, agsain current is zero.
   Such current, which alternates in direction once in a cycle, is called alternating current.
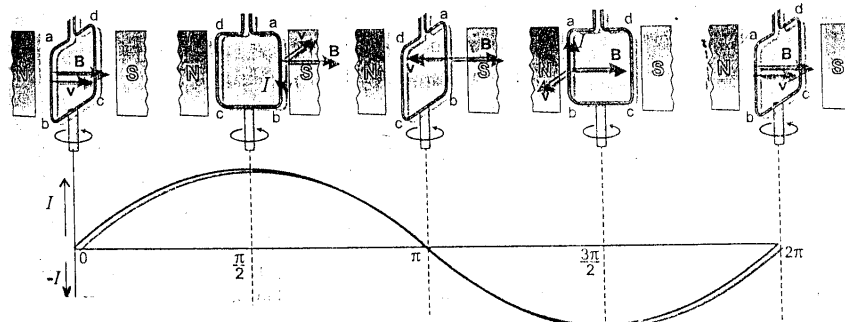

D.C. Generator:

D.C. generator is similar to the A.C. generator in construction with the difference that "slip rings" are replaced by "split rings". The split rings are two halves of a ring that act as a commutator. It is shown in the figure.

When the current in the coil is zero and is about to change direction the split rings also change the contacts with the carbon brushes, so that output remains in the same direction, although the current is not constant in magnitude.

To make emf in the outer circuit constant, multiple coils are wound around a cylindrical core to form the armature.
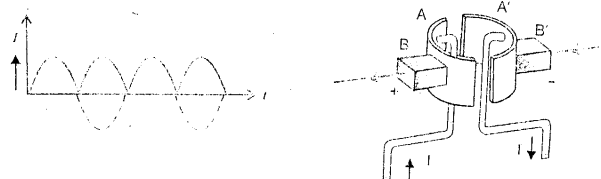
Transformer

Definition:
"A transformer is an electrical device to change a given alternating emf into a larger or smaller alternating emf".

Principle:
It works on the principle of <u>mutual induction</u> between two coils. A changing current in the primary coil induces an emf in the secondary coil.

Construction:
It consists of two coils of copper, electrically insulated from each other, wound on the same iron core. The coil to which A.C. power is supplied is called <u>primary</u> and that from which power is delivered to the circuit is called the <u>secondary.</u>

Working:

There is no electrical connection between the primary and secondary coils, but they are magnetically linked. When an alternating emf is applied to the primary, there will be back emf induced in the primary, which will oppose the applied voltage. If the resistance of the coil is negligible then the back emf is equal and opposite to applied voltage.
When two coils are tightly coupled, the flux through the primary will also pass through the secondary. So the electrical power in a transformer is transformed from its primary to the secondary coil by means of changing flux.

Transformer equation:

Transformer symbol

We have from Faraday's law,

$$\varepsilon = -N \, (\Delta\phi / \Delta t) \qquad \ldots\ldots(1)$$

for self induced emf in the primary coil,

$$\varepsilon_{self} = -N_p \, [\Delta\phi / \Delta t] \qquad \ldots\ldots(2)$$

if the resistance of the coil is negligible, then

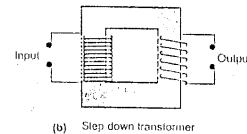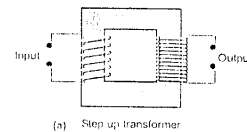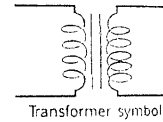$$V_p = - \text{back emf} = N_p \, [\Delta\phi / \Delta t] \qquad \ldots\ldots(3)$$

Since the two coils are coupled,

$$V_s = N_s \, [\Delta\phi / \Delta t] \qquad \ldots\ldots(4)$$

From eqs. (3) & (4), we have

$$\boxed{\frac{V_s}{V_p} = \frac{N_s}{N_p}} \qquad \ldots\ldots(5)$$

which is transformer equation.

(a) Step up transformer

(b) Step down transformer

Step up transformer:
A transformer in which voltage across secondary is greater than the primary voltage.
In equation (5). if $N_s > N_p \Rightarrow V_s > V_p$

Step down transformer:
A transfomer in which voltage across secondary is less than the primary voltage.
In equation (5). if $N_s < N_p \Rightarrow V_s < V_p$

Power & its transmission:

For an ideal case in the transformer,

$$P_{in} = P_{out}$$

or $\quad V_p I_p = V_s I_s \qquad [P = V I$

or $\quad \dfrac{V_s}{V_p} = \dfrac{I_p}{I_s} \qquad\qquad .....(6)$

Thus the currents are <u>inversely</u> proportional to the respective voltages.

This principle is used in the electric supply network where transformer increases the voltage and reduces the current so that it can be transmitted over long distances without much power loss. When current I passes through resistance R, the power loss due to heating effect is $I^2 R$.

Power losses:

Only in an <u>ideal transformer</u> the output power is nearly equal to the input power. But in an <u>actual transformer</u>, the output is always less than input due to power losses. There are two main causes of power loss, namely—<u>eddy currents</u> and <u>magnetic hysteresis</u>.

Efficiency:

The efficiency, E of a transformer is defined as:

$$E = \dfrac{P_{output}}{P_{input}} \times 100$$

To increase the efficiency, power losses should be minimized.

The core should be assembled from <u>laminated sheets</u> of a material whose <u>hysteresis loop area is very small.</u>

The insulation between lamination sheets should be perfect so as to <u>stop the flow of eddy currents.</u>.

The <u>resistance</u> of the two coils should be <u>minimum</u>.

The primary and secondary coils should be wound in such a way that <u>flux coupling</u> between them is <u>maximum</u>.

Applications:

1. For long distance power transmission, current I is reduced. As reducing R by using thick copper wire is uneconomical. $[P = I^2 R]$
   At generating power station, the voltage is stepped up to several thousand volts and power is transmitted at low current.
2. In electronic devices, such as, transistor radio and mobile set, transformers are used ; e.g. from 250 volts to 9volts / 3 volts.
3. Used in computers, TV and radio sets where several different voltages are required.

R-C and R-L Series Circuits

Definitions:

Resistor: A component included in the electric circuit because of its resistance.

Capacitor: A combination of conducting plates separated by an insulator and used to store an electric charge.

Inductor: A coil or other piece of apparatus, possessing inductance and selected for use because of that property.
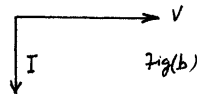
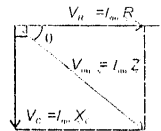In purely resistive a.c. circuit, current and voltage are in phase.
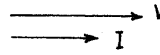
In purely capacitive a.c. circuit, current leads the voltage by 90°

In purely inductive a.c. circuit, current lags behind the voltage by 90°

fig.(a)

fig.(b)

Taking the current as reference, combining fig. (a) & fig. (b), we get

fig.(c)

Applying Kirchhoff's voltage rule to fig. (d), we have

fig.(d)

$$V_{rms} - V_R - V_C = 0$$
$$\text{or } V_{rms} = V_R + V_C \qquad .....(1)$$

equation (1) can be solved by a variety of techniques: a trigonometric analysis, a graphical analysis using vectors, and a differential analysis.

Using graphical analysis to fig. (e), we have
$$V_{rms} = I_{rms} Z$$

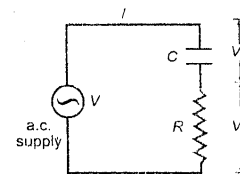To find V_{rms}, applying Pythagorean theorem to the fig, we have

$$V_{rms} = \sqrt{(I_{rms} R)^2 + (I_{rms} X_C)^2}$$

$$\text{or } V_{rms} = \sqrt{(I_{rms} R)^2 + (I_{rms} / \omega C)^2} \qquad [\text{we define: } X_C = 1 / \omega C$$

$$\text{or } V_{rms} = \sqrt{(I^2_{rms} [R^2 + (1 / \omega C)^2]}$$

$$\text{or } V_{rms} = I_{rms} \sqrt{R^2 + (1 / \omega C)^2}$$

$$\text{or } Z = V_{rms} / I_{rms} = \sqrt{R^2 + (1 / \omega C)^2} \qquad [\text{ Impedance : } Z = V_{rms} / I_{rms}$$

To find angle θ

$$\tan\theta = V_C / V_R$$
$$= \mp X_C / \mp R$$
$$= X_C / R$$

or $\theta = \tan^{-1} (1 / \omega CR)$ $[X_C = 1 / \omega C$

To calculate the Impedance of R-L series circuit

Taking current as reference and knowing

$$\tan\theta = V_L / V_R$$
$$= \mp X_L / \mp R$$
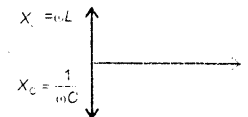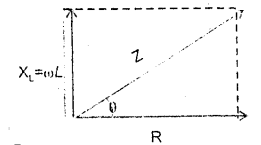$$= X_L / R$$

combining fig. (a) & (c), gives the fig. Shown

Applying Pythagorean theorem to the fig.

$$Z^2 = R^2 + X_L^2$$

or $Z = \sqrt{R^2 + (\omega L)^2}$ [we define: $X = \omega L$

     & $\theta = \tan^{-1} \omega / R$

Comparing the impedance diagrams of R-C and
R-L series circuits, shows that the vector lines of
reactances X_C and X_L are directed opposite to
each other with R as reference.
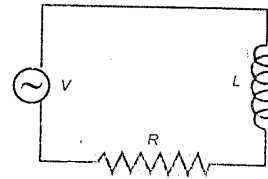
Resonance

Definition:
"The condition in an a.c. circuit in which the inductive reactance and capacitive reactance are
equal and cancel each other. $X_L = X_C$ "

R-L-C series resonance circuit sometimes called **acceptance circuit.** It is used as a tuning
circuit with the antenna of a radio/TV receiving station.

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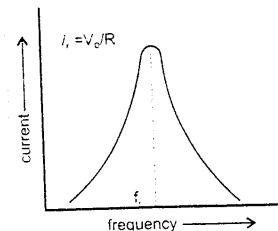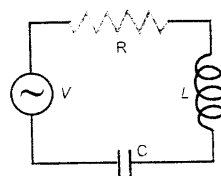
In fig. (a) R-L-C series circuit is shown, and in fig. (b) is its impedance diagram.
The inductive reactance: $X_L = \omega L$ .....(1)

and capacitor reactance: $X_C = 1 / \omega C$ .....(2)

the both are directed opposite to each other.
<u>At low frequencies</u> from equations (1) & (2)

$$X_C \gg X_L$$

So capacitance dominates and circuit behaves like an R-C circuit.

