Modern Physics

for

B.Sc. Students

Ross Nazir Ullah

PREFACE

This book is written for the B.Sc. students. It covers practically all the topics dealt with in the B.Sc. course. In addition to it few topics have been added just to make understand the topics mentioned in the syllabus or for maintaining the interest in the subject.

This book is made self-contained and to some extend self explanatory, to encourage students a feeling of achievement. At the end of each chapter questions, solved and unsolved problems are added to give the reader insights into understanding the nature of subject and its basic principles. Few appendices are added alongwith index for reader's convenience.

An effort has been made to minimize errors and difficult mathematical steps, but they will surely be found. The teachers or students' assistance in pointing them out will be greatly appreciated. Further comments or suggestions for improvement are also invited.

Govt. College, Shakargarh.
September 15, 1992.

ROSS NAZIR ULLAH

C O N T E N T S

Chapter 1

ORIGIN OF QUANTUM THEORY

1.1 INTRODUCTION

We have seen that the description of the motion of a body given by the classical mechanics has proved to be inadequate when the speed of the body approaches the speed of light. For this case, the limitations of the classical mechanics force us to adopt the relativistic mechanics. Another limitation on the classical mechanics occurs when we study matter of very small dimensions---within the microscopic world of atomic and nuclear structure and of the elementary particles. It is here that the remarkably successful quantum mechanics, and its modern version called quantum field theory, takes over the story from classical approaches.

According to the quantum theory, the radiant energy of a system or the exchange of radiant energy between different systems occurs not in a continuous fashion permitting all possible values, as demanded by the wave theory, but in a discrete quantified form, as integral multiples of an elementary quantum of energy. This means that energy should be considered atomic in structure, like matter.

1.2 CLASSICAL THEORIES OF LIGHT

In order to have before us a historical background which will make us appreciate the need that was felt for the introduction of this new theory, we shall first briefly state the different classical theories of light.

The ancient corpuscular theory

It was advocated by Newton towards the end of the 17th century, held that all sources of light must shoot off rapidly moving minute particles or corpuscles, some of which enter the eye and produce the sensation of light. As this theory could not satisfactorily account for the observed phenomena, such as refraction, interference, diffraction and polarization of light, it had to give place to the wave theory.

The Wave theory of light

According to it, light was considered as a wave disturbance in a medium, called luminiferous ether, that filled all apace uniformly, adequately explained all the then known phenomena, as rectilinear propagation, refraction, interference, diffraction and polarization with the basic idea that light was propagated from place to place when the particles of the medium, ether, were set in simple harmonic vibration in a direction at right angles to the direction of propagation and the resulting disturbance or energy travelled out as a transverse wave motion. But a serious difficulty about the existence of ether was faced.

The Electromagnetic theory of light

Maxwell, in 1864, as a result of his investigations on the electromagnetic field, proposed the electromagnetic theory of light, according to which light is considered as the result of rapidly alternating "displacement current" in the medium which gives rise to magnetic effect, similar to those associated with conduction currents. The two fields, electric and magnetic, inseparably associated, the one varying proportionately with the other and the variation of one giving rise to the other, urge each other forward with a finite velocity, viz., that of light.

Some important conclusions of the electromagnetic theory:

a) Unification of the different types of radiation;

The electromagnetic radiations whose origin are the periodic motion of electric charges contained in atoms and molecules, cover a very wide range of frequencies, which are called electromagnetic spectrum given in appendix.

b) An accelerated charge must radiate energy;

It follows that any accelerated charge must radiate energy, acceleration of a charge cannot take place without radiation of energy.

c) Continuity of action;

According to the theory, the electrica~~l~~ and magnetic field vary continuously with time, the flow of energy in an electromagnetic wave should be continuous over the entire wavefront.

The last two conclusions, (b) and (c) of the electromagnetic theory were soon found to be contradicted by a number of experimental observation and hence constituted serious drawbacks of the theory.

1.3 BASIC CONCEPTS OF QUANTUM MECHANICS

The appearance of vacuum apparatus, the development of radio and television techniques, and the improvement of other technical aids to study physical phenomena led at the end of the nineteenth century to the discovery of electrons, X-rays, and radioactivity. The possibility arose of studying separate atoms and molecules. It then turned out that classical physics was not able to explain the properties of atoms and molecules and their interactions with electromagnetic radiation. A study of the conditions of equilibrium between matter and electromagnetic radiation by Planck in 1900 (see section 1.10) and of photoelectric phenomena by Einstein in 1905 (section 2.10 led to the conclusion that electromagnetic radiation posessed both a wave character and a corpuscular character. It was established that electromagnetic radiation is absorbed and emitted in separate portions---quanta which now-a-days called photons.

On the other hand, interference and diffraction phenomena, widely used in a number of optical apparatuses, show the wave character of electromagnetic radiation. It turned out to be impossible to consider the wave properties of radiation as a manifestation of the collective motions of a large number of photons, in the same way as sound waves correspond to the motion of a large number of molecules of the air, of a liquid, or of a solid.

Franck and Hertz (see section 3.13) showed in 1914 the discrete character of the atomic energy states in their experiments on the ionization potentials of gases. The discrete character of atomic levels became also apparent from the study of the optical spectra of atoms. Stern and Gerlach showed in 1922 (see section 3.27) in their experiments on the deviation of a beam of atoms in an inhomogeneous magnetic field that the component of angular momentum along the direction of a magnetic field takes on discrete values.

The year 1913 saw the first successful attempt to explain the properties of the hydrogen atom by Niels Bohr, who intoduced some special postulates (see section 3.8). These postulates were in essential contradiction to the established rules of classical physics.

The experiments of Davisson and Germer in 1927, in which diffraction of electrons on reflection by thin metallic foils was observed, were very important for an explanation of the properties of electrons. These experiments

confirmed de Broglie's hypothesis in 1924 (see section 4.3) about the wave properties of all particles of small mass.

The theory explaining the basic properties of atomic and nuclear phenomena is quantum mechanics which was founded by the work of Bohr, Schrodinger, Heisenberg, Dirac, Fock and Pauli. Quantum mechanics is the theory which is the basis of the explanation of the properties of atoms, molecules, and atomic nuclei, that is, phenomena occurring in volume elements of linear dimensions of the order of 10^{-6} to 10^{-13} cm. Systems of such dimensions (we can call them microworld systems) cannot directly be perceived with our sense organs. We can only investigate them by using "apparatus", that is, by using macroscopic systems which translate the action of micro-objects into a macroscopic language.

Quantum mechanics shows that many other concepts of classical physics also have a limited domain of applicability. For example, it turned out that it is impossible to define the velocity of a particle as the derivative dr/dt. Quantum mechanics often merely gives probability statements. It enables us to evaluate the probability of the influence of atomic objects under well-defined macroscopic conditions upon macroscopic apparatus. Quantum mechanics is a new vigorously developed branch of theoretical physics.

In conclusion, we see that, it is clearly unsatisfactory to have both radiation and matter being treated sometimes as waves and sometimes as particles is an apparently arbitrary manner, and discrete energy levels in atoms being produced by ad-hoc rules. What is needed is a basic reformulation of the theory in such a way that both classical concepts which remain correct, and the Planck-Bohr-de Broglie rules, shall appear as natural consequence. This is quantum theory, which we set up in consequent chapters.

1.4 BLACK BODY RADIATION

We have seen that the description of the motion of a body given by the classical mechanics has proved to be inadequate when the speed of the body approaches the speed of light. For this case, the limitations of the classical mechanics force us to adopt the relativistic mechanics. Another limitation on the classical mechanics occurs when we study matter of very small dimensions---within the microscopic world of atomic and nuclear structure and of the elementary particles. It is here that the remarkably successful quantum mechanics takes over the story from classical approaches.

The first indication that all was not well with the classical wave picture of electromagnetic radiation followed from the failure of the wave theory to explain the observed spectrum of thermal radiation---that type of electromagnetic radiation emitted by all objects due to their temperature. Other experiments followed the photoelectric effect and the Comptom effect (see sections 2.10 & 2.11) in which the wave theory failed to explain them.

An experimental arrangement is shown in fig.1-1. An object if maintained at a temperature T_i . The radiation emitted by the object is detected by an apparatus that is sensitive to the wavelength of the radiation. We may take a dispersive medium, for example, with a prism, different wavelengths will appear at different angles θ. By moving the radiation detector to different angles θ we can measure the intensity of the radiation at a specific wavelength. We measure the radiant intensity for different values of λ . Then, we plot the data as a function of λ , and the result might be similar to fig. 1-2.

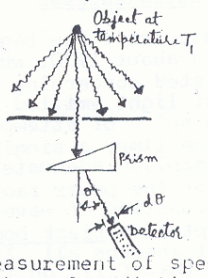


Fig.1-1: Measurement of spectrum of thermal radiation.

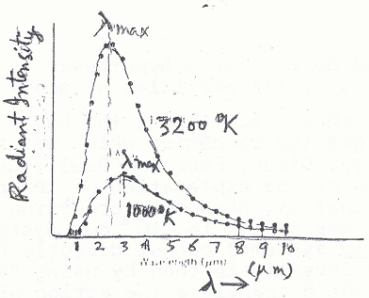


Fig. 1-2: A possible result of the measurement of the radiant intensity over many different wavelengths. Each different temperature of the emitting body gives a different peak .

1.5 STEFAN-BOLTZMANN LAW & WIEN'S DISPLACEMENT LAW

The experimental arrangement of fig.1-1 is for temperature T_1 . Increasing the temperature to T_2, we repeate the measurement and obtain the result shown in fig. 1-2. Repeating the measurement a number of times we learn two important details of the properties of thermal radiation.

1. The total radiant intensity, or monochromatic emissive power, E, over all wavelengths increases as the fourth power of the temperature T. i.e.

$$E = \sigma T^4 \qquad \ldots\ldots (1.1)$$

which is Stefan-Boltzmann Law.
where σ is Stefan-Boltzmann constant $= 5.6697 \times 10^{-8}\ J/m^2 s.^oK$

2. The wavelength of maximum intensity (λ_m) is inversely proportional to the absolute temperature of the black body, i.e., λ_m decreases as we raise the temperature, so that

$$\lambda_m \propto 1/T$$

$$\lambda_m \times T = \text{Constant} \qquad \ldots\ldots (1.2)$$

which is Wien's displacement law. where Wien's const $= 2.898 \times 10^{-3}\ m.^oK$

Ordinary objects are seen because of the light that is reflected by them. At room temperature the thermal radiation is mostly in the infra-red region of the spectrum ($\lambda_m \simeq 10\,\mu m$), where our eyes are not sensitive. As we heat such objects, they begin to emit visible light; according to eq.1.2, as T increases, λ_{max} decreases, and for relatively modest temperatures λ_{max} will move down to the visible region. For example, a piece of metal first begins to glow with a deep red colour, and as the temperature is increased the colour becomes more yellow.

1.6 CAVITY RADIATION

We may trace the beginning of quantum concepts to about 1900, when an unexplained puzzle existed concerning the wavelength spectrum of light emitted by heated solid bodies. In spite of attempts by noted physicists at the time, a single classical theory could not explain adequately the shape of the curve of the power radiated from a black-body as a function of wavelength. The idealized concept of a black body, a theoretical object that absorbs all light of whatever wavelength that falls on it, was conceived

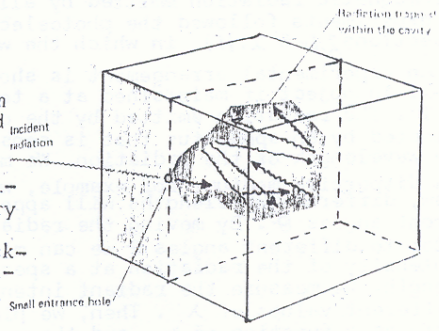


Fig.1-3: The small entrance hole is like blackbody.

to simplify the problem. An experimental approach to such an object is to use a small hole in the wall of a cavity that is heated as shown in fig.1-3. The light emitted through this hole is close to that which would be emitted by an ideal, heated black body. We define:

"A substance which emits radiations of all frequencies, and can also absorb completely radiations of all frequencies is called a black body".

1.7 WIEN'S LAW OF ENERGY DISTRIBUTION

The intensity of radiation emitted by a black body is not uniformly distributed over the whole range of wavelengths involved. The problem to be solved to see how the energy of total radiation from a black body is distributed among the different wavelengths at various temperatures, in other words, to find how E_λ for a black body depends on λ and T.

Wien, in 1893, tackled the problem by applying the principles of thermodynamics to the black body radiation. He showed that the form of the curve (fig.1-4) could be represented empirically by a formula

$$E_\lambda = \frac{c_1 \lambda^{-5}}{e^{c_2/\lambda T}} \qquad \ldots\ldots (1.3)$$

where e is the base of natural logrithms.

c_1 & c_2 are unknown constants to be determined.

E_λ is monochromatic emissive power.