<u>At high frequencies,</u>

$$X_L \gg X_C$$

So inductance dominates and circuit behaves like R-L circuit.

In between these frequencies, there will be a frequency ω_r for which
$$X_L = X_C \quad .....(3)$$

The above condition is called <u>resonance.</u>
At resonance a.c. voltage across L [V_L] and across C [V_C] are equal and $180°$ out of phase or in **anti phase.**
The ratio across inductance (or capacitor) to voltage across resistor or applied voltage <u>at resonance</u> is called **voltage magnification factor** of the series circuit.

Calculating resonance frequency [from eqs. (1), (2) & (3)]

$$\omega_r L = 1 / \omega_r C \quad \Rightarrow \quad \omega_r = \sqrt{1 / LC}$$

or $\quad f = \dfrac{1}{2\pi\sqrt{LC}}$ $\quad [\ \omega = 2\pi f$

Properties

1. The resonance frequency f_r is

$$f_r = \dfrac{1}{2\pi\sqrt{LC}}$$

2. The current and voltage are in phase. The power factor is 1.
3. The impedance will be minimum and is equal to R.
4. Fig. (c) shows the variation between frequency and current.
5. At resonance, V_L and V_C will be very large from V_{source}

Magnetic Properties of Solids

Magnetism:

"A property that all materials posses as a result of the motion of their electrons".

Origin:

Magnetism is due to the spin and orbital motion of the electrons surrounding the nucleus and is a property of all substances. Each electron orbiting the nucleus behaves like an atomic sized loop of current that generates a small magnetic field. Also each electron possesses a spin that also gives rise to a magnetic field. An atom in which there is a resultant magnetic field, behaves like a tiny magnet and is called a magnetic dipole.

Types:

1. **Paramagnetic substances**
   The substances in which, the orbits and the spin axes of the electrons in the atom are so oriented that their fields support each other and the atoms behaves like a tiny magnet.
   e.g. transition metals (such as scandium, vanadium, etc.) and rare earth elements (such as lanthanium, neodymium, etc.)

2. **Diamagnetic substances**
   The substances in whose atoms, there is no resultant field as the magnetic fields produced by both orbital and spin motions of the electrons might add up to zero.
   e.g. atoms of water, copper, bismuth and antimony.

3. **Ferromagnetic substances**
   The substances in which, the atoms cooperate with each other in such a way so as to exhibit a strong magnetic effect.
   e.g. iron, cobalt, nickel, chromium dioxide, and Alnico.
   [Alnico: Alloy for making magnets; an alloy of iron, aluminum, and nickel together with one or more of cobalt, copper, and titanium, used for making strong permanent magnets.]

Domains:

"A region inside a ferromagnetic material in which all the atomic magnetic fields point the same way".

Domains exist in ferromagnetic substances. Their size are of the order of millimeter or less but large enough to contain 10^{12} to 10^{16} atoms. Each domain is magnetized to saturation. They behave as a small magnet with its own north and south poles. In unmagnetized iron, the domains are oriented in a disorderly fashion. When the specimen is placed in an external magnetic field, the entire specimen becomes saturated. The combination of a solenoid and a specimen of iron I side it thus makes a powerful magnet and is called an electromagnet.

Soft magnetic material:

Such material in which domains are easily oriented on applying an external field and also readily return to random positions when the field is removed. For example iron. This is desirable in an electromagnet and also in transformers.

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Hard magnetic material:

Such material in which domains are not so easily oriented to order. They require very strong external fields, but once oriented, retain the alignment. For example, steel and Alnico V, makes a good permanent magnet.

Effect of temperature:

Curie temperature

"The temperature above which a ferromagnetic substance loses its ferromagnetism".

Thermal vibrations tend to disturb the orderliness of the domains. Ferromagnetic materials preserve the orderliness at ordinary temperatures. When heated, they begin to loose their orderliness due to increased thermal motion. This process begins at Curie temperature. For iron its value is 750 °C. Above this temperature iron is paramagnet but not ferromagnetic.

Hysteresis & Hysteresis Loop:

Hysteresis

The lagging of magnetization of ferromagnetic material behind the magnetizing force.

Hysteresis loop

The loop formed by magnetic hysteresis.

Saturation

The magnetic flux density increases from zero and reaches a maximum value. At this stage the material is said to be magnetically saturated.

(a) **Hysteresis loop of steel**

Remanence (or Retentivity)

i) The residual magnetic flux density in a substance when the magnetizing field strength is returned to zero.
ii) When substances are applied forces for producing magnetization and then force removed then power of retaining their original magnetization is called retentivity.
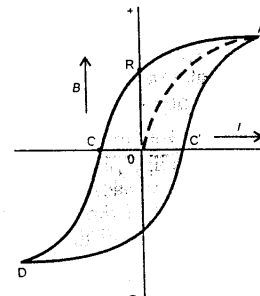
Coercivity

Degree of reversed magnetizing force required to deprive the metal of the whole of its original magnetization.
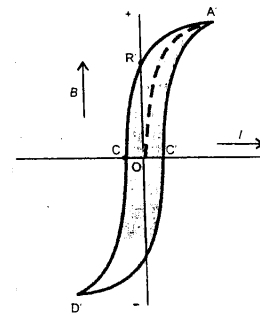
Hysteresis loss

The dissipation of energy that occurs, due to magnetic hysteresis, when the magnetic material is subjected to cyclic changes of magnetization.

(b) **Hysteresis loop of soft iron**

OR = Retentivity
OC = Coercivity

Area of the loop

The area of the loop is a measure of the energy needed to magnetize and demagnetize the specimen during each cycle of the magnetized current. This is the energy required to do work against internal friction of the domains. Hard magnetic materials (such as steel) have large loop area and soft magnetic materials (such as iron) have small loop area.
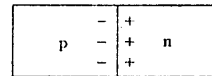
Semi-conductor Diode

Definition:
"It is a semi-conductor device, made up of a junction between p-type and n-type materials'.

Explanation:
Semiconductor diode contains donor impurities on one side and acceptor impurities on the other side of a single crystal of Germanium or Silicon. A diffusion of electrons and holes takes place across the junction which stops after setting up a potential difference at the junctions, called depletion or barrier region.
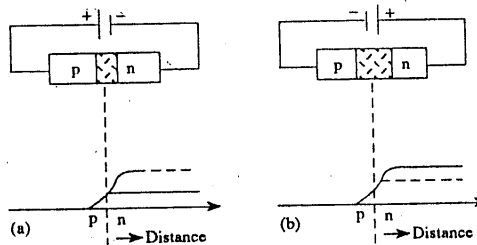
Properties:
The semiconductor diode allows the current to flow only in one direction.

Forward Bias
If we connect p-type material of the junction with +ve terminal of the battery and n-type material with –ve terminal as shown in the figure(a)The height of the potential barrier is reduced and it gives easy flow of current.

Reverse Bias
When n-type material of the junction is connected to +ve terminal of the battery and p-type with –ve terminal, the height of the potential barrier at the junction increases and it makes the flow of current difficult across the p-n junction.

## Diode as a Rectifier:
Rectifier
"An electrical conductor that allows current to flow through it in one direction only, thus enabling the conversion of a.c. to d.c.".
The diode easily passes current in the forward direction with forward bias. The flow of current is practically zero in reverse direction, with reverse biased.
It can act as a switch. Due to this property it is used for rectification.

Half Wave Rectifier:
The following figure (a) shows half wave rectifier circuit with input voltage [fig. (b)] and ouput voltage [fig. (c)].
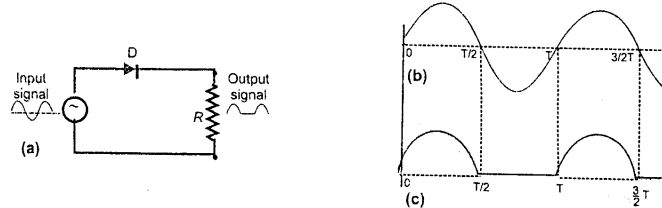
During +ve half cycle, the current flows in the circuit as a pulse of half sine wave form. During the next negative half cycle, the anode is –ve and it is reverse biased. No current exists in the load as shown in fig. (c).

Full Wave Rectifier:
The full wave rectification is obtained by using two diodes and centre tapped transformer as shown in the figure (a)
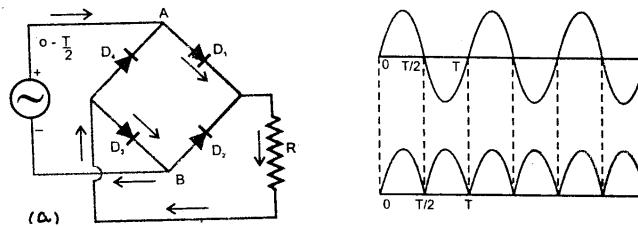
In the first half cycle, diode D_1 is forward biased and conducts a current pulse to the load, but D_2 is reverse biased and appears as an open circuit.
In the second half cycle, the transformer polarity reverses and D_2 is forward biased and allows current pulse to the load, but D_1 is reverse biased and open. Thus direction of current through load R remains the same when either of the diode conducts.

Specially Designed p-n Junctions:

1. **Light emitting diode**
   A semiconductor diode, made from certain materials (e.g. gallium arsenide), in which light is emitted in response to the forward-bias current.
2. **Photo diode** A semiconductor diode that produces a significant photo-current when illuminated.
3. **Photo voltaic cell** An electronic device that uses the photovoltaic effect to produce an emf. For example a solar cell.

Applications of photo diodes are:

i)   Visible & invisible detection,
ii) Automatic switching,
iii) Logic Circuits,
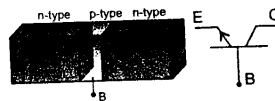iv) Optical communication equipments.

Transistors

Definition:

__Transistor__

i) A transistor consists of a single crystal of germanium or silicon which is grown in such a way that it has three regions.

ii) A semiconductor device used as a substitute for vacuum tubes in electronic operations.

Types & Symbol:

1. __npn transistor__

2. __pnp transistor__
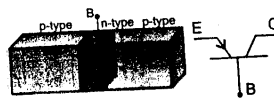
Construction:

The central region is known as base and the other two regions are called emitter and collector. Usually the base is very thin $\sim 10^{-6}$ m. The collector is comparatively larger than the emitter.

Operation:
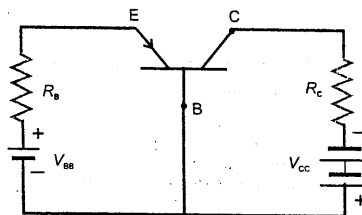
For normal operation, batteries V_{BB} and V_{CC} are connected in such a way that its emitter-base junction is forward biased and its collector-base junction is reverse biased. V_{CC} is of much higher value than V_{BB}.

Fundamental equations:

In npn transistor <u>conventional current</u> I_E flows from base to emitter. A very small part of it, current I_B flows in the base, the rest of it I_C flows in the collector.

From the above figure,

$$I_E = I_C + I_B$$

It is found that the current gain β is constant for given transistor.

$$\beta = I_C / I_B$$

Transistor as an amplifier:

Transistors are basically used as amplifiers in major electronic circuits. It is the building block of every complex electric circuit.
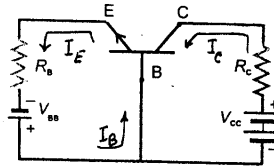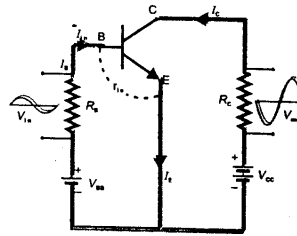
In the figure,

$$V_{CC} \gg V_{BB}$$

V_{BB} is forward biased

V_{CC} is reverse biased

$\Delta I_B = I_{in}$ --the change in base current

& $\Delta I_C = I_{out}$ --the change in collector current

r_{ie} is internal resistance between base and emitter

so $I_{in} = V_{in} / r_{ie}$ as $R_B \gg r_{ie}$

& $I_{out} = V_{out} / R_C$

we have

$$\beta = \frac{\Delta I_C}{\Delta I_B} = \frac{I_{out}}{I_{in}} = \frac{V_{out} / R_C}{V_{in} / r_{ie}}$$

or voltage gain $= \dfrac{V_{out}}{V_{in}} = \beta \dfrac{R_C}{r_{ie}}$

For typical values, $\beta = 50$, $R = 10\ K\Omega$, $r\ 1\ K\Omega$; the gain is 500.

Operational Amplifier

Definition:

1. "A high gain integrated voltage amplifier".
2. "A very high gain voltage amplifier, with very high input impedance, usually having output voltage proportional to much greater voltage difference between its two inputs".

It is sometimes used to perform mathematical operations electronically.
An op-amp is usually a multistage device designed for insertion into other equipment. It is supplied as a complete packaged unit, commonly as a single integrated circuit.

Symbol:
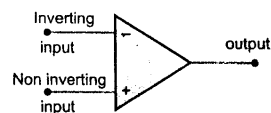
Symbol with voltage supplies:

Signals:
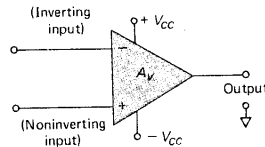
Inverting:
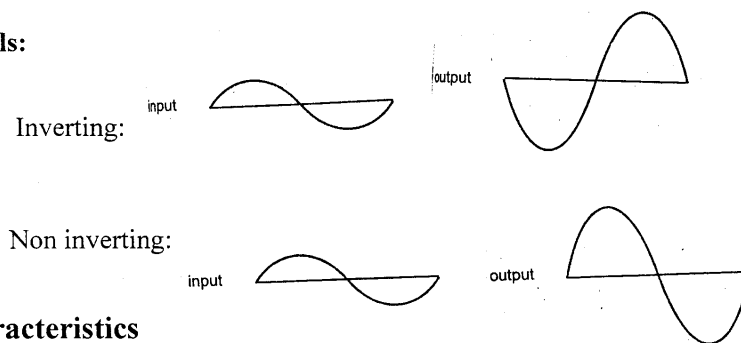
Non inverting:

Characteristics

1. Input Resistance (R_{in})

The resistance between the inverting and non-inverting inputs of the op-amp. Typically, this value is in excess of 1 MΩ and may be as high as 10^{12} Ω. Due to high R_{in}, no current flows between the two input terminals.

2. Output Resistance (R_o)

The resistance between the output terminal and ground. Typically, this value is less than 50 Ω.

3. Open Loop Gain (A_{OL})

It is the ratio of output voltage V_o to the voltage difference between inverting and non-inverting inputs when there is no external connection between the output and the inputs. Mathematically,

$$A = \frac{V}{V_+ - V_-} = \frac{V_o}{V_i}$$

4. Input voltage Range (V_i)

The maximum positive and negative voltage that may be supplied to either input terminal and maintain proper bias of the input transistor.

5. Output Voltage Swing (V_o)

The maximum excursion (positive and negative) of the output voltage specified for specific power supply voltages. Typically the swing is within one or two V_{BE} of supply voltage.

Inverting Amplifier

The op-amp shown wired as an inverting amplifier.
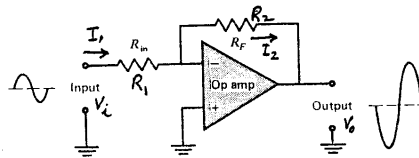
Note the use of a feedback resistor R_F (or R_2) and an input resistor R_{in} (or R_1). The signal is applied to the inverting (-) input. The waveform signal has been amplified.

Virtual ground:

A condition called 'virtual ground' exists at the inverting terminal. The virtual ground is unlike a typical ground. Here (-) terminal is at ground potential even though there is no physical connection between the inverting and ground. Because the voltage is zero at the inverting terminal even though the impedance at the terminal is not zero, no current flows into the inverting terminal.

$$\text{So } V_- \approx 0$$

Closed loop gain:

In the figure:
The non-inverting terminal is grounded.
The concept of a virtual ground leads:

$$I_1 \text{ (current through } R_1) = \frac{V_i - V_-}{R_1} = \frac{V_i - 0}{R_1}$$

$$\text{or} \quad I_1 = V_i / R_1 \qquad \qquad .....(1)$$

& I_2 (current through R_2) $= \dfrac{V_- - V_o}{R_2} = \dfrac{0 - V_o}{R_2}$

or $I_2 = - V_o / R_2$ .....(2)

Applying Kirchhoff's current rule, as no current flows between (-) & (+) terminals.

$$I_1 = I_2$$

From equations (1) & (2), we have

$$\dfrac{V_i}{R_1} = - \dfrac{V_o}{R_2}$$

or $$\dfrac{V_o}{V_i} = - \dfrac{R_2}{R_1}$$

or $$G = - \dfrac{R_2}{R_1}$$

It shows that the output signal is $180°$ out of phase w.r.t. input signal, and the gain depends only upon externally connected resistances R_1 & R_2 and is independent of happening inside the amplifier.

The **voltage gain** of the op amp is set by the value of the resistors R_1 & R_2 .
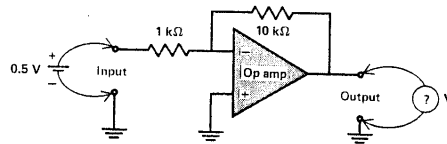
For $R_1 = 1000\ \Omega$, $R_2 = 10,000\ \Omega$

$G = - R_2 / R_1 = 10,000 / 1000 = -10$

Suppose the input voltage V is +0.5V. the output voltage V would be calculated as

$V_{out} = G \times V_{in} = - 10 \times 0.5$

or $V_{out} = -5V$

The output voltage is negative because the inverting input to the op amp was used, causing a polarity reversal.

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Non-Inverting Amplifier

A non-inverting amplifier circuit using an op amp is shown in the diagram.
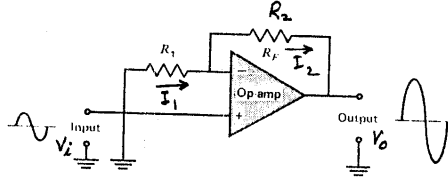
Note that the signal goes into the non-inverting input.

R_F and R_1 are still connected in the same way as for inverting amplifier.
The waveform shows voltage gain without change of phase.

Closed loop Gain (G):

$$A_V = \frac{R_1 + R_F}{R_1} = \frac{1000 + 10,000}{1000} = \frac{11,000}{1000} = 11$$

Voltage gain = 11
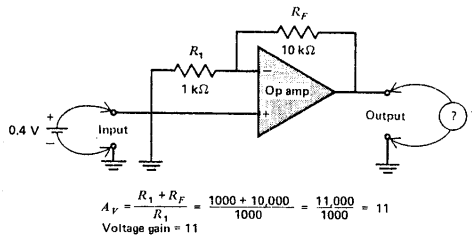
since (-) and (+) inputs are virtually at the same potential,

$$V_- \approx V_+ \approx V_i$$

So from the above figure

$$I_1 = \frac{0 - V_-}{R_1} = \frac{0 - V_i}{R_1} = -\frac{V_i}{R_1} \qquad .....(1)$$

& $$I_2 = \frac{V_- - V_o}{R_2} = \frac{V_i - V_o}{R_2} \qquad .....(2)$$

Applying Kirchhoff's current rule, as no current flows between (-) & (+) terminals.

$$I_1 = I_2$$

or $$= \frac{-V_i}{R_1} = -\frac{V_i - V_o}{R_2}$$

or $$= \frac{V_o}{R_2} = \frac{V_i}{R_2} + \frac{V_i}{R_1}$$

or $$V_i \left(1/R_1 + 1/R_2 \right) = V_o / R_2$$

or $$G = V_o / V_i = 1 + R_2 / R_1$$

It shows that G is independent of internal structure of op amp & depends only on R_1 & R_2

Logic Gates

Logic gate (or Logic circuit)

i) An electronic circuit that can be used to perform simple logical operations.
ii) An electronic circuit designed to perform a particular logical function based on the concepts of "and", "either-or", "neither-nor", etc. Normally these circuits operate between two discrete voltage levels, i.e. high and low logic levels, and described as binary logic circuits.