Wien selected these constants so that his theoretical curve coincided with the experimental curve.

Fig. 1-4 shows that Wien's formula is excellent for short wavelengths but not for long wavelengths. Also this formula does not tell anything about the nature of radiation.

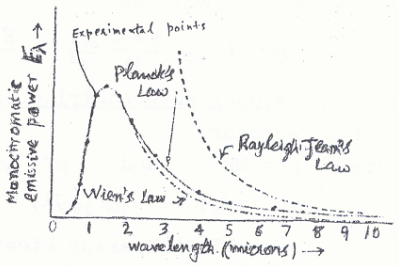


Fig.1-4: Graphs of radiation laws.

1.8 RAYLEIGH-JEANS LAW

Rayleigh-Jeans tackled the problem of black body on the basis of classical electromagnetic theory of radiation.

The classical calculation of the radiant energy emitted at each wavelength now breaks down into several steps. Without proof, here are the essential parts of the derivation.

1. The cavity radiator is filled with electromagnetic standing waves.

If the walls of the box (cavity) are metal, radiation is reflected back and forth with a node at each wall. This is the same condition that applies to other standing waves, like those on a stretched string or a column of air in an organ pipe. Now standing waves in a stretched string can be set up only if its length is equal to the integral multiple of $\lambda/2$ (fig. 1-5)

2. The number of standing waves with wavelengths between λ and $\lambda+d\lambda$ is

$$N(\lambda)d\lambda = \frac{8\pi V}{\lambda^4} d\lambda \qquad \ldots.(1.4)$$

where V is the volume of the box.

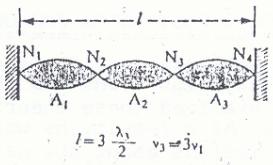


Fig. 1-5: Standing waves.

For one-dimensional standing waves,

$$L = n\frac{\lambda}{2} \quad \text{or} \quad \lambda = \frac{2L}{n} \quad \text{or} \quad n = \frac{2L}{\lambda}$$

The number of possible standing waves with wavelengths between λ_1 and λ_2 is

$$n_2 - n_1 = \frac{2L}{\lambda_2} - \frac{2L}{\lambda_1}$$

and in the interval from λ to $\lambda+d\lambda$,

$$N(\lambda)\,d\lambda = \frac{2L}{\lambda+d\lambda} - \frac{2L}{\lambda} = 2L\left(\frac{\lambda-(\lambda+d\lambda)}{\lambda(\lambda+d\lambda)}\right)$$

$$\text{or} \quad N(\lambda)\,d\lambda = -\frac{2L}{\lambda^2}d\lambda \quad (\text{neglecting } \lambda d\lambda, \text{as } d\lambda \text{ is very small})$$

$$\text{or} \quad N(\lambda)\,d\lambda = \frac{2L}{\lambda^2}d\lambda \quad \left\{\begin{array}{l}\text{neglecting -ve sign, as } \lambda \text{ increses,} \\ \frac{2L}{\lambda^2} \text{ decreases}\end{array}\right\}$$

Extending this result to three dimensional electromagnetic waves,

We have, area of a sphere of radius L is $4\pi L^2$

volume contained in spherical shell per wavelengths area is $\frac{4\pi L^2}{\lambda^2}$

$$N(\lambda)\,d\lambda = \frac{2L\,d\lambda}{\lambda^2} \times \frac{4\pi L^2}{\lambda^2} = \frac{8\pi L^3}{\lambda^4}d\lambda$$

Therefore, the number of standing waves per unit volume in the wavelength range λ and $\lambda+d\lambda$ will be

$$N(\lambda)\,d\lambda = \frac{8\pi L^3}{\lambda^4}\frac{d\lambda}{L^3} = \frac{8\pi}{\lambda^4}d\lambda \qquad \ldots\ldots (1.5)$$

3. Each individual wave contributes an energy of kT to the radiation in the cavity radiator.

since $p = NkT$ and $p = \frac{1}{3}mN\overline{v^2}$

so $NkT = \frac{2}{3}N(\frac{1}{2}m\overline{v^2})$

or $kT \propto$ average kinetic energy

Combining these components,

radiant intensity = (number of waves per unit volume) x (energy per wave)

$$\text{or} \qquad E_\lambda = \frac{8\pi}{\lambda^4}kT \qquad \ldots\ldots (1.6)$$

where k is the Boltzmann constant. This equation is shown in fig.1-4.

The curve fits well for long wavelengths, but it leads to infinity at short wavelengths. The failure of the Rayleigh-Jeans law at short wavelengths is known as the ultraviolet catastrophe and represents a serious problem for classical physics, so a new kind of physical theory was needed.

1.9 PLANCK'S POSTULATES FOR QUANTUM THEORY OF RADIATION

Being well aware of the shortcomings of both the Rayleigh-Jeans and the Wien's law, Planck examined these theories for some changes. As a result, he was led to certain assumptions about the nature of the electromagnetic oscillators that are in equilibrium with the energy density

Fig.1-6: Max Karl Planck (1858-1947)

within the black body cavity. This new concept changed the whole outlook of physicists who believed in the continuous behaviour of radiation upto the end of nineteenth century. Max Planck for the first time introduced the concept of discontinuity in the realm of energy. He assumed radiation as being emitted in little bursts of energy rather than continuously. His postulates which have become the foundation of the quantum theory of radiation are as follows:

1. The atoms that make up the walls of the cavity radiator behave like tiny electromagnetic oscillators, each with a characteristic frequency of oscillation. These oscillators can emit and absorb the waves not continuously but only in the process of changing their amplitudes.
2. The amount of energy emitted or absorbed by an oscillator is proportional to its frequency. Calling the constant of proportionality h, we then write the change in oscillator energy as

$$E = h\nu$$

3. An oscillator cannot have an arbitrary energy, but must occupy one of a discrete set of energy states given by

$$E = nh\nu \quad , \quad n = 1,2,3,\ldots \qquad \ldots\ldots(1.7)$$

where n is an integer or zero. It was assumed that the ground state corresponded to the zero energy state. h is now called Planck's constant.

1.10 PLANCK'S RADIATION LAW

Max Planck using his postulates arrived at a formula for the distribution of energy in a black body radiation spectrum which is given below:

$$E_\lambda = \frac{8\pi}{\lambda^4} \frac{h\nu}{e^{h\nu/kT} - 1} \qquad \ldots\ldots (1.8)$$

The curve (fig.1-4) representing this equation fits well with the experimental curve.

Here the exponential term in the denominator forces the energy density to zero at extremely short wavelengths. This is the distribution that is actually observed in laboratory measurements of black body spectra.

This Planck's energy distribution law was presented on October 19,1900 at a meeting of the Berlin Physical Society. When he announced his results, no great shock waves were felt in the world of accepted physical theories. Even Planck himself did not believe he had done anything more than to find an ad-hoc explanation for a single physical phenomenon. It was another five years until Einstein, in his analysis of the photoelectric effect, showed that Planck's result was not just a curiosity associated with cavity radiation but was, in fact, a fundamental property of electromagnetic waves that led to a new and unexpected way of looking at the physical world.

Without proof, here are some parts of the derivation of Planck's Law:

The number of standing waves per unit volume per wavelength interval, from eq.(1.5) is:

$$N(\lambda)\, d\lambda = \frac{8\pi}{\lambda^4} d\lambda \qquad \ldots\ldots (1.5)$$

The number of photons having the energy in the range E to E + dE is, according to the Bose-Einstein distribution

$$P(E)\, dE = N(E) f_{BE}\, dE$$

The total radiant energy carried by photon of energy E is just Ep(E) so

$$U(E)\, dE = E P(E)\, dE$$
$$= E N(E) f\, dE \qquad \ldots\ldots (1.9)$$

In eq.(1.9), converting from photon energy E to wavelength

$$U(\lambda)\, d\lambda = h\nu\, N(\lambda)\, f_{BE}\, d\lambda$$

substituting the values from eqs. (1.5) & (1.11), we get

$$U(\lambda)\, d\lambda = E_\lambda = h\nu \times \frac{8\pi}{\lambda^4} \times \frac{1}{e^{E/kT} - 1}$$

$$\text{or} \quad E_\lambda = \frac{8\pi}{\lambda^4} \frac{h\nu}{e^{h\nu/kT} - 1} \qquad \ldots\ldots (1.10)$$

which is Planck's Law.

Looking to fig. 1-4 , it is easily shown that Planck's formula reduces to a form of Wien's Law near one end of the spectrum, and to the Rayleigh-Jeans Law near the other end.

Particles that do not obey the Pauli principle are those with integral spins (0,1,2,.... in units of $\hbar$) and are known collectively as bosons. The distribution function for bosons is known as the Bose-Einstein distribution and has the form

$$f_{BE}(E) = \frac{1}{A\, e^{E/kT} - 1} \qquad \ldots\ldots (1.11)$$

Particles of half-integral spin ($\frac{1}{2}, \frac{3}{2}$, ...) which obey the Pauli principle, like electrons or nucleons, are known as fermions, and the correct distribution function is the Fermi-Dirac distribution:

$$f_{FD}(E) = \frac{1}{A\, e^{E/kT} + 1} \qquad \ldots\ldots (1.12)$$

where A is normalization coefficient.

1.11 DERIVATION OF RAYLEIGH-JEANS LAW FROM PLANCK'S LAW

We have from Planck's Law

$$E_\lambda = \frac{8\pi}{\lambda^4} \frac{h\nu}{e^{h\nu/kT} - 1} \qquad \ldots\ldots (1.10)$$

for small frequencies $h\nu/kT$ is small and we obtain by Binomial series expansions

$$e^x = 1 + x + \cdots \quad \text{if } x \text{ is small}$$

$$e^{h\nu/kT} = 1 + h\nu/kT + \cdots$$

so eq. (1.10) becomes

$$E_\lambda = \frac{8\pi}{\lambda^4} \frac{h\nu}{(1 + h\nu/kT + \cdots) - 1} = \frac{8\pi}{\lambda^4} \frac{h\nu}{h\nu/kT}$$

$$\text{or} \quad E_\lambda = \frac{8\pi}{\lambda^4} kT$$

which is Rayleigh-Jeans Law.

1.12 DERIVATION OF WIEN'S LAW FROM PLANCK'S LAW

From Planck's Law, we have

$$E_\lambda = \frac{8\pi}{\lambda^4} \frac{h\nu}{e^{h\nu/kT} - 1}$$

since $c = \lambda\nu$ or $\nu = c/\lambda$, so

$$E = \frac{8\pi}{\lambda^5} \times \frac{hc}{e^{hc/kT} - 1}$$

putting $8\pi hc = c_1$, and $hc/k = c_2$ treated as constants, we get

$$E_\lambda = \frac{c_1}{\lambda^5} \frac{1}{e^{c_2/\lambda T} - 1}$$

for short wavelengths we have $e^{c_2/\lambda T} - 1 \simeq e^{c_2/\lambda T}$, so $E_\lambda = \frac{c_1}{\lambda^5} \frac{1}{e^{c_2/\lambda T}}$

$$\text{or} \quad E_\lambda = \frac{c_1 \lambda^{-5}}{e^{c_2/\lambda T}}$$

which is Wien's Law.

QUESTIONS

1.1 Explain the flow in Rayleigh-Jean law which failed to explain black body radiation spectra.

1.2 What are the fields of classical physics on which the classical theory of black body radiation is based ? Why don't we believe that the "ultraviolet catastrophe" suggests that something is wrong with one of those classical theories?

1.3 How does the total intensity of thermal radiation vary when the temperature of an object is doubled?

1.4 In what region of the electromagnetic spectrum do room temperature objects radiate? What problems would we have if our eyes were sensitive in that region?

1.5 Record the basic assumptions of Planck's quantum theory of radiation. How does this theory fit into the experimental curves obtained with a black body radiator?

1.6 Deduce Wein's displacement law and Rayleigh-Jean law from Planck's radiation law.

PROBLEMS

1-1 a) What will be the wavelength shift of the peak of the black body radiation curve when a star whose surface temperature is T_1 °K cools to a temperature T_2?

b) What is the fractional wavelength change when T_1 = 5800 °K and T_2 = 4350 °K ?

Solution:

We have from eq.(1.2)

$$\lambda_m \times T = \text{const.}$$

$$\text{or } \lambda_{m_1} T_1 = \lambda_{m_2} T_2$$

$$\text{or } \lambda_{m_2} = (T_1/T_2)\,\lambda_{m_1}$$

$$\text{or } \Delta\lambda_m = \lambda_{m_2} - \lambda_{m_1} = (T_1/T_2)\lambda_{m_1} - \lambda_{m_1}$$

$$\text{or } \Delta\lambda_m = \left(\frac{T_1}{T_2} - 1\right)\lambda_{m_1} \quad \ldots\ldots \text{(1.2-b)}$$

b) from eq. (1.2-b), we have

$$\frac{\Delta\lambda_m}{\lambda_m} = (T_1/T_2 - 1)$$

putting T_1 = 5800 °K and T_2 = 4350 °K

$$\frac{\Delta\lambda_m}{\lambda_m} = \left(\frac{5800}{4350} - 1\right) = 1/3$$

1-2 If an object is heated from 100 °C to 200 °C, by what factor will the energy lost by thermal radiation increase ?