OR gate:

Definition:
A circuit with two or more inputs and one output whose output is high if any one or more of the inputs are high.

Symbol:

Truth Table

A	B	Output
0	0	0
0	1	1
1	0	1
1	1	1

Mathematical notation is: $X = A + B$

AND gate:

Definition:
A circuit with two or more inputs and one output in which the output signal is high if and only if all the inputs are high simultaneously.

Symbol:

Truth Table

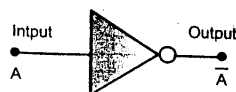| A	B	Output
0	0	0
0	1	0
1	0	0
1	1	1

Mathematical notation is: $X = A \cdot B$

NOT gate (or Inverter):

Definition:
A circuit with one input whose output is high if the input is low and vice versa.

Symbol:

Truth Table

A	Output
0	1
1	0

Mathematical notation is: $X = \overline{A}$

NOR gate:

Definition:

A circuit with two or more inputs and one output, whose output is high if and only if all the inputs are low.

Symbol:
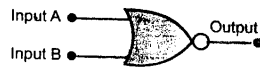
Truth Table

A	B	Output
0	0	1
0	1	0
1	0	0
1	1	0

Mathematical notation is: $X = \overline{A + B}$

NAND gate:

Definition:

A circuit with two or more inputs and one output, whose output is high if any one or more of the inputs is low, and low if all the inputs are high.
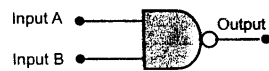
Symbol:

Truth Table

A	B	Output
0	0	1
0	1	1
1	0	1
1	1	0

Mathematical notation is: $X = \overline{A \cdot B}$

Exclusive OR gate(XOR):

Definition:

A circuit with two or more inputs and one output whose output is high if any one of the inputs is high.

Symbol:

Truth Table

A	B	Output
0	0	0
1	0	1
0	1	1
1	1	0

Construction by combining AND, OR and NOT gates.

Mathematical notation is: $X = A\overline{B} + \overline{A}B$

Exclusive NOR gate(XNOR):

Definition:

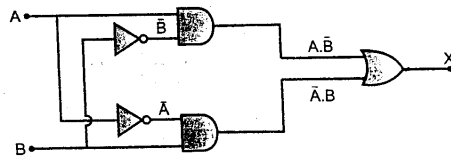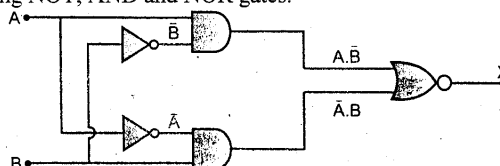A circuit with two or more inputs and one output, whose output is high if two inputs are identical and low when two inputs are different.

Symbol:

Truth Table

A	B	Output
0	0	1
0	1	0
1	0	0
1	1	1

Construction by combining NOT, AND and NOR gates.

Mathematical notation: $X = A\bar{B} + \bar{A}B$

Applications:

Logic gates are widely used in **control systems.**

They control the function of the system by <u>monitoring</u> some physical parameters, e.g. temperature, pressure, sound, light, etc. with the help of <u>sensors.</u>

[**Sensors** are the devices which convert the physical quantities into electrical voltage.]

1. **LDR**—to convert light intensity into electrical voltage for <u>night switch.</u>
2. **Thermister**—to convert temperature into electric voltage for <u>sensitive thermometers.</u>
3. **Microphone**—to convert sound into electrical voltage in <u>audio amplifiers</u>.
4. **Level sensors**—to convert level of liquid, to attain certain limit.

A Practical Example

Sensors are used to monitor the pressure and temperature of a <u>chemical solution stored in a container.</u>

A	B	C
0	0	1
0	1	0
1	0	0
1	1	0

T = Temperature sensor
P = Pressure sensor
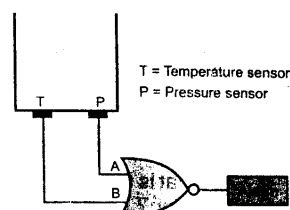
The designed NOR Gate should be such that when either the temperature or pressure are within the specified limits, the circuitry for each sensor procuess a HIGH (1), i.e..
when A = B = 0
When the limit of T and P is exceeded i.e.
when A = 0, B = 1 or when A = 1, B = 0, or when A = B = 1, the output of the circuit is LOW (0), the alarm is activated.

Theory of Relativity

Reference Frames and Relativity

To see things from someone else's point of view is sometimes difficult.

If you by pass a slowly moving car, you might notice that the other car is moving backward. The other person in other car might claim that your car is moving forward.

Describing events from different reference frames is the subject of **relativity.** That word usually implies Eintein's theories of special and general relativity. The special theory of relativity show up only when an object is moving nearly the speed of light. The general theory of relativity accounts for what happens in reference frames that are accelerating with respect to our own.

If you are sitting in a car and that is racing forward on start off, are you accelerated forward or backward ? The observer on the road would say that you are certainly accelerated forward. You yourself know that you are thrown backward—against your seat. If your car is turning around a corner to the left, are you thrown toward the door or in the other direction. An observer watching from a roadway above would conclude that you started to travel along a line tangential to the car's curved path. In both these cases, it appears that the direction of forces and the consequent accelerations depend on the reference frame in which they are described.

Newton's three laws of motion apply only within the reference frame at rest or moving with constant velocity with respect to fixed points. Such a frame is called _inertial reference frame—a reference frame in which law of inertia hold._

In accelerated reference frame or _non-inertial reference frame_ is that in which law of inertia does not hold.

Special theory of Relativity

The Special theory of relativity treats problems involving inertial or non-accelerating frames of reference.

General theory of relativity treats problems involving accelerated or non-inertial frames of reference.

Postulates of Special theory of relativity :

Special theory of relativity based on Eintein's two postulates:

1. The laws of Physics are the same in all inertial frames of reference.
2. The velocity of light in free space is a constant c regardless of the state of motion of the source.

Results of Special theory of relativity

There are four distinct and astonishing results of special theory of relativity.

1. **Increase of mass**

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

2. **Length contraction**

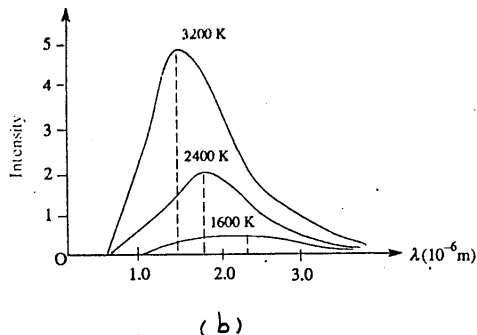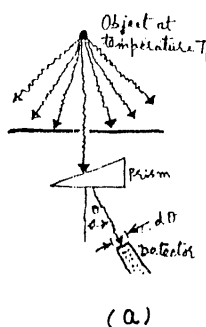$$\ell = \ell_0 \sqrt{1 - v^2/c^2}$$

3. **Time dilation**

$$t = \frac{t_0}{\sqrt{1 - v^2/c^2}}$$

4. **Mass-Energy conversion**

$$E = m c^2$$

Black Body Radiation

The classical wave theory of electromagnetic radiation failed to explain the observed spectrum of thermal radiation, emitted by all objects due to their temperature.

(a)

(b)

An experimental arrangement is shown in fig. (a). And the results are shown in fig. (b).
An object if maintained at a temperature T_1 . The radiation emitted by the object is detected by an apparatus that is sensitive to the wavelength of the radiation.
The following two laws give details of the properties of thermal radiation.

Wien's Displacement Law:

"The wavelength of maximum intensity (λ_m) is inversely proportional to the absolute temperature of the black body". Mathematically

$$T \times \lambda_m = constant$$

Stefan Boltzmann Law:

"The total amount of heat radiated by a perfectly black body per second per unit area is directly proportional to the fourth power of its absolute temperature". Mathematically

$$E = \sigma T^4$$

Energy distribution formulae

The intensity of radiation emitted by a black body is not uniformly distributed over the whole range of wavelengths involved. The problem is to be solved, to see how the energy of total radiation from a black body is distributed among the different wavelengths at various temperatures.

Wien's Law:

According to Wien, energy is distributed among different wavelengths according to the following formula:

$$E_\lambda = \frac{c_1 \lambda^{-5}}{e^{c_2/\lambda T}}$$

It is excellent for short wavelength but not for long wavelength.

Rayleigh-Jeans Law:

According to Rayleigh-Jeans:

$$E_\lambda = \frac{8 \pi kT}{\lambda^4}$$

The curve fits well for long wavelengths, but it leads to infinity at short wavelengths.
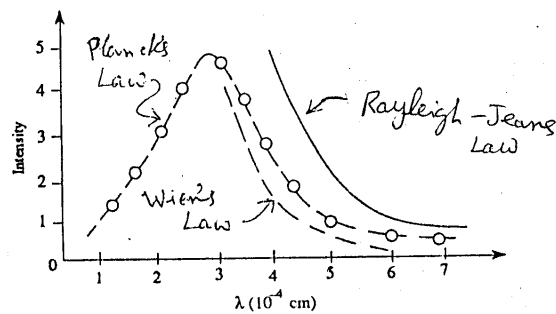
Planck's Radiation Law:

According to Planck, the formula for the distribution of energy in a black body radiation spectrum is given below:

$$E_\lambda = \frac{8 \pi}{\lambda^4} \frac{hf}{e^{hf/kT} - 1}$$

The curve shown in the figure representing this equation fits well with the experimental curve.

Photoelectric Effect

Statement:

"The emission of electrons by a substance when illuminated by electromagnetic radiation".

Explanation:

It is observed that the intensity of the spark between the two high voltage electrodes is increased by illuminating the electrodes by ultraviolet light. This increase in intensity is due to electrons ejected from the metal surface, which is photoelectric effect.

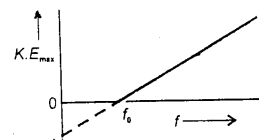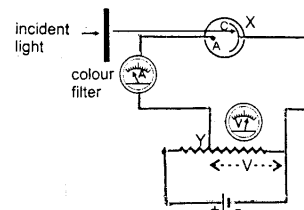## Experimental Observations:

Experimental observations by different workers are summarized here.

1. The rate of emission of photoelectrons is directly proportional to the intensity of the incident light.
2. The kinetic energy of photoelectrons does not depend upon the intensity of light, but it depends upon the frequency of the light.
3. Maximum energy of photoelectrons increases with increasing frequency above the threshold frequency.
4. Photoelectrons are emitted from the given metal surface when the frequency of the incident light is above a certain critical value f_o, which is called threshold frequency.
5. No photoelectric emission occurs if the frequency of light is less than that of the threshold frequency, even if the intensity of light is very high.
6. The photoelectrons are emitted instantaneously even when the weakest possible beam of light having frequency above the threshold frequency falls on the metal plate.

The above first three observations are called the three laws of photoelectric emission.

The classical electromagnetic theory of light completely failed to explain last two observations.

Work function (ϕ_o)

"The minimum energy required to remove an electron from the surface of a material and send it into field-free space".

Einstein's Explanation:

Einstein proposed that light is propagated in the form of photons, bursts of energy. Each photon of frequency carries an energy $E = hf$. When a beam of photons in the incident light strikes on the metallic surface, each photon imparts all its energy to one and only one electron. If energy of the photon hf is greater than or equal to the work function ϕ_o, only then the photon is capable of ejecting out the electron.
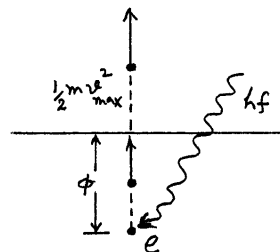
Einstein's Photoelectric Equation:

If photon energy hf is greater than work function ϕ, than the electron having mass m is ejected out of the metal surface with velocity v_{max}.
We have (from the figure)

$$hf = \phi + \tfrac{1}{2} m v^2_{max}$$

or $\boxed{hf - \phi = \tfrac{1}{2} m v^2_{max}}$

which is Einstein's photoelectric equation.

Compton Effect

Statement:

"The phenomenon in which an x-ray photon is scattered from an electron, the scattered photon having the smaller frequency than the incident photon"

Explanation:

According to the classical theory, the wavelength of x-rays scattered by electrons must be equal to the wavelength of primary x-rays. But a change in wavelength has been observed by experiments. This is called the Compton effect.

Illustration:

Suppose an x-ray photon of energy hf collides with an electron at rest and is then changed in direction and frequency as shown in the figure.
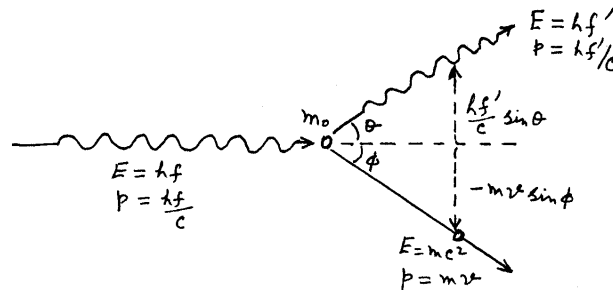
Applying Law of Conservation of Energy, we get

$$E_{initial} = E_{final}$$
$$E + m_0 c^2 = E + E_e$$
$$\text{or} \quad hf + m_0 c^2 = hf' + m c^2$$
$$\text{or} \quad hf - hf' = (m - m_0) c^2 \quad\quad .....(1)$$
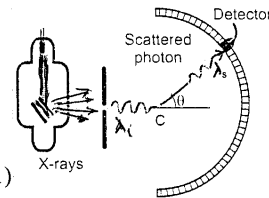
Applying Law of Conservation of Linear momentum in the direction of the incident photon, we have

$$(p_x)_{initial} = (p_x)_{final}$$
$$\text{or} \quad \frac{hf}{c} + 0 = \frac{hf'}{c} \cos\theta + mv \cos\varphi \quad\quad ..... (2)$$

Applying Law of Conservation of Linear Momentum in the direction at right angle to the incident photon,

$$(p_y)_{initial} = (p_y)_{final}$$
$$\text{or} \quad 0 = \frac{hf'}{c} \sin\theta - mv \sin\varphi \quad\quad ..... (3)$$

From equations (1), (2) & (3) after some calculations [given in foot note], we get

$$\frac{1}{f'} = \frac{1}{f} + \frac{h}{m_0 c^2}(1 - \cos\theta)$$

$$\left[\lambda = \frac{c}{f}\right.$$

$$\text{or} \quad \lambda' = \lambda + \frac{h}{m_0 c}(1 - \cos\theta) \quad - - - - - - - (4)$$

$$\text{or} \quad \lambda' = \lambda + \frac{2h}{m_0 c}\sin^2\frac{\theta}{2}$$

This relation shows that the wavelength of the scattered photon is greater than the wavelength of the incident photon. Or the frequency of the scattered photon is smaller than the incident photon, which is Compton effect.

From eq. (4) we have

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}(1 - \cos\theta) - - - - - - (5)$$

Which give change in wavelength $\Delta\lambda$, known as Compton Shift or Compton wavelength.

To evaluate the change in frequency
let us eliminate ϕ from eqs ② & ③
Taking eqs ② & ③, we have

$$m\,v\,c\,\cos\phi = h(f - f'\cos\theta)$$

$$m\,v\,c\,\sin\phi = h f' \sin\theta$$

Squaring and adding, we get

$$m^2 v^2 c^2 = h^2(f^2 - 2ff'\cos\theta + f'^2\cos^2\theta + f'^2\sin^2\theta)$$

$$\text{or} \quad m^2 v^2 c^2 = h^2(f^2 - 2ff'\cos\theta + f'^2) \quad - - - - - - - - - (6)$$

from eq(1), we have

$$mc^2 = h(f - f') + m_0 c^2$$

$$\text{or} \quad m^2 c^4 = h^2(f^2 - 2ff' + f'^2) + 2h(f - f')m_0 c^2 + m_0^2 c^4 \quad - - - - - - (7)$$

Subtracting (6) from (7)

$$m^2 c^2(c^2 - v^2) = -2h^2 ff'(1 - \cos\theta) + 2h(f - f')m_0 c^2 + m_0^2 c^4$$

$$\text{or} \quad \frac{m_0^2 c^2}{1 - v^2/c^2}(c^2 - v^2) = m_0^2 c^4 = \quad // \qquad // \qquad //$$

$$\text{so} \quad 2h(f - f')m_0 c^2 = 2h^2 ff'(1 - \cos\theta)$$

$$\text{or} \quad \frac{f - f'}{ff'} = \frac{h}{m_0 c^2}(1 - \cos\theta)$$

$$\text{or} \quad \boxed{\frac{1}{f'} = \frac{1}{f} + \frac{h}{m_0 c^2}(1 - \cos\theta)}$$

Davisson-Germer Experiment

Introduction:
In 1926 Clinton Davisson and Lestor Germer made first experimental confirmation of the wave nature of electrons followed soon after de Broglie's hypothesis.
They carries out scattering experiments of the <u>low energy</u> electrons off the surface of a metal crystal.

The Apparatus and Observations:

The apparatus is shown in the figure. A beam of electrons from a heated filament is accelerated through a potential difference V. Passing through a slit, the beam strikes a single crystal of nickel. The electrons scattered at various angles were detected by the detector. The whole apparatus was enclosed in a vacuum chamber. It was observed that at certain angles, the intensity of the scattered beam was large and at other angles it was small.
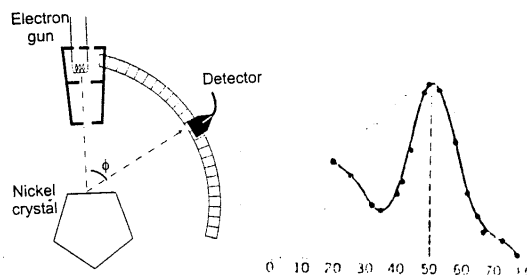
Results:

The results obtained from the experiment were explained by treating the beam of electrons as wave of wavelength given by de Broglie's expression. The diffraction of electrons from the crystal was similar to that of X-rays from crystals.

G. P. Thomson Experiment

G.P.Thomson extended the research on electron waves to high speed electrons. Davison and Germer used slow electrons, but G.P.Thomson used <u>fast electrons</u> having energies up to 60,000 electron volts. He used a fine beam of electrons, that was transmitted through a thin film of gold. Gold consists of a large number of micro crystals oriented at random. The electrons, diffracted by crystals making the same glancing angle, will move in a cone and produce a circular ring on the photographic plate.
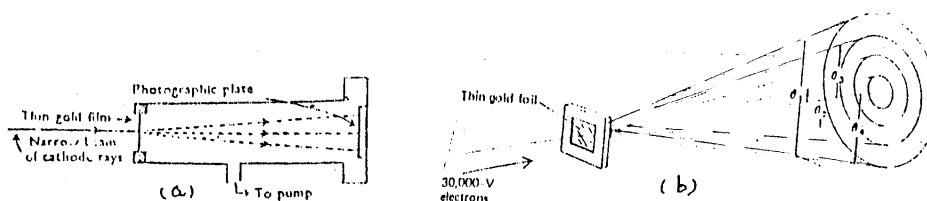
Several rings were formed on the plate for different values of n and θ in Bragg equation,

$$2d \sin\theta = n\lambda$$

and λ can be calculated from the de Broglie formula

$$\lambda = h / mv$$

De Broglie Wavelength

The de Broglie wavelength for a cricket ball of mass 0.3 kg moving with velocity 40 m/s will be

$$\lambda = \frac{h}{mv} = \frac{6.6 \times 10^{-34}}{0.3 \times 40} = 5.5 \times 10^{-33} \text{ m}$$

It is impossible to measure such a small wavelength.
Instead, the wavelength attached with an electron moving with velocity 10×10^{6} m/s will be

$$\lambda = \frac{h}{mv} = \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 10 \times 10^{6}} = 0.72 \times 10^{-10} \text{ m}$$

This wavelength is in the short x-rays range and can be measured experimentally.

Uncertainty Principle

Statement:

"It is impossible simultaneously to determine exactly both the position of an object and its momentum"

Explanation:

One can determine either the position or the momentum of a particle exactly, but both cannot be known exactly at the same time. The more we know one, the less we can know of the other.

Illustration:

To see an electron, we use a light of wavelength λ. The light consists of photons, having momentum h / λ. When one of these photons hits the electron, the photon will be scattered and the original momentum of the electron will be changed.