Soln: We have from eq.(1.1),

$$E = \sigma T^4 \text{ or } \frac{E_2}{E_1} = \left(\frac{T_2}{T_1}\right)^4 = \left(\frac{473\,^\circ K}{373\,^\circ K}\right) = 2.6$$

1-3 A photon has a wavelength of 4000 Å. What is its energy ? Also calculate momentum.

Soln:

We have from eq.(1.7),

$$E = h\nu = \frac{hc}{\lambda} = \frac{6.6\times10^{-34}\times3\times10^{8}}{4000\times10^{-10}} = 4.95\times10^{-19}\ J = 3.1\ eV.$$

Also $$p = \frac{E}{c} = \frac{h}{\lambda} = \frac{6.6\times10^{-34}}{4000\times10^{-10}} = 1.65\times10^{-27}\ kg\text{-}m/s.$$

1-4 a) At what wavelength does a room temperature (T = 20 °C) object emit the maximum thermal radiation ?

b) To what temperature must we heat it until its peak thermal radiation is in the red region of the spectrum ?

Soln:

a) We have from eq. (1.2),

$$\lambda_m \times T = Const.$$

The value of Wien's constant is

$$\lambda_m \times T = 2.898\times10^{-3}\ m\ °K$$

$$\text{so } \lambda_m = \frac{2.898\times10^{-3}}{T} = \frac{2.898\times10^{-3}}{293} = 9.89\ \mu m$$

b) for red light $\lambda \simeq 6500$ Å

$$\text{so } T = \frac{2.898\times10^{-3}}{6500\times10^{-10}} = 4460\ °K$$

1-5 A spherical black body 1.0 meter in radius is held at a constant temperature. Calculate the temperature, if the energy radiated by the body is 100 J/sec.

Soln:

Area of the body = $A = 4\pi r^2 = 12.57\ m^2$

so energy flux, $E = \frac{100}{12.57}\ J/m^2\text{-}sec.$

from eq.(1.1), we have

$$E = \sigma T^4$$

$$\text{or } T = \left(\frac{E}{\sigma}\right)^{1/4} = \left(\frac{100}{12.57}\times\frac{1}{5.6697\times10^{-8}}\right)^{1/4} = 108.8\ °K.$$

1-6 Assuming that the Sun is a perfect black body and the temperature of the Sun is 10^4 °K, Calculate wavelength of the maximum emittance.

Soln:

We have from eq.(1.2), $\lambda_m \times T = const.$

$$\text{or } \lambda_m = \frac{const.}{T} = \frac{2.898\times10^{-3}}{10^4} = 2879.8\ Å$$

1-7 Calculate the energy in a quantum of radiation having a wavelength 5000 Å.

Soln:

$$E = h\nu = \frac{hc}{\lambda} = \frac{6.624\times10^{-34}\times3\times10^{8}}{5000\times10^{-10}} = 1.56\times10^{-19}\ J.$$

1-8 A radio transmitter operates at a frequency of 97 M Hz with a power output of 200 KW. How many photons are emitted per second ?

Soln:

Each photon has energy, E, equal to

$$E = h\nu = 6.6\times10^{-34}\times97\times10^{6} = 6.4\times10^{-26}\ J$$

so the rate of photon emission is $= \frac{\text{output power}}{\text{energy per photon}} = \frac{200\times10^{3}}{6.4\times10^{-26}} = 3.13\times10^{30}$ photons/s.

UNSOLVED PROBLEMS

1:1 The fireball of a nuclear device has a radius of 1 m and a surface temperature of 10^7 °K.

a) Assuming it a black body, what is the wavelength of the peak radiation ?

b) What is the total power radiated?

1:2 Assuming that the Sun is a black body. If $\lambda_m = 5000$ Å for the solar spectrum, what is the surface temperature of the sun?

1:3 Find the wavelength and frequency of a photon whose energy is

a) 5 eV b) 1 keV c) 1 GeV.

1:4 A photon has a frequency of 1 G Hz. Find its energy and wavelength.

1:5 An FM radio transmitter has an output of 100 kilowatts at a frequency of 95 M Hz. How many quanta are emitted per second by the transmitter ?

Chapter 2

ELECTRONS AND PHOTONS

2.1 CONDUCTION OF ELECTRICITY THROUGH SOLUTIONS

The atomic nature of electricity was observed, when electricity is passed through aqueous solutions of chemical compounds. It is called electrolysis. Faraday stated two laws.

1. The mass of an element deposited when a current of electricity is passed is proportional to the quantity of charge that passes through the solution.
2. The mass of an element deposited by the passage of the same quantity of electricity is proportional to the chemical equivalent of the element.

Measurements was made for the amount of electricity needed to deposit one chemical equivalent of the element and found equal to 96,500 coulombs or one 'faraday'. One faraday of electric charge liberates (or deposits) in one gram atomic weight of copper (63.54 grams) the number of atoms equal to Avogadro's number N_o, that is 6.02×10^{23} .

Faraday concluded from the experimental results that one and the same amount of electricity is associated in the process of electrolysis with one atom of these substances. He thought of this charge as carried by the atom, (or molecule), and he called the atom with its charge an ion. In electrolysis the current results from the movement of positive and negative ions. This results an elementary unit of electricity, that is, atom of electricity. Then Stoney (1874) gave the idea of a natural unit of electricity, and suggested the name electron for this quantity.

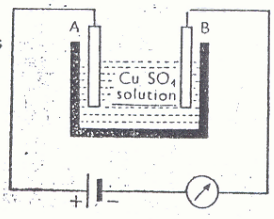


Fig.2-1:Electrolysis cell.

2.2 ELECTRICITY THROUGH GASES AT NORMAL PRESSURE

At normal pressure and temperature gases are insulators. But the gases can pass electricity by subjecting them some external agencies, for example, ultraviolet light, X-rays, electric sparks, are found to be conducting. This conductivity does not remain long lived. The cause of this conductivity is ionisation. When gases are subjected to the certain agents, a large number of ions are produced, and they help in conducting the current. In the absence of ionising agent we need high voltage upto 50,000 volts. At this potential difference sparking takes place in the gas. At the start we require more potential then to maintain the spark. Similarly the distance between the electrodes is shorter at start than to maintain it.

2.3 ELECTRICITY THROUGH GASES AT LOW PRESSURES

Electricity through gases at low pressures gives interesting phenomena. Careful studies gave rise to the discovery of X-rays, cathode rays, positive rays, and electrons and some properties of atoms.

A simplest form of the discharge of electricity through gases at low pressure is with the discharge tube arrangement. It consists of a glass tube nearly 0.3 m long with a circular electrode sealed into each end, as shown in fig. 2-2. A smaller side tube is sealed into it for vacuum pump and a pressure gauge. The two electrodes are connected with a potential difference of the order of 50,000 volts. Electrode A is connected to the positive side of a source of high potential, and electrode C is connected to the negative side. When the pressure of the air inside the tube is reduced to a few millimeters of mercury the electric discharge fills the entire space between the electrodes with a pink or reddish glow. Spectroscopic studies show that it consist of a series of lines characteristic of the gases within the tube.

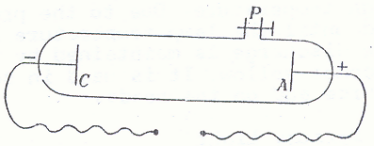


Fig.2-2: Discharge tube

When the pressure of the air within the tube is reduced to about 0.1 mm of mercury, the discharge is as follows: a bluish glow around the cathode C; then a dark space, called the Crookes dark space; then the negative glow and then the Faraday dark space; then the striated positive column upto the anode and around the anode the anode glow.

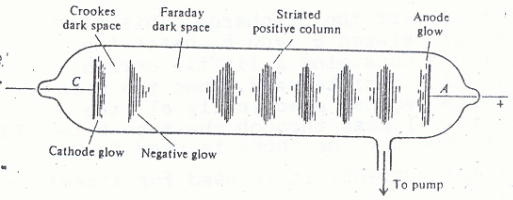


Fig.2-3: Appearance of electric discharge for 0.1 mm of mercury P.

At a pressure of about 0.01 mm, Crookes dark space practically fills the whole tube, when a new phenomenon is being observed. The walls of the tube begin to glow with a bluish or greenish light depending upon the material of the glass, which is known as fluorescence. Further lowering of pressure results difficulty in function. And fina-lly the phenomenon stops, because a perfect vacuum is non conducting for any value of potential difference between the electrode.

2.4 APPLICATIONS OF THE DISCHARGE TUBE

From many applications we will discuss only three of practical use.

1- Sodium Vapour Lamp

In this discharge lamp, low pressure is used at ordinary temperature. Argon or neon is used as a catalyser to start the discharge. Fig. 2-4 shows a practical sodium vapour lamp. A U-tube A with electrodes EE is the actual discharge tube. Sodium drops are in the inner walls along with a small amount of neon gas. The lamp usually functions at 220 volts A.C. Starting voltage is higher which is supplied by a transformer. The lamp requires an operating temperature of 300 °C.

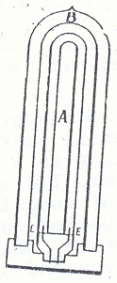


Fig.2-4:Sodium vapour lamp.

Therefore a double-walled vacuum tube B is enclosed for insulating high temperature. Due to the presence of neon, the light is at first red until the lamp temperature rises at which sodium evoporates. Then the discharge is maintained by the sodium vapours when the light becomes yellow. It is used in science laboratory as a monochromatic source and on the roads.

2- Mercury Light

This light consists of a mercury vapour lamp. It consists of a discharge tube which contains mercury along with small quantity of argon. The argon is used to start the discharge. Then mercury vaporises and maintain the discharge. It is used at 220 volts A.C. In fig. 2-5 the electrode E_3 is very near to E_1 and is used to sta-rt the discharge. This glow discharge between E_1 and E_3 heats the electrodes consuming a lit-tle current due to high resistance R. When the main discharge begins, practically all the current will pass through the tube between E_1 and E_2. The choke is used to limit the current. It is used for street lights.

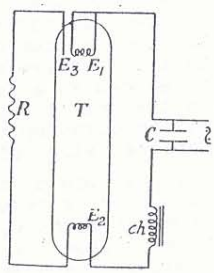


Fig.2-5: Mercury vapour lamp

It gives pale blue light. In its spectrum we see green, blue and yellow colours and no red colour. Also there is ultra-violet ray, not visible to the eye which play an important part in fluorecent tube lights.

3- Fluorecent Tube Light

The fluorescent tube lights are modifications of the mercury vapour lamp. Here invisible ultra-violet rays are utilized to produce bright light by coating the inner surface of the discharge tube with a thin layer of some fluorescent material, such as, zinc sulphate, or cadmium borate. The ultraviolet rays are produced, when fast moving electrons collide with mercury vapours. So unused and wasted energy is converted into useful light. So it gives three times more light than that of the ordinary elecric bulb for the same amount of energy consumed. The desired coloured light is obtained by changing the fluorescent material. It is possible to poroduce a light near to natural daylight.

In fluorescent tube light, there is a small quantity of mercury which vaporizes to a low pressure of nearly 1 mm. Two electrodes EE are made up of tungsten. These electrodes are heated for few seconds with a "starter". This starter S is of glow type and consists of two metallic strips inside a glass bulb

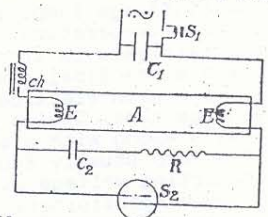


Fig.2-6: Fluorescent lamp.

filled with helium. When A.C. 220 volts is applied across the starter between the strips a glow discharge is produced. The discharge heats up the strips, then these strips bend towards each other and finally touch which helps the tube electrodes to be heated. When the strips are in contact the voltage between them drops to zero, the glow discharge is destroyed and the strips get cooled and separate again. Meanwhile the hot electrodes have main discharge started between themselves. The condenser C is in the starter circuit to by-pass radio frequency and resistance R prevents the starter strips from welding together. A choke is used in the circuit to limit the current to the required value.

2.5 CATHODE RAYS

The detailed analysis of the results obtained in the discharge tube experiments has shown the existence of three types of radiations, viz., cathode rays, positive rays (Section 2.6) and X-rays(Sect. 3.31) Cathode rays were the first to be discovered by Plucker in 1859.

When a gas is enclosed in the discharge tube with high voltage, electricity passes through the gas, and the gas becomes illuminated. At pressure below 0.1 mm of mercury the illumination gradually disappears. At very low pressure of about 10^{-5} mm of mercury, a fluorescent glow appears on the wall of the glass tube opposite the cathode.