Incident
photon

Scattered
photon

Electron
before
collision

Electron
after
collision
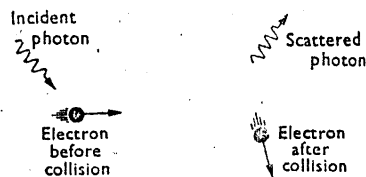
One cannot calculate the exact change in momentum (Δ mv) of the electron. But the change of momentum of the electron will be of the same, so

$$\Delta p \;=\; \Delta(mv) \approx h / \lambda. \qquad\qquad \ldots\ldots (1)$$

This equation gives the uncertainty in the momentum. In order to reduce the uncertainty in momentum, one must use light of large wavelength.

Now we find that the accuracy with which we can locate the position of an object depends upon the wavelength of the light used. The shorter the wavelength, the more fine details of an object can be observed with it.

long λ short λ
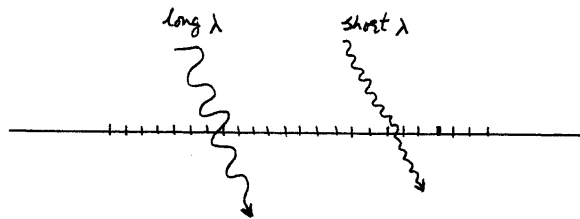
There is some error or uncertainty in the measurement of x. It is denoted by Δx. The uncertainty Δx will be of the order of λ, so

$$\Delta x \approx \lambda \qquad \qquad \dots\dots (2)$$

To reduce the uncertainty in position, one must use light of short wavelength.

Multiplying the two equations,

$$\boxed{\Delta p \, \Delta x \approx h} \qquad \qquad \dots\dots (3)$$

Eq. (3) is one form of <u>uncertainty principle</u>.
According to it, <u>the product of the uncertainty Δp in the momentum of a body at some instant and the uncertainty in its position Δx at the same instant is approximately equal to Planck's constant</u>.

Another form of Uncertainty Principle:

We have
$$E = hf$$

or $\quad \Delta E \approx h \, \Delta f \qquad \qquad \dots\dots (4)$

Also
$$t = 1/f$$

or $\quad \Delta t \approx 1/\Delta f \qquad \qquad \dots\dots (5)$

Combining the two equations, we get

$$\boxed{\Delta E \, \Delta t \approx h} \qquad \qquad \dots\dots (6)$$

which is another form of uncertainty principle.

Eq. (6) states that the <u>product of the uncertainty in a measured quantity of energy and the interval of time during which it is measured is approximately equal to Planck's constant</u>. This means that the more <u>precise</u> we are in our measurement of time (i.e. smaller Δt), the less precise we will be in the determination of energy.

Bohr's Model

Bohr applied the quantum theory of radiation as developed by Planck and Einstein to the Rutherford nuclear atom. His theory is based on the following postulates.

1. An electron, bound to the nucleus in an atom, can move around the nucleus in certain circular orbits without radiating. These orbits are called the discrete stationary states of the atom.
2. Only those stationary orbits are allowed for which orbital angular momentum is equal to an integral multiple of $h / 2\pi$, i.e.

   $mvr = nh / 2\pi$, $\quad$ n being principal quantum number, having values $1, 2, 3, \ldots\ldots$
3. Whenever an electron makes a transition, i.e., jumps from high energy state E_n to a lower energy state E_p, a photon of energy hf is emitted so that

$$hf = E_n - E_p$$

These postulates are a combination of some ideas taken over from classical physics together with others in direct contradiction to classical physics.

The first postulate rejects the claim that an accelerated charge must radiate in atomic systems, in spite of its validity in the ordinary world. In the second postulate the angular momentum is quantized in atomic systems. The third postulate provides the link with Planck's theory of radiation.

Quantized Radii

From the equation,

$$mvr = nh / 2\pi \qquad \ldots\ldots(1)$$

for n^{th} orbit with velocity v_n,

$$mv_n r_n = nh / 2\pi \qquad \ldots\ldots(2)$$

$$\text{or} \quad v_n = \frac{nh}{2\pi mr_n} \qquad \ldots\ldots(3)$$

We have

$$F_{electrostatic} = \frac{ke^2}{r_n^2} \qquad \ldots\ldots(4)$$

$$F_{centripetal} = \frac{mv_n^2}{r_n} \qquad \ldots\ldots(5)$$

from equations (4) & (5), we have

$$\frac{mv_n^2}{r_n} = \frac{ke^2}{r_n^2}$$

$$\text{or} \quad mv_n^2 = \frac{ke^2}{r_n} \qquad \ldots\ldots(6)$$

from eq. (2)

$$v_n = \frac{n h}{2\pi m r_n} \qquad .....(7)$$

from eq. (6) & (7)

$$\frac{m [nh]^2}{[2\pi m \ r_n]^2} = \frac{k e^2}{r_n}$$

or $\quad \dfrac{n^2 h^2}{4\pi^2 m \ r_n} = k e^2$

or $\quad r_n = \dfrac{n^2 h^2}{4\pi^2 k m e^2} \qquad .....(8)$

or $\quad r_n = n^2 \ r_1 \qquad .....(9)$

where $\quad r_1 = \dfrac{h^2}{4\pi^2 k m e^2}$

eq. (8) gives the radius of the nth "non radiating" orbit. The electron remains normally in the first orbit for which n = 1.

So for first orbit, r_1 ,
   n = 1, h = 6.63 x 10^{-34} J. s , m = 9.1 x 10^{-31} kg
   k = 9 x 10^9 N.m /C^2 , e = 1.6 x 10^{-19} C

so r_1 = 0.53 x 10^{-10} m = 0.53 A = 0.053 nm .....(10)
which is called the <u>Bohr radius</u> . This result agrees well with the radius of the atom.

Eq. (9) shows that the radii of the orbits for stationary states are also quantized and are given by

$$r_n = r_1 , 4 r_1 , 9 r_1 , 16 r_1 , 25 r_1 , ...$$

These radii are proportional to the square of the integer number n, called <u>principal quantum number.</u>

From eqs. (7) & (8) , the speed of electron in the nth orbit is

$$v_n = \frac{2\pi k e^2}{n h} \qquad .....(11)$$

54

Quantized Energies

Electric potential energy $= \Delta U = $ Work $= $ F x displacement

$$= k\frac{q_1 q_2}{r^2} \times r = k\frac{q_1 q_2}{r}$$

potential energy in the nth orbit is

$$P.E. = k\frac{(+e) \times (-e)}{r_n} = -k\frac{e^2}{r_n} \quad .....(12)$$

$$and \quad K.E. = \frac{1}{2} m v_n^2 \quad .....(13)$$

from eqs. (**6**) & (13)

$$K.E. = \frac{1}{2} k e^2 / r_n$$

So total energy is

$$E_n = KE + PE = \frac{1}{2} k e^2 / r_n - k e^2 / r_n$$

$$or \quad E_n = \frac{-k e^2}{2 r_n} \quad .....(14)$$

putting the value of r_n from eq. (8)

$$E_n = -\frac{k e^2}{2 [n^2 h^2 / 4\pi^2 k m e^2]}$$

$$or \quad E = E_n = -\frac{2\pi^2 k^2 m e^4}{h^2} \left(\frac{1}{n^2}\right) \quad , n = 1, 2, 3, \quad .....(15)$$

where the negative sign indicates a bound system.

Putting the values in eq. (15) , as

$$m = 9.11 \times 10^{-31} \text{ kg} \quad , e = 1.60 \times 10^{-19} \text{ Coulomb}, h = 6.626 \times 10^{-34} \text{ J.s}$$
$$\pi = 3.14, \quad k = 9 \times 10^9 \text{ N.m} / C^2$$

we get

$$E_n = -\frac{13.6}{n^2} \text{ eV}, \text{ for } n = 1, 2, 3, ... \quad .....(16)$$

The state of lowest energy or ground state corresponds to $n = 1$, and its energy is

$$E_1 = -13.6 \text{ eV}$$

From eq. (15), it can easily be seen that if the atom is in its ground state, 13.6 eV will necessary to liberate the electron from the atom. Therefore, the binding energy (BE) or ionization energy for the hydrogen atom in its ground state is

$$BE = E_i = 13.6 \text{ eV}$$

X-RAYS

Discovery:

There is probably no subject in pure science which illustrates better than X-rays the importance to the entire world of scientific research. In the autumn of 1895, in an experiment designed to study cathode rays, Professor Wilhelm Roentgen carefully shielded a discharge tube with black cardboard. When the room was darkened and a discharge passed through the tube, Roentgen was somewhat startled to see a wavering light across the room near a workbench. He repeated the procedure and found the same. He lit a match and discovered that the source was the fluorescence of a small platinum-barium-cyanide screen on the bench. It was observed that these x-rays as Roentgen called them had very great penetrating power.

Inner shell transitions:

In heavy atoms, the inner shell electrons are tightly bound and large amount of energy is required for their displacement from their normal energy levels. After excitation, when an atom returns to its normal state, photons of larger energy are emitted. Thus transition of inner shell electrons gives rise to the emission of high energy photons or X-rays.

Production:

When an electron is accelerated or decelerated, it radiates photons. When fast moving electrons strike a target they are slowed down and finally come to rest with the emission of photons. X-rays are produced by;

i) Gas tube
ii) Coolidge tube
iii) Betatron

We will discuss only the Coolidge tube.

Coolidge tube:

The arrangement of producing X-rays is shown in the fig.
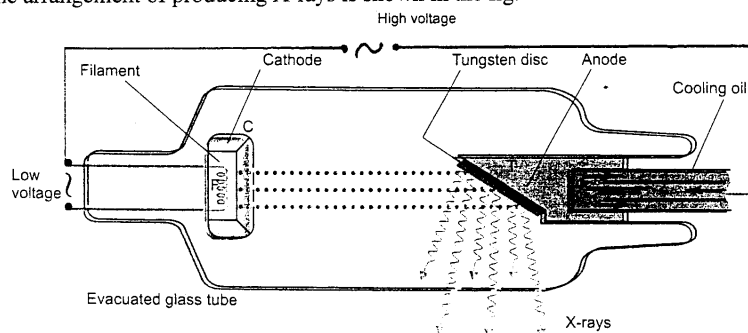
The apparatus consists of a high vacuum tube called X-ray tube. When the filament F, from an adjustable battery, heats the cathode it emits electrons, which are accelerated towards the anode T. The current of electrons is controlled independently of the applied voltage. The anode is made of tungsten or molybdenum is attached with copper rod whose end outside the tube is kept cool by means of copper discs immersed in oil.

The usual potential difference is from 50,00 to 100,000 volts for the current of 50 to 100 mA. Major power received by anode is converted into heat. Only 1 % is converted into X-rays. The KE of striking electrons is;

$$KE = V e \qquad .....(1)$$

If one of the electron in K shell is removed, then the electron from L shell jumps down by emitting a photon of energy $hf_{k\alpha}$ called K_α X-rays.

$$hf_{k\alpha} = E_L - E_K \qquad .....(2)$$

The possibility is also that the electron from M to K shell might come,

$$hf_{k\beta} = E_M - E_K \qquad .....(3)$$

The photons emitted in such inner shell transitions are called characteristic X-rays.

Intensity Measurement:

The intensity of a beam of x-rays may be measured by any one of its effects, e.g., effect on photographic plate, penetration through matter, rise of temperature of the target, ionization of gases. The ionization chamber is commonly used for measuring the intensity of x-rays. When a beam of x-rays is passed through a gas, it get ionized.

The greater the intensity of x-rays the greater the number of ions produced in a given time. The apparatus is shown in the figure below.
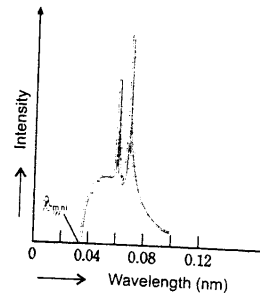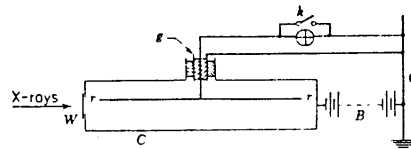
X-ray Spectrum:

The fast moving bombarding electrons are suddenly slowed down to impact with the target. The **continuous spectrum** is obtained due to deceleration of impacting electrons. This continuous distribution of x-rays is called **bremsstranhlung** (German for braking or decelerating radiation). It is different from discrete x-ray energies.
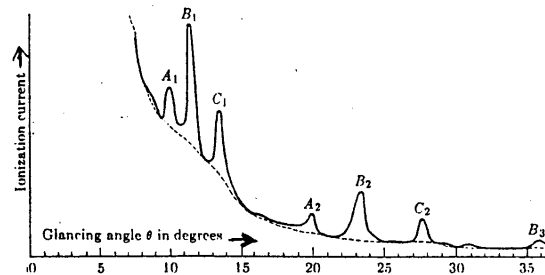
Characteristic X-Rays:

The spectra of x-rays contains a continuous spectrum upon which is superimposed a line spectrum. The wavelengths of the continuous spectrum are found to be independent of the material of the target and dependent upon the potential difference across the tube. On the

other hand the line spectrum is characteristic of the element used as target. These are known as characteristic x-rays. These can be produced by two methods.

i) In x-ray tube, using the material as the target, whose characteristic x-rays are required.

ii) From the x-ray tube, the x-rays are made to fall on the material which later on emits its characteristic x-rays.

Properties:

The following are the main properties of x-rays.

1. Many substances are more or less transparent to x-rays.
2. X-rays can expose photographic plates and films.
3. X-rays are not deflected by electric or magnetic fields.
4. X-rays can produce ionization in gases.
5. X-rays are scattered after colliding with electrons. (Compton effect)
6. The produced x-rays when scattered by a substance, then that substance emits new rays its characteristic x-rays.
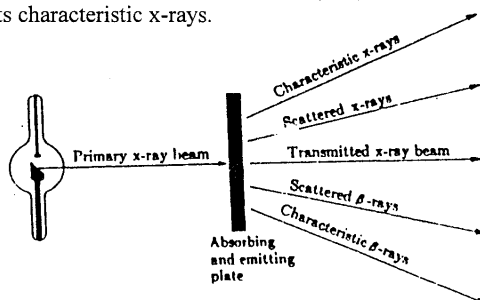
Uses:

1. Used to determine the **atomic number** of an atom from Moseley law, which states that 'the frequency of a spectral line in an x-ray spectrum varies as the square of the atomic number of the element emitting it'.

2. In **medicine** use in hospitals for surgical operations. In x-ray picture of some part of human body, shadow of bones appears lighter than the surrounding flesh. A crack or fractured bone allows greater amount of x-rays to pass.

2(b) Used for selective destruction of cancer cells.

3. In **industry** the defects in structural steel can be seen as interiors of the materials opaque to ordinary light but a crack or air bubble can be traced.

4. Diffraction of x-rays is used for determining **crystal structure.**

5. **CAT Scanner**
   It is a computerized system for scanning shadow photographs of the internal structure of the body with direct focus on an organ to eliminate confusing shadow. Tumors and other anomalies much too small to be seen with older techniques can be detected.

Hazardous Effects:

X-rays cause damage to living tissue.

1. When absorbed in tissues, they break molecular bonds and create reactive free radicals, which can disturb the genetic tissues.

2. A cell may damage [during selective destruction of cancer cells] by radiation but survive and continue dividing and produce generation of defective cells.

3. Excessive radiation exposure can cause changes in the productive system of the organism.

LASERS

The name LASER stands for Light Amplification by Stimulated Emission of Radiation.

Definition:

"A device which is able to produce a beam of radiation with unusual properties. generally the beam is coherent monochromatic, parallel with high intensity".

Laser Principle:

The light is produced in a process in which de-excitation of an atom is caused by an incident photon with the emission of a second photon of the same energy which is coherent with the original photon, shown in the figure.
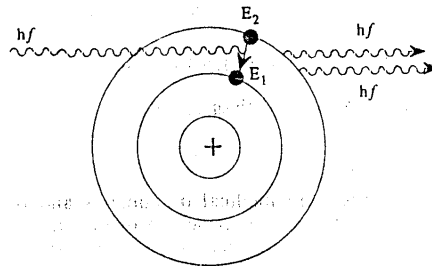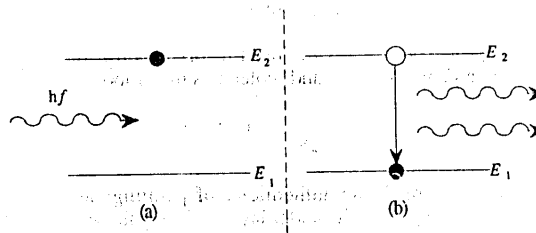

Figure (a) shows a photon is incident on an atom in excited state.
Figure (b) shows stimulated emission.

Stimulated means that to produce greater activity. The photons, shown in the above figure, stimulate other atoms to emit photons in a chain of similar events.

Laser Operation:

The operation of laser depends upon the existence of temporarily stable called metastable state in the atoms of some substances.

When light is made incident on such a substance, there is a net absorption of photons due to more atoms in the ground state than in the excited state. It is only if the situation can be inverted. There should be more atoms in excited state than in ground state then a net emission of photons will result, called population inversion. Because of population inversion, the rate of energy radiation by stimulated emission exceeds the rate of absorption and the result is an amplification of ordinary light.

Mirrors at the two ends are used to confine the emitted photons in the assembly. One end is totally reflecting and the other is partially transparent. Photons move back and forth and stimulate other excited atoms to emit photons. As the number of photons multiplies, the resulting radiation is more <u>intense</u> and <u>coherent</u> than light from ordinary sources.
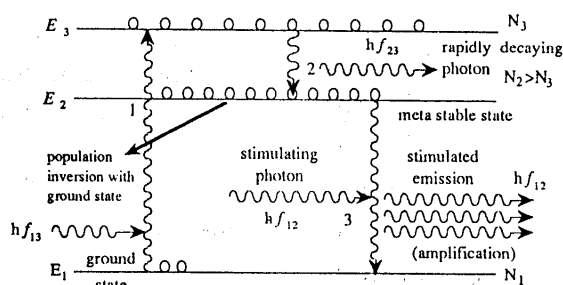
Kinds of Lasers:
The lasers are classified into three major kinds.

1. **Solid Lasers**
   Here a fluorescent crystal, e.g., ruby, glass or a semiconductor is used as light amplifying substance.

2. **Liquid Lasers**
   These lasers make use of a dye dissolved in methanol or a similar liquid.

3. **Gas Lasers**
   These lasers contain a gas or mixture of gases as their light amplifying substance. Helium-neon, argon ion and CO_2 gas lasers are widely used.

Laser Applications:
   1. In surgery, it is used for bloodlessly removing small tumours, cutting, coagulating at the same time and welding of detached retina to the eye.
   2. Precision cutting, welding and drilling of tiny holes into hard materials.
   3. Production of three dimensional images called holograms.
   4. Used as sensitive detectors of pollutants in the atmosphere.
   5. Used to induce nuclear fusion reaction.
   6. Solid state detectors are used in electrical devices.
   7. Used for photographic recording of output data of a computer.
   8. Used to fragment gallstones and kidney stones.
   9. Used in surveying over large distances, in optical fibres, and developing hidden finger prints.

Mass Defect & Binding Energy

Mass Defect:

"The arithmetic difference between the mass of a nucleus and the larger combined mass of its constituent particles".

Nuclear Binding Energy:

"The energy needed to break up a nucleus into its constituent particles, and it is equivalent to the mass defect".

The binding energy of the nucleus is calculated from mass-energy relationship, which is in Einstein's special theory of relativity.,

$$E = (\Delta m) c^2$$

Where Δm is the decrease in mass which is converted into energy E at the time of formation of the nucleus.

Illustration:

We have for helium ($_2He^4$) nucleus:

Mass of 2 protons & 2 neutrons, as free particles = 4.0331 amu

Mass of the nucleus   (2 protons + 2 neutrons)   = 4.0028 amu

Difference ('missing' mass Δm)   = 0.0303 amu

Which is equivalent to 28.2 Mev of energy.

$$E = (\Delta m) c^2 = 0.0303 \times 1.66 \times 10^{-27} \times 9 \times 10 \text{ joules } = x$$

$$[1 \text{ amu} = 1.66 \times 10^{-27} \text{ kg }]$$

$$= \frac{x}{1.66 \times 10^{-19}} \text{ eV } = y$$

$$= \frac{y}{10^{-6}} \text{ MeV } = 28.2 \text{ MeV}$$

This means that when two protons and two neutrons combine to form a helium nucleus 28.2 MeV of energy will be released.