Early workers suspected that the glow was caused by some kind of ray that traveled from one electrode to the other, and a number of tubes were devied in which obstructions could be manipulated in and out of the beam by the experimenter. From the shadows produced by these obstructions it was proved that the rays emitted from the cathode. And so these rays were called cathode rays. These are simply a stream of fast-moving negatively charged particles, called electrons. Their velocity is about 5×10^9 cm/sec.

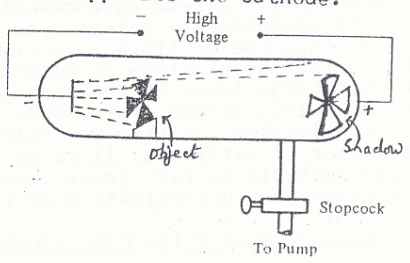


Fig.2-7: Gas discharge tube.

The cathode rays were found to have several interesting properties.

1. The rays travel in straight lines. They cast shadows of objects placed in their path. (fig. 2-7)
2. The rays can penetrate small thickness of matter. In gold or aluminium foil of about .0025 mm they can pass into open air. And in air they can penetrate upto 5 cm of the thickness.
3. The rays carry negative electric charge, and move with a velocity 5×10^9 cm/sec.
4. The rays leave surfaces normally.
5. The rays produce fluorescence on some substances which they strike.
6. The rays produce ionization when collided with neutral atoms.
7. The rays possess momentum and can move a light object placed in their path.

8. The rays are deflected by electric and magnetic fields.
9. The rays are able to heat bodies on which they fall.
10. When the rays strike on a heavy metal, such as, platinium, X-rays are produced.

2.6 POSITIVE RAYS

After the discovery of cathode rays, in 1886 Goldstein observed that when the cathode of a discharge tube was pierced by narrow holes, streamers of light appeared behind it. He assumed that the luminosity was caused by rays which traveled in the opposite direction to the cathode rays and passed through the holes in the cathode. These rays, which were called canal rays, as they proceeded canalised by the holes in the cathode. They were deflected by electric and magnetic fields, and from the directions of the deflections it was concluded that the rays consist of positively charged particles. This result gave rise to the more generally used term positive rays. Further investigations showed that they produce phosphorescence, affect photographic plates, penetrate thin aluminium foils, and disintegrate metals upon which they fall. These results suggested that the positive rays are streams of positive ions produced by the ionization of atoms or molecules in the strong electric field of the discharge tube. So they can give direct information as to the masses of molecules and atoms. This information should refer to the atoms or molecules individually rather than give average values for a large number of particles. It is on this account that a precise positive ray analysis is very great importance. These measurements prove the existence and determination of isotopes.

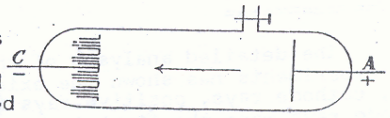


Fig.2-8: Canal rays.

2.7 CHARGE TO MASS RATIO FOR CATHODE RAYS BY J.J.THOMSON'S METHODE

In 1897, J.J. Thomson established the fact that cathode rays consist of negative particles, which were soon to be called electrons. Thomson determined the ratio e/m, by means of an apparatus of the type shown in fig. 2-10. Most of the electrons emitted from the cathode C strike the anode A_1. From where some of them pass through a small hole in the anode A_1 and are further accelerated and collimated by A_2. The tube is highly evacuated, so the particles after leaving A_2 will travel with velocity v and finally strike the fluorescent screen at S.


Fig.2-9: Sir Joseph John Thomson (1856-1940)

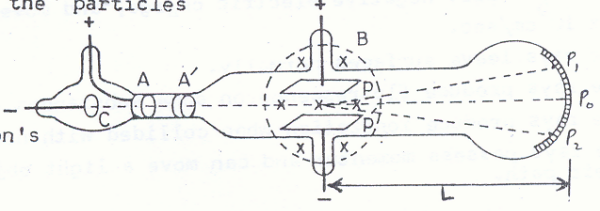


Fig.2-10: J.J.Thompson's Experiment.

The narrow stream of cathode rays can be deflected in the vertical plane by an electric field E between the parallel metal plates. They can also be deflected in the same plane by a magnetic field B perpendicular to the paper. In the present case electric field E is directed downward and magnetic field B is directed into the plane of paper, within the region indicated by the dotted circle in fig. 2-10.

Now the magnetic force on a single electron is given by

$$\vec{F}_{mag.} = e(\vec{v}\times\vec{B}) \quad \ldots\ (2.1)$$

or $\vec{F} = eBv\sin\theta$

or $F = Bev \quad [\sin\theta = 1] \quad \ldots\ (2.2)$

Also centripetal force is given by

$$\vec{F}_c = \frac{mv^2}{R} \quad \ldots\ (2.3)$$

And force due to electric intensity E is given by

$$\vec{F}_{elect.} = q\vec{E} \quad \ldots\ (2.4) \left[\vec{E} = \frac{\vec{F}}{q}\right]$$

or $F = eE \quad \ldots\ (2.5) \left[q = e\right]$

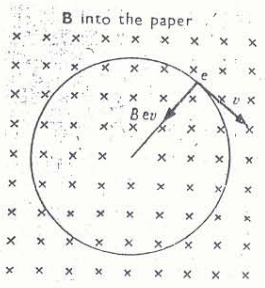


Fig.2-11: An electron projected into a magnetic field will move along a circle.

Since magnetic force F provides necessary centripetal force, so from eqs. (2.2) and (2.3) we get

$$Bev = \frac{mv^2}{R}$$

$$\text{or } e/m = \frac{v}{BR} \quad \ldots\ (2.6)$$

From equations (2.2) and (2.5) we get

$$Bev = eE$$

$$\text{or } v = \frac{E}{B} \quad \ldots\ (2.7)$$

Putting the value of v from eq.(2.7) into eq. (2.6), we have

$$e/m = \frac{E}{B^2R} \quad \ldots\ (2.8)$$

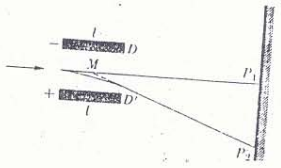


Fig.2-12: Deflection of the electrons in an electric field.

Now in fig. 2-10, we see

$$\sin\theta = \frac{s}{L}$$

$$\text{Also } \sin\theta = \frac{\ell}{R}$$

$$\text{so } \frac{s}{L} = \frac{\ell}{R}$$

$$\text{or } R = \frac{\ell L}{s} \quad \ldots(2.9)$$

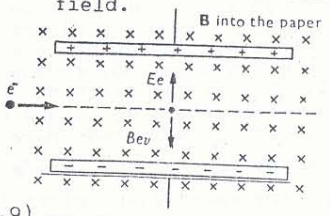


Fig.2-13: F_e & $F_{mag.}$ are balanced on an electron

From equations (2.8) and (2.9), we have

$$e/m = \frac{E\,s}{B^2 \ell L} \qquad \ldots\ldots \quad (2.10)$$

Since the ratio e/m is the only unknown quantity in Eq. (2.10), it can be readily determined.

$$e/m = 1.76 \times 10^{11} \text{ coulombs/kilogram.}$$

The design of Thomson's cathode ray tube is used in modern oscilloscopes and television sets.

2.8 THE CHARGE ON AN ELECTRON BY MILLIKAN'S METHOD

R.A. Millikan measured the charge of the electron in 1909. His oil-drop method is an accurate method for determining the charge on an electron. In this method, minute droplets of oil will reach a terminal velocity of only a few millimeters per second when allowed to fall freely in air. When they are confined to a draft-free chamber and illuminated such droplets can be observed with a microscope and their velocities may be easily measured. The mass of a given oil droplet can be calculated from its observed terminal velocity. If this droplet acquires a charge, it can be accelerated by means of an applied electric field until it achieves its terminal velocity in the upward direction. From the terminal velocities, the mass of the droplet, and the value of the electric field, the amount of charge on the drop can be determined.

awarded the 1923 Nobel Prize.

Fig.2-14: Robert A. Millikan (1868-1953)

Fig. 2-15 shows the Millikan's apparatus. The oil droplets are produced by an atomizer. The air between the plates is ionized by X-rays and is strongly illuminated by a suitable light. The droplets appeared as tiny twinkling stars in the dark night when observed with a strong microscope. With a little practice the observer can single out a droplet and measure its velocity of free fall by means of a scale in the eyepiece of the microscope. Before the droplet falls out of the field of view, the electric field is turned on. And if that droplet is charged then its velocity is measured. Here we assume that:

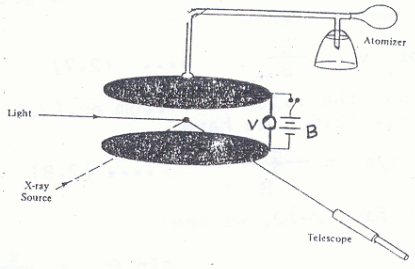


Fig.2-15: Millikan's oil drop app.

1. The number of ions is the same as the number of drops.
2. The drops are exactly spherical in form.
3. The density of a single drop is the same as that of the oil in bulk.

4. There is no decrease in mass due to evaporation.

5. The Stoke's law is applicable.

Now let

ρ = density of oil

ρ_o = density of air

a = radius of the droplet

g = acceleration due to gravity

v_o = terminal velocity or maximum velocity of the droplet

v = velocity of the droplet corresponding to charge q

η = coefficient of viscosity

Also we define,

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$\text{or mass} = \text{density} \times \text{volume}$$

And

$$\text{Volume} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi a^3 \quad \ldots\ldots (2.11)$$

$$\text{Weight} = \text{mass} \times g = \rho \times \frac{4}{3}\pi a^3 \times g \quad \ldots\ldots (2.12)$$

Archimedes Principle:

"The buoyant force (or upward thrust) which a liquid exerts on a body placed in it is equal to the weight of the fluid the body displaces".

And Stoke's law:

$$F = 6\pi\eta a v \quad \ldots\ldots (2.13)$$

Now from Archimedes Principle, we have

$$\text{upward thrust} = \frac{4}{3}\pi a^3 \rho_o g \quad \ldots\ldots (2.14)$$

So from eqs. (2.12) and (2.14), we have

$$\text{Net weight or effective weight} = \frac{4}{3}\pi a^3 \rho g - \frac{4}{3}\pi a^3 \rho_o g$$

$$\text{or} \qquad W = \frac{4}{3}\pi a^3 (\rho - \rho_o) g \quad \ldots (2.15)$$

We have from Stoke's law

$$F = 6\pi\eta a v_o = W \quad \ldots\ldots (2.16)$$

from eq. (2.4), we have

$$F = qE \quad \ldots\ldots (2.17)$$

Now from eqs. (2.13), (2.16) & (2.17), we have

$$6\pi\eta a v = \text{Net force} = W + qE$$

Multiplying and dividing by v_o, we have

$$6\pi\eta a v_o \frac{v}{v_o} = W + qE \quad \ldots\ldots (2.18)$$

from eq. (2.16) and (2.18), we have

$$W \frac{v}{v_o} = W + qE$$

$$\text{or} \quad qE = W \frac{v}{v_o} - W$$

$$\text{or} \quad qE = W\left(\frac{v}{v_o} - 1 \right)$$

$$\text{or} \quad q = \frac{W}{E}\left(\frac{v}{v_o} - 1 \right) \qquad \ldots\ldots \quad (2.19)$$

From eqs. (2.15) and (2.19), we get

$$q = \frac{4\pi a^3 g}{3E}(\rho - \rho_o)\left(\frac{v}{v_o} - 1 \right) \qquad \ldots\ldots \quad (2.20)$$

Since all the quantities on the right hand side of eq.(2.20) can be measured, so q can be calculated.

From the results of thousands of measurements, Millikan always found the same value of q. It is

$$e = 1.60 \times 10^{-19} \text{ coulombs}$$

Using this value of e and the value of e/m, we can easily calculate the value of m, the mass of the electron,

$$m = 9.1084 \times 10^{-31} \text{ kg.}$$

2.9 BUCHERER'S EXPERIMENT

The charge to mass ratio for cathode rays was measured by J.J. Thomson. The velocity of electrons of cathode rays is slower as compared to β-particles. And relativistic effects was negligible. According to special theory of relativity the mass of the particle is increased with the increase of velocity. In 1909 Bucherer verified the relativistic dependence of mass on velocity. In this experiment β-particle are first of all sorted out according to their velocity by means of suitably adjusted electric and magnetic fields and are then subjected to a magnetic field alone. So they are deflected in different arcs according to their velocities. And then they are traced on a photographic film. Finally the e/m values for electrons having different velocities are calculated.

The experimental set up is shown in fig. 2-16. We have two parallel discs very close to each other. The source of β-particles is placed at their center. The discs a-re surrounded by a coaxial cylindrical photographic film. There is vacuum inside the chamber. An electric field is maintained between the discs. A uniform magnetic field is applied in whole the chamber. This magnetic field is parallel to the plane of the

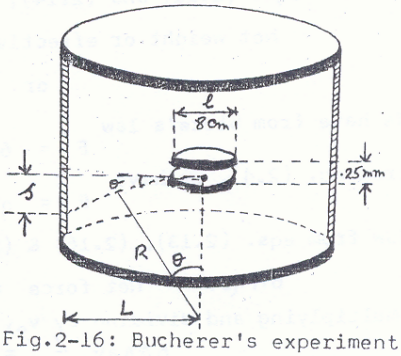


Fig.2-16: Bucherer's experiment

discs. The β-particles coming out from the source are subjected to the electric force as well as magnetic force. Since the discs are very close to each other (0.25 mm apart), the β-particles cannot escape from between the discs and strike the photographic film. They only move and strike the film when electric and magnetic fields are equal and opposite to each other. They will emerge at different angles having different velocities.

The force due to electric field is

$$\vec{F} = q\vec{E}$$

$$\text{or} \quad F = eE \qquad \ldots\ldots (2.21)$$

and the force due to magnetic field is

$$\vec{F} = e(\vec{v} \times \vec{B})$$

$$\text{or} \quad F = Bev \sin\theta \qquad \ldots\ldots (2.22)$$

The centripetal force is

$$F = \frac{mv^2}{R} \qquad \ldots\ldots (2.23)$$

Also in fig. 2.16, we have

$$\sin\theta = \frac{s}{L}$$

$$\text{and} \quad \sin\theta = \frac{l}{R}$$

$$\text{so} \quad \frac{s}{L} = \frac{l}{R}$$

$$\text{or} \quad R = \frac{lL}{s} \qquad \ldots\ldots (2.24)$$

Now, from equations (2.21) and (2.22), the balance of electric and magnetic forces requires,

$$Bev\sin\theta = eE \qquad \ldots\ldots (2.25)$$

$$\text{or} \quad v = \frac{E}{B\sin\theta} \qquad \ldots\ldots (2.26)$$

This equation (2.26) shows that the velocity of the β-particles depends on angle θ, as E and B are constant. So the β-particles do not emerge from the plates with different velocities, but they will emerge in different directions. In this way β-particles of different velocities are sorted out.

As the particles move in circular arc, so from equations (2.22) and (2.23), balancing magnetic field with centripetal force,

$$Bev\sin\theta = \frac{mv^2}{R} \qquad \ldots\ldots (2.27)$$

$$\text{or} \quad e/m = \frac{v}{RB\sin\theta} \qquad \ldots\ldots (2.28)$$

putting the values of v and R from eqs. (2.26) and (2.24), we get

$$e/m = \frac{Es}{lLB^2\sin^2\theta} \qquad \ldots\ldots (2.29)$$

In this eq. (2.29) substituting all the values in R.H.S., we can calculate the charge to mass ratio. We see the variation of e/m

for β-particles as predicted by special theory of relativity. Bucherer measured e/m for different velocities from the relativistic formula.

$$m = \frac{m_o}{\sqrt{1 - v^2/c^2}}$$

$$\text{or} \quad e/m = \left(\frac{e}{m_o}\right)\sqrt{1 - \frac{v^2}{c^2}} \qquad \ldots\ldots (2.30)$$

the value of e/m_o was calculated which was found to be constant and in good agreement with that obtained for cathode rays. It shows that β-particles are ordinary electrons emitted from the nucleus, as they are created at the time of emission. The velocities upto 0.9c are verified.

2.10 PHOTOELECTRIC EFFECT

We define the Photo-electric effect as:

"The emission of electrons by a substance when illuminated by electromagnetic radiation".

Such emission of electrons are called photo-electrons, the name so called due to the source of their excitation. The quantum or particle nature of light dominates its wave naturein this phenomenon.

Attempts were made to explain the observed phenomenon, but these were not successful. Experimental observations by different workers are summarized here.

1. The rate of emission of photo-electrons is directly proportional to the intensity of the incident light.
2. The kinetic energy of photo-electrons does not depend upon the intensity of light, but it depends upon the frequency of the light.
3. The maximum energy of photo-electrons increases with the increasing frequency above the threshold frequency.
4. Photo-electrons are emitted from the given metal surface when the frequency of incident light is above a certain critical value ν_o, which is called threshold frequency. No photo-electric emission occurs if the frequency of light is less than that of the threshold frequency, even if the intensity of light is very high. Threshold frequency

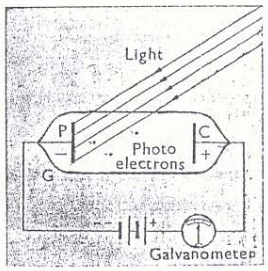


Fig.2-17: Photoelectric effect.

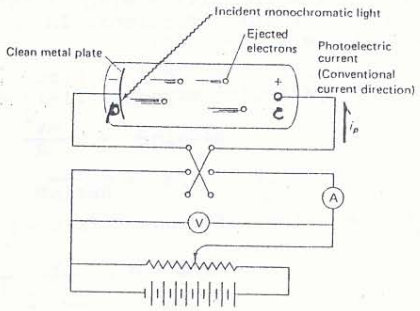


Fig.2-18: Photoelectric Expt.

depends upon the metal surface.

5. The photo-electrons are emitted instantaneously even when the weakest possible beam of light having frequency above the threshold frequency falls on the metal plate.

The above first three observations are called the three laws of photo-electric emission.

One thing of particular notice is the fact that even when electrons are emitted, their kinetic energy does not depend on the intensity of the radiation, but only on its frequency. This is completely unintelligible on the classical picture of a continuous exchange of energy as defined before. However, it is quite simple to understand on Planck's Photon hypothesis.

In 1905, Einstein tried to explain the photo-electric effect following the Planck's concept of quantization of energy. His simple explanation was so inspiring that he was awarded the Nobel prize. Einstein supposed that light is propagated in the form of photons, bursts of energy. Each photon of frequency ν carries an energy $E = h\nu$. Some energy is required to take an electron out from the metal surface. We define work function (ϕ) as, "the minimum energy required to remove an electron from the surface of a material and send it into field-free space".

If energy of the photon $h\nu$ is greater than or equal to the work function ϕ, only then the photon is capable of ejecting out the electron. That is why, the photo-emission is not observed if frequency of the light is less than some critical frequency. If whole of the energy $h\nu$ of a photon is absorbed by the electron and $h\nu$ is greater than or equal to ϕ, then electron is ejected out and photoelectric effect is observed.

We define the threshold frequency, ν_0, as, "minimum frequency required for a photon of radiation to remove an electron from the surface of a material".

We have, $\phi_0 = h\nu_0$

If $h\nu$ is greater than ϕ_0, then the energy $(h\nu - \phi_0)$ or $(h\nu - h\nu_0)$ appears as kinetic energy of the photoelectron. If the mass of the electron be m and it is ejected out of the metal surface with velocity v, then

$$\tfrac{1}{2}mv^2 = h\nu - h\nu_0 \qquad \ldots\ (2.31)$$

Equation (2.31) is known as Einstein's photo-electric equation.

Suppose that light from the source falls on a metal plate P (fig.2-18) connected to the negative terminal of the battery, enclosed in an evacuated glass tube. The electrons emitted from the plate P are collected by a collector C, connected to the positive terminal of the battery. When the light falls on the plate P, galvanometer indicates a current flowing in the circuit due to photo-electrons. If we reverse the polarity of the battery and make the collector negative with respect to the plates the current goes on decreasing with the increase in applied potential. Only those electrons can reach the electrode C which possess energy greater than eV, where 'V' is the voltage across the tube and 'e' is the charge of an electron. If the stopping potential for electrons i.e., the potential for which no electron can

reach the plate be V_o, then the maximum kinetic energy of the photo-electrons can be determined by $\left[V = \frac{W}{q}\right]$

$$\tfrac{1}{2}mv_{max}^2 = eV_o \quad \ldots\ldots (2.32)$$

where e and m are the electronic charge and mass respectively and v_{max} is the maximum velocity of ejected electrons. Thus eq.(2.31), can be written as:

$$\tfrac{1}{2}mv_{max}^2 = eV_o = h\nu - \phi_o \quad \ldots\ldots (2.33)$$

The equation (2.33) can be used to determine any of the quantities, Planck's consta-nt h, frequency of the incident radiation or the threshold frequency ν_o if the remaining are known and V_o is measured experimentally.

In 1914 Millikan produced the first direct experimental proof of the equation developed by Einstein, eq. (2.33), and at the same time made the first direct photo-electric determination of Planck's constant h.

2.11 COMPTON EFFECT

We define the Compton effect as:

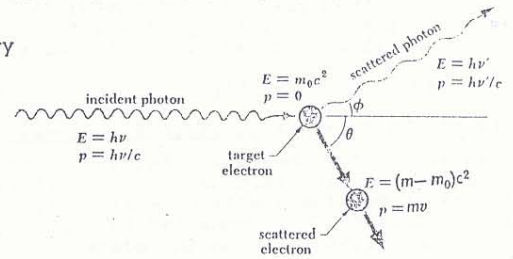
"The phenomenon in which a photon is scattered by an electron and the scattered photon has a frequency less than its original frequency."

According to the classical theory, the wavelength of X-rays scattered by electrons must be equal to the wavelength of primary X-rays. But a change in wavelength has been observed by experiments. This is called the Compton effect.

The Compton effect was discovered by A.H. Compton in 1923 during the course of his studies of the scattering of X-rays by matter with improved techniques.

The scattering process is analyzed simply as an interaction (a "collision" in the classical sense of particles) between a single photon and an electron, which we assume to be at rest.

Fig.2-19: The geometry of Compton scattering.



Suppose an X-ray photon of energy $h\nu$ collides with an electron at rest and is then changed in direction and frequency as shown in the figure 2-19.

Initially, the photon has energy E given by

$$E = h\nu = hc/\lambda \quad \left[c = \lambda\nu\right]$$

and linear momentum p, $p = \frac{E}{c} = \frac{h\nu}{c}$ $\left[\begin{array}{l} E = mc^2 \\ \quad = mc \times c = pc \\ \text{or } E = pc \text{ or } p = E/c \end{array}\right.$

The electron, at rest, has rest energy $m_o c^2$.

After the scattering, the photon has energy E' and momentum p', and moves in a direction at an angle ϕ with respect to the direction of the incident photon. The electron has total energy E_e and momentum p_e and moves in a direction at an angle θ with respect to the initial photon. (To allow for the possibility of high-energy incident photons giving energetic scattered electrons, we use relativistic kinematics for the electron). The usual conditions of conservation of energy and momentum are then applied.

$$E_{initial} = E_{final}$$

$$E + m_o c^2 = E' + E_e \quad \ldots\ (2.34)$$

$$\text{or} \quad h\nu + m_o c^2 = h\nu' + mc^2$$

$$\text{or} \quad h\nu - h\nu' = (m - m_o)c^2 \quad \ldots\ (2.35)$$

And

$$(p_x)_{initial} = (p_x)_{final}$$

$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c}\cos\phi + mv\cos\theta \ldots (2.36)$$

Also

$$(p_y)_{initial} = (p_y)_{final}$$

$$\text{or} \quad 0 = \frac{h\nu'}{c}\sin\phi - mv\sin\theta \ldots (2.37)$$

To evaluate the change in frequency, let us eliminate θ from (2.36) and (2.37), taking these equations, we have

$$mvc\cos\theta = h(\nu - \nu'\cos\phi)$$

$$mvc\sin\theta = h\nu'\sin\phi$$

Squaring and adding, we get

$$m^2v^2c^2 = h^2\nu^2 + h^2\nu'^2\cos^2\phi - 2h^2\nu\nu'\cos\phi + h^2\nu'^2\sin^2\phi$$

$$\text{or} \quad m^2v^2c^2 = h^2(\nu^2 - 2\nu\nu'\cos\phi + \nu'^2) \quad \ldots\ (2.38)$$

from eq. (2.35), we have

$$mc^2 = h(\nu - \nu') + m_o c^2$$

squaring

$$m^2c^4 = h^2(\nu^2 - 2\nu\nu' + \nu'^2) + m_o^2c^4 + 2h(\nu - \nu')m_o c^2 \quad \ldots\ (2.39)$$

subtracting eq.(2.38) from eq.(2.39), we get

$$m^2c^2(c^2 - v^2) = -2h^2\nu\nu'(1 - \cos\phi) + 2h(\nu - \nu')m_o c^2 + m_o^2c^4 \qquad \left[m = \frac{m_o}{\sqrt{1 - v^2/c^2}}\right.$$

$$\text{or} \quad \frac{m_o^2c^2(c^2 - v^2)}{1 - v^2/c^2} = \quad " \quad " \quad " \quad "$$

$$\text{or} \quad \frac{m_0^2 c^2 (c^2 - v^2)}{c^2 - v^2} = -2h^2 \nu\nu' (1 - \cos\phi) + 2h(\nu - \nu') m_0 c^2 + m_0^2 c^4$$

$$\text{or} \quad m_0^2 c^4 = -2h^2 \nu\nu' (1 - \cos\phi) + 2h(\nu - \nu') m_0 c^2 + m_0^2 c^4$$

$$\text{or} \quad \frac{\nu - \nu'}{\nu\nu'} = \frac{h}{m_0 c^2} (1 - \cos\phi)$$

$$\text{or} \quad \frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{m_0 c^2} (1 - \cos\phi)$$

$$\text{or} \quad c\left(\frac{1}{\nu'} - \frac{1}{\nu}\right) = \frac{h}{m_0 c} (1 - \cos\phi)$$

$$\text{or} \quad \lambda' - \lambda = \frac{2h}{m_0 c} \sin^2 \phi/2 \qquad \ldots\ldots (2.40)$$

$$\text{or} \quad \lambda' = \lambda + \frac{2h}{m_0 c} \sin^2 \phi/2$$

This relation shows that the wavelength λ' of the scattered photon is greater than the wavelength λ of the incident photon.