This energy equivalent of the mass defect in a nucleus is its binding energy, and is a measure of the stability of the nucleus.

Graph of E_B /nucleon verses Z

The binding energy per nucleon, arrived at by dividing the total binding energy of a nucleus by the number of nucleons it contains, is a most interesting quantity. The binding energy per nucleon is plotted as a function of mass number Z in the figure.
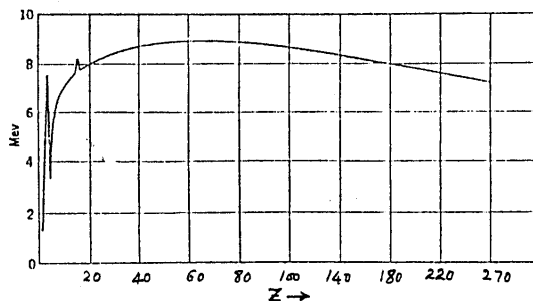
The figure shows that binding energy of the element with Z > 82 decreases with increasing mass number, and there are no stable nuclei over this range.

Radiation Detectors

Five distinct methods have been developed for the detection of single atomic particles, e.g., α-particles, β-particles, fast moving protons, γ-rays.

These can be classified as photographic, cloud chamber, bubble chamber, optical and electrical method. Three of them viz. Wilson Cloud Chamber, Geiger-Muller Counter and Solid State detector will be discussed here.

Wilson Cloud Chamber:

Definition: It is a device that shows the visible path of an ionizing particle.

Principle: Here a super-saturated vapour condenses into droplets around the ions.

Construction: It consists of a glass-fronted cylinder containing a mixture of air and water vapour.

Working: When the piston is moved back rapidly, the vapours around incoming ions cools adiabatically and a trail of particles can be observed.

The α-particles leave thick, straight and Continuous **tracks** due to intense ionization produced by them, β-partcles form thin and discontinuous tracks extending in erratic manner, and γ-rays leave no definite tracks along their path.

Placing the chamber in a strong magnetic field and the bending of paths will **provide information** about charge, mass and energy of the radiating particle, which has helped in the discovery of **new particles**.
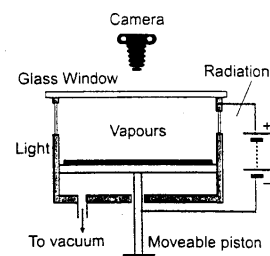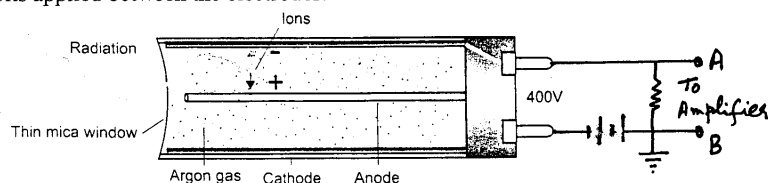

Geiger-Muller Counter:

Definition: Instrument used for the detection and measurement of radioactivity. It is gas-filled radiation detector operated at high voltage in which the gas amplification effect produces a large discharge pulse after each primary ionizing event.

It is the most widely used detector of single particles. Here the **principle** of ionization chamber is used. The discharge in the tube results from the ionization produced by the incident radiation.

This counter, developed by Geiger and Muller (1913), is very simple to **construct** and is extremely sensitive to the passage of charged particles. It is usually worked with about 400 volts applied between the electrodes.

The above figure shows the **essential parts** of a Geiger-Muller Counter. It consists of a long glass tube containing two electrodes. The stiff central wire is very thin and is the anode in a hollow metal cylinder acting as a cathode. The counter is usually filled with 90 % argon and 10 % ethyl alcohol at a pressure of about 0.1 atmospheric pressure.

Geiger-Muller Counter or G-.M- Counter has been designed for the measurement of α-, β-, γ-rays, and X-rays. In the form described, they are used mainly for β- and γ-rays.

The discharge that takes place on entry of a particle is a **corona discharge** along the whole length of the wire. For high accuracy the temperature of the counter should be controlled. Amplification of the order of 10^8 is produced which leads to a sufficiently large ion current to be recorded.

[Corona discharge: Small regions of glowing air usually accompanied by small sounds and are caused by the ionization of the air.]

Quenching: The process of preventing a continuous discharge in a Geiger counter so that the incidence of further ionizing radiation can cause a new discharge.

A gas or vapour (called **Quenching gas**) such as ether or a halogen gas is introduced into the tube of Geiger counter to prevent a continuous discharge. The gas quenching is called **self quenching** because atoms readily recombine into molecules again for the next pulse.

G.M. counter is not suitable for fast counting. It is because of its relatively long "dead time" of the order of more than a millisecond which limits the counting rate to a few hundred counts per second.

[Dead time: In any electrical device, the time interval immediately following a stimulus during which it is insensitive to another stimulus.]

Solids State Detector:

Definition: It is a semi conducting crystal in which specially designed p-n junction operating under a reversed bias in which electron-hole pairs are produced by the incident radiation to cause a current pulse to flow through the external circuit. Then the electrical pulse is amplified.

Three **types** of solid-state detectors are noteworthy.

1. Surface barrier detectors, 2. Lithium drifted detectors, 3. Crystal conduction counters.

We will discuss only the first one.

Surface barrier detector

The surface barrier detector is used for charged particles.
Principle: When a charged particle is passed through the semiconductor, its conductivity increases due to formation of "holes" and "free electrons".
Construction: It consists of silicon layer which is of N-type material. On the surface a thin P-type layer of gold is deposited. The two conducting surfaces are connected with resistance R and to a battery.
Working: When some ion is entered, a pulse of current is produced across R and is amplified and then recorded by a scalar unit.

The energy needed to produce an electron-hole pair is about 3 eV to 4 eV which makes the device useful for detecting low energy particles. The collection time of electrons and holes is much less than gas filled counters.
Advantge: It can count very fast. It is much smaller in size than any other detector and operates at low voltage. This type is **used** for detecting α- or β-particles and a specially designed device can be used for γ-rays.
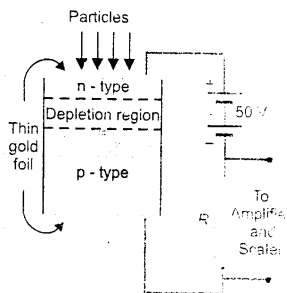
Nuclear Reactor

Definition:

"A device in which the controlled fission of radioactive material produces new radioactive substances and energy".

Natural uranium contains 0.7 % of U^{235} and 99.3 % of U^{238} . The fuel must be enriched by increasing the proportion of U^{235} , as it has an effective role in nuclear reaction.

Principle:

The controlled chain reaction is the principle of nuclear reactor. The fission reaction is controlled in such a way that only one neutron out of 2.5 neutrons released on the average is used to induce fission in another atom.

Main Parts:

1. Core—contains uranium and one of the moderator.
2. Moderators—Graphite is generally used as moderator, i.e., to slow down fast neutrons in a fission reaction. Also uranium rods are used for this purpose.
3. Control rods—of boron or cadmium are used, which are effective neutron absorbers.
4. Cooling System—It consists of pipes along which the coolant is pumped in the form of gas or water.
5. Outer Shield—It is wall of concrete which absorbs radiations due to fission.

Types of Reactors:

The following features are used to describe them.
1. Neutron energies
   i) High energy, ii) Intermediate, iii) Low energy (thermal)
2. Purpose
   i) Research, ii) Breeder or Pu^{239} production, iii) power generation
3. Fuel Moderator
   i) Homogeneous (liquid form), (ii) Heterogeneous (solid or lumps of U^{238})
4. Moderator
   i) Graphite, ii) Water, ii) Beryllium
5. Coolant
   i) Air, CO_2 or He, ii) Water or liquid, ii) Liquid metal

The first nuclear reactor was constructed in 1942 in Chicago University. It was originally a thermal, natural uranium, graphite moderated, heterogeneous, air cooled research reactor.

Power Reactor:

A power reactor is a device to make use of the natural heat developed in a uranium reactor as source of huge power.

Nuclear power is an important substitute for the world's energy supplies.

The figure shows the difference between a nuclear power plant and a conventional one.
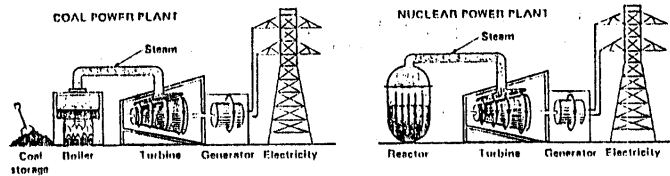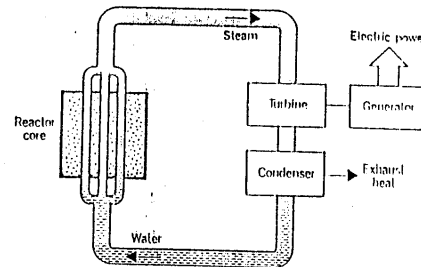

The reactor is designed for the extraction of the fission energy to produce usable power in the form of electricity.

There are three systems for extracting usable fission energy.

1. **Boiling Water Reactor**
   In it water circulates through the core. The heat turns water to steam, which is used to generate electricity.

2. **Pressure Water Reactor**
   Here heat is extracted in two step process, as shown in the figure.
   Firstly high pressure water circulates, then this hot water heats second water system to deliver steam to the turbine.
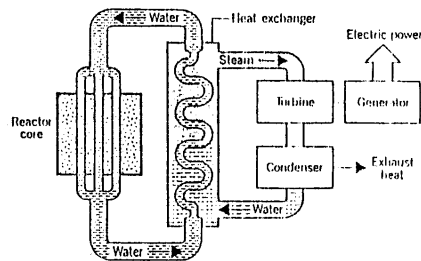
3. **Liquid Metal Reactor**
   Liquid sodium which has high heat capacity, replaces the pressurized water in the above figure.