From eq.(2.40), we have

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c} (1 - \cos\phi) \qquad \ldots\ldots (2.41)$$

which gives the change in wavelength $\Delta\lambda$, known as Compton shift. It is interesting to note that Compton shift $\Delta\lambda$ depends only upon the scattering angle ϕ and is independent of the frequency of the incident photon. Compton effect can be explained only on the basis of quantum theory of electro-magnetic radiations. This effect strongly supports the view that a photon acts as a particle.

Putting the values in eq.(2.41), we have

$$h = 6.6 \times 10^{-27}$$

$$m_0 = 9.1 \times 10^{-28}$$

$$c = 3 \times 10^{10}$$

so $$\Delta\lambda = 0.0242 (1 - \cos\phi) \times 10^{-8} \text{ cm}$$

$$= 0.0242 (1 - \cos\phi) \text{ Å}$$

2.12 DUAL NATURE OF ELECTROMAGNETIC RADIATION

Certain experiments invoving interference and diffraction effects, show that electromagnetic radiation interacts as waves. But other experiments invoving photoelectric effect and compton effect indicate that electromagnetic radiation consists of particles, which are called photons. So it has dual nature, sometimes it behaves like particles and sometimes as waves.

The particle and wave natures are not same. Particles deliver their energy in packets, and the energy of a wave is spread uniformly over the entire wavefront. There is idea of collision in case of particles but in waves we see the idea of superposition.

We regard the electron as particle in the experiments for measuring e/m. But the electron diffraction experiments of Davisson and Germer, and of G.P. Thomson show that the electron must be regarded as a wave. So matter have also a dual "wave-particle" nature.

Light emitted by a source must travel either as waves or as particles. Suppose we place a double-slit apparatus on one side of the source and a photoelectric cell on the other side. Light emitted toward the double slit behaves like a wave and light emitted toward the photocell behaves like particles. How did the source know in which direction to aim the waves and in which direction to aim the particles? Let us repeat this experiment with light coming from a distant galaxy. Suppose the light has beam travelling toward us for a time 15×10^9 years (the age of the universe). Our experiment could not be signalled back in the time it takes us to remove the double-slit apparatus with photo electric apparatus. Yet we find that the light from distant stars can show interference effect and also photoelectric effect.

We conclude that light shows its particle aspect or the wave aspect, depending on the kind of experiment we are doing. A particle-type experiment shows the particle nature, while the wave-type experiment shows the wave nature.

The dilemma of wave-particle duality cannot be resolved with a simple explanation. Both the wave and particle aspects of photons are required for the description of light is known as 'Bohr's principle of complementarity'. That is, the wave and particle aspects complment each other. We can observe only one property in a given experiment, and not both at the same time. For example, we observe the wave property in a diffraction experiment, but not the particle property. Also we observe the particle property in the Compton effect, but not the wave property. Like a coin in which we cannot observe both the faces at the same time. It is not possible to perform a simple experiment in which the electron will behave as a particle and a wave.

QUESTIONS

2.1 Explain in detail how cathode rays are produced ? Describe some experiments to demonstrate the properties of cathode rays.

2.2 How are the cathode rays generated? Give reasons for believing that they are streams of negatively charged particles. In what ways do the cathode rays differ from X-rays?

2.3 Describe J.J. Thomson's method of determinimg the ratio e/m for cathode rays.

2.4 How was electronic charge determined by Millikan?

2.5 Describe Bucherer's experiment to study retalivistic variation of mass.

2.6 How is the wave nature of light unable to account for the observed properties of the photoelectric effect?

2.7 Why doesn't the photoelectric effect work for free electrons?

2.8 What might be the effects on a photoelectric effect experiment if we were to double the frequency of the incident light? If we were to double the wavelength? If we were to double the intensity?

2.9 What is Compton effect? How can we explain this effect on the quantum theory?

2.10 A beam of photons passes through a block of matter. What are the three ways that the photons can lose energy in interactimg with the material?

PROBLEMS

2-1 A potential difference of 300 volts is applied to parallel plates of a capacitor 5×10^{-3} m apart. Calculate the force on an oil drop carrying a charge 32×10^{-19} C.

Soln: We have

$$\Delta V = E \cdot \Delta \ell$$

$$\text{or } V = dE$$

$$\text{or } E = \frac{V}{d} = \frac{300}{5\times10^{-3}} = 6\times10^{4}\ V/m$$

$$\text{So } F = qE = 32\times10^{-19}\times6\times10^{4} = 1.92\times10^{-13}\ N.$$

2-2 What is the threshold wavelength for sodium, whose work function is 3.69×10^{-19} J ?

Soln: We have

$$\phi_0 = h\nu_0 \qquad [c = \lambda\nu]$$

$$\text{or } \phi_0 = \frac{hc}{\lambda_0}$$

$$\text{or } \lambda_0 = \frac{hc}{\phi_0} = \frac{6.6\times10^{-34}\times3\times10^{8}}{3.69\times10^{-19}} = 5420\ \text{Å}$$

2-3 It is determined experimentally that the threshold wavelength for the photoemission of electrons from lithium is 5865 Å. What is its work function?

Soln: We have $\phi_0 = \frac{hc}{\lambda_0} = \frac{6.6\times10^{-34}\times3\times10^{8}}{5865\times10^{-10}} = 3.41\times10^{-19}$ J.

2-4 a) What are the energy and momentum of a photon of red light of wavelength 6500 Å?

b) What is the wavelength of a photon of energy 2.4 eV?

Soln: a) $E = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^{8}}{6500\times10^{-10}} = 3.06\times10^{-19}$ J

$p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}}{6500\times10^{-10}} = \frac{3.06\times10^{-19}}{1.6\times10^{-19}} = 1.91$ eV.

b) $\lambda = \frac{hc}{E} = \frac{6.63\times10^{-34}\times3\times10^{8}}{2.4\times1.6\times10^{-19}} = 5170$ Å

2-5 X-rays of wavelength 0.24000 Å are Compton scattered and the scattered beam is observed at an angle of 60° relative to the incident beam. Find

a) the wave-length of the scattered X-rays,
b) the energy of the scattered X-ray photons,
c) the kinetic energy of the scattered electrons.

Soln: a) We have from eq.(2.41)

$$\lambda' = \lambda + \frac{h}{m_0 c}(1-\cos\phi)$$
$$= 24000\times10^{-10} + \frac{6.63\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}}(1-\cos60^\circ) = 2.412\ \text{Å}$$

b) We have $E' = \frac{hc}{\lambda'} = \frac{6.63\times10^{-34}\times3\times10^{8}}{2.412\times10^{-10}\times1.6\times10^{-19}} = 5141$ eV.

c) From eq.(2.34), we have

$$E_e = (E-E') + m_0c^2 = K_e + m_0c^2$$
$$\text{or } K_e = E - E'$$
$$= \frac{hc}{\lambda} - \frac{hc}{\lambda'}$$
$$= \frac{6.63\times10^{-34}\times3\times10^{8}}{24000\times10^{-10}} - \frac{6.63\times10^{-34}\times3\times10^{8}}{2.412\times10^{-10}} = 5167-5141 = 26\text{ eV}.$$

2-6 An X-ray photon is scattered from a stationary electron with $\theta = 35°$. Calculate energy loss by the photon.

Soln: We have

$$\Delta\lambda = \frac{h}{m_0c}(1-\cos\phi) = \frac{6.63\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}}(1-\cos35^\circ) = 4.39\times10^{-13}\ m.$$

$$\Delta E = h\Delta\nu = \frac{hc}{\Delta\lambda} = \frac{6.63\times10^{-31}\times3\times10^{8}}{4.39\times10^{-13}\times1.6\times10^{-19}} = .29\ \text{MeV}$$

UNSOLVED PROBLEMS

2:1 A stream of electrons travelling with a velocity of 3×10^7 m/sec is deflected in passing through an electric field of 0.06 N/C perpendicular to their path. If the radius of the deflected path is 3 m, calculate the value of e/m for the electron.

2:2 A water drop of radius 10^{-7} m is charged with one electron. Calculate the electric field required to keep it stationary.

2:3 If the photoelectric threshold wavelength of sodium is 5420 Å, calculate the maximum velocity of photoelectrons ejected by photons of wavelength 4000 Å.

2:4 Ultraviolet light of wavelength 2500 Å is incident upon potassium What is the stopping potential for photoelectrons if the work function of potassium is 2.21 eV?

2:5 X-rays having a wavelength of 0.3 Å are Compton scattered. Find the wavelength of a photon scattered at 60° and the energy of the scattered electron.

Chapter 3

ATOMIC STRUCTURE

3.1 INTRODUCTION

Ancient philosophers started with the concept of splitting matter into elementary parts. Analyzing the light was not clear to them. For example, the explanation of the rainbow was not known. Sir Isaac Newton was first who gave the idea of analyzing the light into different colours. In 1672 Newton performed the experiment that sun light consists of seven colours called the spectrum of sunlight. This experiment was the start of spectroscopy.

Fig.3-1: Sir Isaac Newton (1642-1727) Born in Lincolnshire, England.

In 1800, William Herchel became interested and observed a rise in temperature in the thermometer placed across a solar spectrum dispersed by a prism. Few more scientists observed that electromagnetic spectrum extends on both sides of the visible ends.

Joseph Fraunhaffer (Bavarian) observed dark lines in the solar spectrum. He built diffraction grating, and made the first accurate measurements of wavelength. In 1859, Kirchoff demonstrated that the lines corresponded to definite elements in the source of light.

3.2 BASIC PROPERTIES OF ATOMS

Here we summarize some of the basic properties of atoms.

1. It is the smallest subdivision of an element which displays all the properties of that element. If it is further subdivided, it will cease to represent the element.
2. Atoms are very small, about 0.1×10^{-9} m in radius. It cannot be seen from the naked eye, because to see an atom using visible light ($\lambda \simeq 500 \times 10^{-9}$ m) is hopeless.
3. Atoms are stable, they do not spontaneously break apart into smaller pieces or collapse, so the internal forces must be in equilibrium.
4. The atoms are composed of a negative part and a positive part.
5. The negative part is formed of electrons which are very light. If we shake or tickle the atoms with sufficient force, electrons are emitted, as in Compton effect (see section 2.11) and photoelectric effect (section 2.10).
6. Atoms are electrically neutral.
7. The positive part must be responsible for the entire mass of the atom, as the electrons are very light.
8. Atoms emit and absorb electromagnetic radiation. This radiation have many forms, e.g., visible light, X-rays, ultraviolet light, infrared portion, heat radiation, radio frequencies, etc.

3.3 THE ATOM MODELS

To account for the experimentally obtained spectroscopic data, several theories have been successively proposed, which are called the atom models. Thus we have the Thomson atom model, Rutherford nuclear atom model, Bohr atom model, Sommerfeld's atom model, and Vector atom model. Which we will study in the proceeding sections.

THOMSON ATOM MODEL

An early model of the structure of the atom was proposed by J.J. Thomson, who was known for his discovery (1897) of the electron. In 1898 he put forward his model. According to this model, the atom was a spherical mass of positive material and the electrons were embedded in it at random. The whole thing looked, more or less, like a fruit cake; the positive part being the cake and the electrons being the fruit pieces. (This model is sometimes known as "plum-pudding" model.) It was assumed that the electrons were just sitting inside the atom. The main features of this model can be summarised as follows:

Fig.3-2: Sir Joseph John Thomson (1856-1940)

1. The atom has a uniform density.
2. There is no empty or free space inside the atom.
3. The electrons are in static equilibrium with the positive charges.

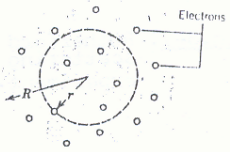
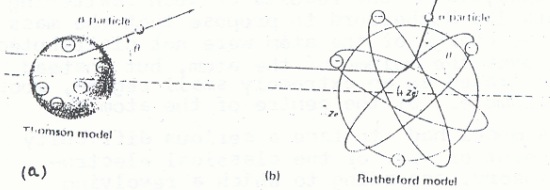


Fig.3-3: The Thomson model of the atom.

After some time, the experimental researches of Lord Rutherford on the scattering of α-particle from radioactive substances by matter definitely proved that the assumption of Thomson about the uniform distribution of positive charges was wrong. As the experimental results show that the positive charges in the atom should be concentrated in a very small region at the centre of the atom.

Fig.3-4: a) The expected deflection is small due to small electrical field within the atom.
b) The positive charge is concentrated, so deflection of α-particle is greater.



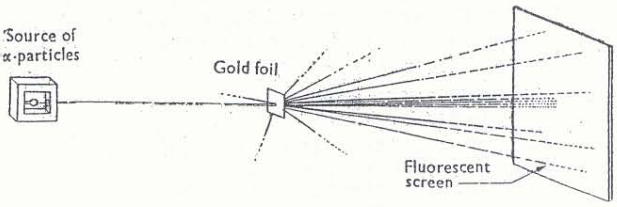
3.4 RUTHERFORD NUCLEAR ATOM MODEL

In 1911 Professor Ernest Rutherford who had been a student of Thomson, and two of Rutherford's students, Hans Geiger and Ernest Marsden, performed a number of experiments on the scattering of α-particles by a very thin gold foil. They obtained a collimated beam of α-particles of known number of particles from Po^{214} source.

Fig.3-5: Sir Ernest Rutherford (1871-1937)

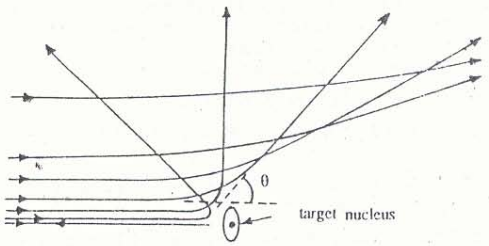
A study of the scattering or deflection angles of α-particles passing through the foil was observed by doing this experiment many times.

Fig.3-6: Only a few α-particles were scattered. A great majority of α-particles crossed the gold foil.



An experiment of this sort was performed in 1910. The results showed some probability of an alpha particle scattering at angles beyond 90°. When it was found that α-particles were being scattered backwards, it was such

Fig.3-7: Rutherford scattering experiment.



a surprise that Rutherford stated:

"It was quite the most incredible event that ever happened to me in my life. It was as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you".

The analysis of the results of such scattering experiments led Rutherford to propose that the mass and positive charge of the atom were not distributed uniformly over the volume of the atom, but instead were concentrated in an extremely small region, about 10^{-14} m in diameter, at the centre of the atom.

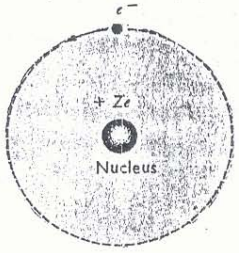


Fig.3-8: Rutherford's model of hydrogen atom.

This model had to face a serious difficulty from the point of view of the classical electromagnetic theory, according to which a revolving electron should radiate energy continuously. This energy can only come from the atomic system, which will therefore steadily lose energy. As a result, the electron will approach the nucleus by a spiral path, and finally fall into the nucleus (in $\sim 10^{-8}$ sec). But the atom is observed very stable. Moreover, when the electron is continuously radiating energy during its fall to the nucleus, a continuous spectrum should be observed. But the emitted atomic spectrum consists of discrete set of lines. It was solved in 1913 by Niels Bohr, who admitting the failure of classical theory, applied the quantum theory to the nuclear atom model.

3.5 ORIGIN OF SPECTRAL LINES

Ever since the spectroscope was discovered, it was obvious that arcs or sparks gave spectral lines of extra-ordinary interest. It was found that each type of atom had its characteristic spectrum, so spectrum analysis became a useful method. Two clearly distinguished types of spectra were observed; the line spectra, which were characteristic of the atoms, and band spectra, characteristic of molecules.

The spectrum of an element consists of a number of series spread over infra-red to ultra-violet regions of the visible spectrum. All series are

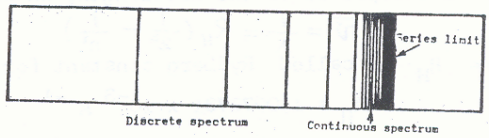


Fig.3-9: A typical spectrum series.

similar in general appearance. They differ only of location on the wavelength scale. In a typical series, the spectral lines are well-apart at one end. They get closer and closer towards the other end till they merge at a point, called series limit. The portion of the spectrum upto the series limit is called discrete spectrum and that beyond the series limit is called continuous spectrum.

3.6 ATOMIC SPECTRA (OR LINE SPECTRA)

The radiation from atoms can be classified into continuous spectra and discrete or line spectra. The radiation from a hot body have continuous spectrum. White light is a mixture of different colours. A white hot body emits light at all frequencies of the visible spectrum. In vapour tubes, e.g. sodium, neon, or mercury, when electric discharge is maintained, light is emitted at a few discrete wavelengths. Sodium vapour street lights give yellowish colour, "neon signs" give red colour line, and mercury vapour street lights give blue and green lines.

When white light is passed through a sample of gas we get absorption spectrum (as certain wavelengths have been absorbed from the light). The solar spectrum observed by Joseph Fraunhoffer showed many dark lines now called Fraunhoffer lines.

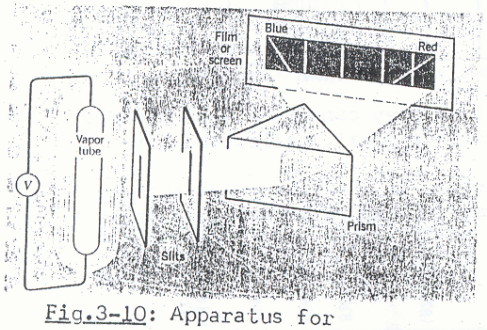


Fig.3-10: Apparatus for observing line spectra.

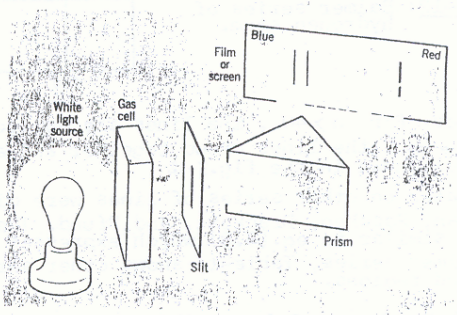


Fig.3-11: Apparatus for observing absorption spectra.

In Section 1.4 of black body radiation we see, in the absence of a theory that explains the data, we try to find a function that fits the data, and then try to find a theory that explains the derived function. A Swiss school teacher, Johannes Balmer in 1885, experimenting on the spectrum of hydrogen, found an empirical formula:

$$\lambda = 364.5 \frac{n^2}{n^2 - 4} \qquad \ldots\ldots (3.1)$$

where λ is wavelength in n m, and $n = 3,4,5, \ldots$

A few years later Rydberg described in a more useful way in terms of the wave number or reciprocal of the wavelength, which is now known as Balmer series.

$$\bar{\nu} = \frac{1}{\lambda} = R_H \left(\frac{1}{2^2} - \frac{1}{n^2} \right) \qquad \ldots\ldots (3.2)$$

where R_H is called Rydberg constant for hydrogen.

$$R_H = 1.0973731 \times 10^{-3} \text{ A}^{-1}$$

$$n = 3,4,5, \ldots$$

We define:

Wave number: It is the number of waves in one centimeter of light path in vacuum.

3.7 THE SPECTRUM OF HYDROGEN

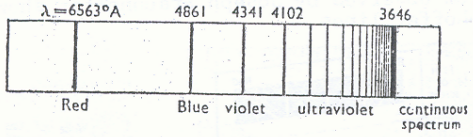
Hydrogen is the lightest of all elements. Its spectrum, owing to the simplicity of the atom with its single electron, exhibits regularities in the spectrum as compared to other elements. Being the simplest, it is expected that it will help in understanding the theory of the optical spectra.

In 1885 Balmer discovered a number of lines in the hydrogen spectrum could be grouped in a series of lines called Balmer series:

$$\frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{n^2} \right) \qquad \ldots\ldots (3.3)$$

If we put $n = 3,4,5, \ldots$, we can find the values of for successive members of the series of lines. The value of λ are quite separate and distinct for small values of n. About 40 lines of the Balmer series have been observed. For $n > 40$, the lines are so close that they cannot be seen clearly. As $n \to \infty$, the lines crowd together and approach a limit, called the series limit as shown in the fig.3-12.

Fig.3-12: Balmer series of hydrogen atom.



Later on other series of lines were discovered in the hydrogen spectrum by Lyman, Paschen, Brackett, and Pfund. These series can be expressed by relations similar to eq.(3.3) which expresses the Balmer series. In addition to the Balmer series, Table-1, shows other series found in the ultraviolet and infrared regions.

Fig.3-13: Lyman, Balmer and Paschen series of hydrogen spectrum.

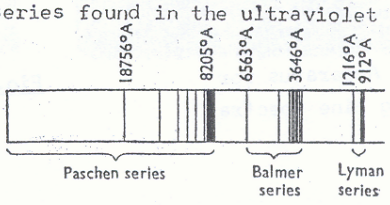


Table-1: Spectral series for hydrogen.

Series	Spectral region	Series Equation	Series limit ($n=\infty$)
Lyman	ultraviolet	$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$, $n = 2, 3, 4, \ldots$	911.27 Å
Balmer	visible	$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$, $n = 3, 4, 5, \ldots$	3645.1 Å
Paschen	infrared	$\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$, $n = 4, 5, 6, \ldots$	8201.4 Å
Brackett	infrared	$\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right)$, $n = 5, 6, 7, \ldots$	14,580 Å
Pfund	infrared	$\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right)$, $n = 6, 7, 8, \ldots$	22,782 Å

Example:

For Balmer series, the value of λ corresponding to the series limit is given by:

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

putting $n = \infty$, & $R = 1.097 \times 10^{-3}$ Å, we get

$$\frac{1}{\lambda} = 109677.581\left(\frac{1}{4} - 0\right) = \frac{1.09677.581 \times 10^{-3}}{4}$$

$$\text{or } \lambda = \frac{4}{1.09677.58 \times 10^{-3}} = 3646 \text{ Å}.$$

3.8 THE BOHR MODEL---POSTULATES

Following Rutherford's proposal that the mass and positive charge are concentrated in a very amall region at the centre of the atom, the Danish physicist Niels Bohr in 1913 suggested that the atom was in fact like a miniature planetary system, with the electrons circulating about the nucleus like planets circulating about the sun. The atom thus doesn't collapse under the influence of the electrostatic Coulomb attraction between nucleus and electrons for the same reason that the solar system doesn't collapse under the influence of the gravitational attraction between sun and planets. In both cases, the attractive force provides the centripetal acceleration necessary to maintain the orbital motion.

Fig.3-14: Niels Henrik David Bohr (1885-1962)

Bohr applied the quantum theory of radiation as developed by Planck and Einstein to the Rutherford nuclear atom. His theory is based on the following postulates.

1. An electron in an atom revolves in a circular orbit around the nucleus under the Coulomb attractive force.
2. An electron moving in one of these allowed orbits does not radiate energy. These states are called stationary states. (See Fig.3-16)

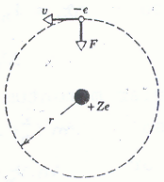


Fig.3-15: The Bohr model of the atom.

3. The only orbits that are allowed are those in which the angular momentum of the revolving electron is an integral multiple of $\frac{h}{2\pi} = \hbar$. The angular momenta of the only permissible orbits are given by

$$\vec{L} = mvr = n\frac{h}{2\pi} = n\hbar, \quad n = 1,2,3, \ldots \qquad (3.4)$$

where h is Planck's constant and

$$\hbar = 1.05 \times 10^{-34} \text{ J.sec.}$$

4. Emission or absorption of radiation occurs only when an electron makes a transition from one orbit to another. If an electron jumps from an orbit of higher energy E_i, to an orbit of lower energy E_f, a photon of frequency is given by:

$$\nu = \frac{E_i - E_f}{h} \qquad \ldots\ldots(3.5)$$

These postulates are a combination of some ideas taken over from classical physics together with others in direct contradiction to classical physics.

The second postulate rejects the claim that an accelerated charge must radiate in atomic systems, in spite of its validity in the ordinary world. In the third postulate the angular momentum is quantized in atomic systems. The fourth postulate provides the link with Planck's theory of radiation, since

$$E_i - E_f = h\nu \qquad \ldots\ldots (3.6)$$

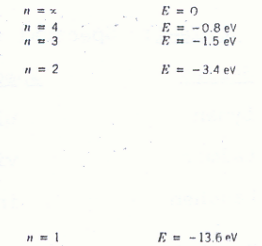


Fig.3-16: The energy levels of atomic hydrogen.

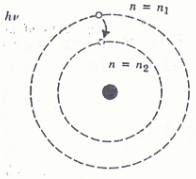


Fig.3-17: An electron jumps from the state n_1 to the state n_2 and emits a photon.

3.9 BOHR'S THEORY OF HYDROGEN ATOM

Bohr's postulates can be justified only if the experimental results agree with those derived from the postulates.

Suppose the electron is revolving about the nucleus with speed v_n in the nth orbit of radius r_n.

The moment of inertia = $I = mr_n^2$ ....(3.7)

also $v_n = r_n\omega_n \quad [v = r\omega]$

$$\omega_n = \frac{v_n}{r_n} \qquad \ldots.(3.8)$$

The angular momentum = $I\omega_n$

so $m r_n^2 \times \frac{v_n}{r_n} = n\frac{h}{2\pi}$

or $m v_n = \frac{nh}{2\pi r_n} \qquad \ldots.(3.9)$

We have

$$F_{\text{electrostatic}} = \frac{ke^2}{r_n^2}, \quad k = \frac{1}{4\pi\epsilon_0} = 9\times10^9 \text{ N-m}^2/\text{C}^2 \qquad \ldots\ldots (3.10)$$

$$F_{\text{centripetal}} = \frac{mv_n^2}{r_n} \qquad \ldots\ldots (3.11)$$

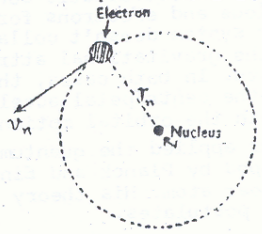


Fig.3-18: The electron is revolving about the nucleus.

From equations (3.10) and (3.11), we get

$$\frac{mv_n^2}{r_n} = \frac{ke^2}{r_n^2}$$

or $$mv_n^2 = \frac{ke^2}{r_n} \qquad \ldots\ldots (3.12)$$

from eq. (3.9), we have

$$m^2v_n^2 = \frac{n^2h^2}{4\pi^2r_n^2}$$

or $$mv_n^2 = \frac{n^2h^2}{4\pi^2r_n^2m} \qquad \ldots\ldots (3.13)$$

from equations (3.12) and (3.13), we get

$$\frac{ke^2}{r_n} = \frac{n^2h^2}{4\pi^2r_n^2m}$$

or $$r_n = \frac{n^2h^2}{4\pi^2mke^2} \qquad \ldots\ldots (3.14)$$

which gives the radius of the nth "non radiating" orbit. The electron remains normally in the first orbit for which $n = 1$.

So for first orbit, r_1,

$$n = 1, \quad h = 6.63 \times 10^{-34}\ \text{J.s}, \quad m = 9.1 \times 10^{-31}\ \text{kg},$$

$$k = 9 \times 10^{9}\ \text{N.m/c}^2, \quad e = 1.6 \times 10^{-19}\ \text{C}$$

so $$r_1 = .53 \times 10^{-10}\ \text{m} = 0.53\ \text{Å} \qquad \ldots\ldots (3.15)$$

which is called the Bohr radius. This result agrees well with the radius of the atom.

From eq. (3.15), we have

$$r_n = n^2 r_1 \qquad \ldots\ldots (3.16)$$

which shows that the radii of the orbits for stationary states are also quantized and are given by r_1, $4r_1$, $9r_1$, and so on. These radii are proportional to the square of the integer number n, called the principal quantum number.

To calculate E_n, energy of the electron in the nth orbit.

We have,

Absolute potential $= V = \frac{kq}{r}$ .....(3.17)

Potential energy $= \frac{kq}{r} \times q'$ .....(3.18)

$$\left[\begin{array}{l} V = \frac{1}{4\pi\epsilon_0}\frac{q}{r_1} \\ \Delta U = q\Delta V \end{array}\right. \quad \left\{\begin{array}{l} U = W_{ork} = Fxd \\ = \frac{1}{4\pi\epsilon_0}\frac{qq'}{r^2} \times r \\ \Delta V = \frac{\Delta U}{q'} = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \\ V = \frac{1}{4\pi\epsilon_0}\frac{q}{r_1} \end{array}\right.$$

Potential energy in the nth orbit is,

$$\text{P.E.} = \frac{k(+e)}{r_n} \times (-e) = -\frac{ke^2}{r_n} \qquad \ldots\ldots (3.19)$$

The -ve sign shows that this amount of energy must be given to the electron in order to take it to infinity where P.E. is taken as zero.

We have, kinetic energy of electron in the nth orbit,

$$\text{K.E.} = \tfrac{1}{2}mv_n^2 \qquad \ldots\ldots (3.20)$$

from equations (3.12) and (3.20), we get

$$\text{K.E.} = \tfrac{1}{2}\left(\frac{ke^2}{r_n}\right) = \frac{ke^2}{2r_n} \qquad \ldots\ldots (3.21)$$

From equations (3.19) and (3.21), we have

$$\text{Total energy} = E_n = \text{P.E.} + \text{K.E.} = \frac{-ke^2}{r_n} + \frac{ke^2}{2r_n}$$

$$\text{or} \quad E_n = -\frac{ke^2}{2r_n} \qquad \ldots\ldots (3.22)$$

from equations (3.14) and (3.22), we get

$$E_n = -\frac{ke^2}{2} \times \frac{4\pi^2 m k e^2}{n^2 h^2} = -\frac{2\pi^2 m k^2 e^4}{n^2 h^2}, \quad n = 1,2,3,\ldots \qquad \ldots\ldots (3.23)$$

$$\text{or} \quad E = E_n = -\frac{2\pi^2 m k^2 e^4}{h^2}\left(\frac{1}{n^2}\right) \qquad \ldots\ldots (3.24)$$

where the negative sign indicates a bound system.

Putting the values in eq. (3.24), as

$$m = 9.11 \times 10^{-31} \text{kg}$$

$$e = 1.60 \times 10^{-19} \text{ Coulomb}$$

$$h = 6.626 \times 10^{-34} \text{ J.s.}$$

$$\pi = 3.14$$

$$k = 9 \times 10^{9} \text{N.m/c}^2$$

We get

$$E_n = -\frac{13.6}{n^2} \text{ eV}, \text{ for } n = 1,2,3, \ldots \qquad \ldots\ldots (3.25)$$

The state of lowest energy or ground state corresponds to n = 1, and its energy is

$$E_1 = -13.6 \text{ eV}$$

From eq. (3.24), it can easily be seen that if the atom is in its ground state, 13.6 eV will be necessary to liberate the electron from the atom. Therefore, the binding energy (BE) or ionization energy for the hydrogen atom in its ground state is

$$BE = E_i = 13.6 \text{ eV}$$

3.10 THE ORIGIN OF ATOMIC SPECTRUM OF HYDROGEN

Considering eq. (3.24), we started the Bohr's postulate of quantization of angular momentum and obtained a result which states that the energy of electron in an atom is quantized, i.e. quantization of angular momentum leads to the quantization of energy. It is evident from eq. (3.24) that lowest allowed value of total energy occurs for the smallest quantum number n = 1. With the increase in 'n' the total energy of the quantum state becomes less negative, approaches to zero as 'n' approaches infinity. Above the line given by $n = \infty$, the energy states have positive energy, $E > 0$, and the spectrum of the states is continuous. The system is then 'unbound', meaning that the electron is free. (see fig. 3-21)

From Bohr's frequency condition, we have

$$\nu = \frac{E_i - E_f}{h} \qquad \ldots\ldots (3.26)$$

from equations (3.23) and (3.26), we get

$$\nu = \frac{2\pi^2 m k^2 e^4}{h^2 \times h}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \qquad \ldots\ldots (3.27)$$

Or from eq.(3.27), $\frac{1}{\lambda} = \frac{\nu}{c} = \frac{2\pi^2 m k^2 e^4}{h^3 c}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

put $R_H = \frac{2\pi^2 m k^2 e^4}{h^3 c}$ .....(3.28)

after substitution of the values of m, k, e, h, & c, this constant is found to be equal to R.

So

$$\frac{1}{\lambda} = R\left(\frac{1}{p^2} - \frac{1}{n^2}\right); \quad n_i \equiv n^{th}\ orbit,\ n_f \equiv p^{th}\ orbit \quad ..... (3.29)$$

which gives all the observed series of lines in the spectrum of hydrogen.

Thus we can have

$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$; p = 1; n = 2,3,4,... Lyman series

$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$; p = 2; n = 3,4,5,... Balmer series

$\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$; p = 3; n = 4,5,6,... Paschen series

$\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right)$; p = 4; n = 5,6,7,... Brackett series

$\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right)$; p = 5; n = 6,7,8,... Pfund series

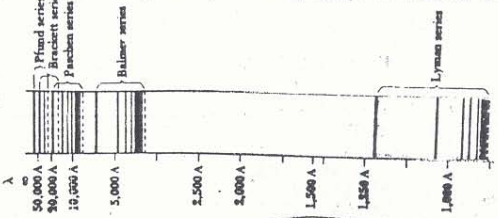


Fig.3-19: Atomic spectrum of hydrogen

3.11 ATOMIC ENERGY STATES

In the light of Bohr's theory and equations (3.14) and (3.23), the spectral lines of the different series may be represented diagrammatically in the following two ways:

1. Orbit diagram:

Fig. 3-20 gives a diagrammatic representation of Bohr's orbits and the transition of the electron from outer orbits to inner ones. The orbits corresponding to n = 1,2,3,... are K,L,M, ...shells.

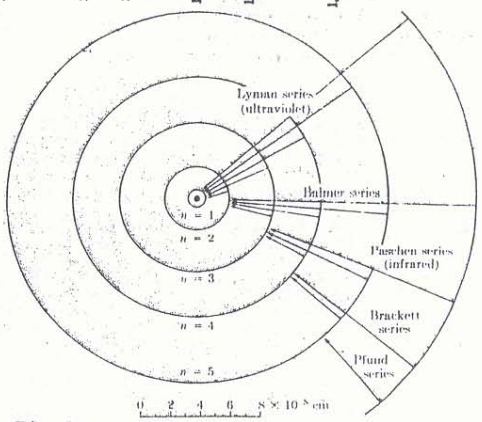


Fig.3-20: Scheme of transitions between stable Bohr orbits in the hydrogen atom.

2. Energy level diagram:

Here different discrete energy states of the atom are represented by horizontal lines and the transition of the atom from one to the other of those states, giving rise to a spectral line, by a vertical line connecting the two states involved. The energy levels crowd together when the value of n becomes great.

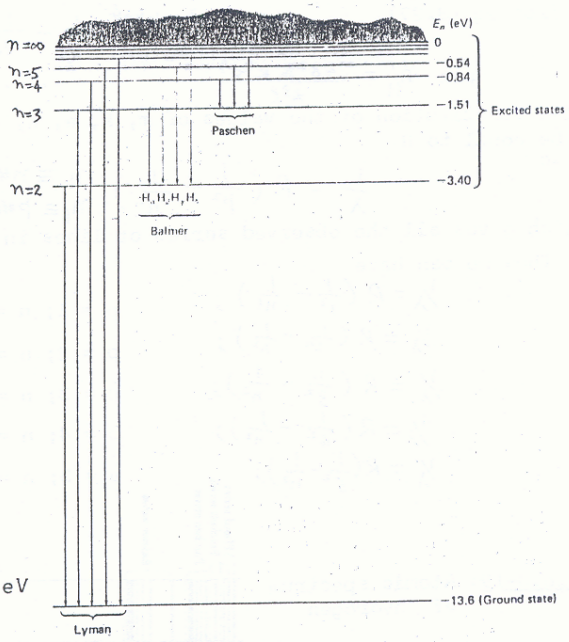


Fig.3-21: Energy level diagram for the hydrogen atom.

3.12 FEW DEFINITIONS

From the fig. 3-21, we can see,

The excitation energy, E_e is the energy that must be supplied to the atom to raise the electron from the ground state to an excited state. For example,

$E_e = -3.40 - (-13.6) = 10.2$ eV

is the excitation energy for the state n = 2 (first excited state). Corresponding to Ve excitation energy the potential V is called excitation potential or resonance potential.

The ionization energy E_i is the energy that we must supply to liberate the electron from the atom when the electron is in the ground state. In fig. 3-21, E_i = 13.6 eV. Corresponding to Ve ionization energy, the potential V is called ionization potential.

The binding energy BE for a given state is the energy that must be supplied to the atom to remove an electron when the electron is in a given excited state. For example, the BE for the state n = 2 is 3.40 eV. If the atom is in the ground state, the BE for that state is equal to the ionization energy (13.6 eV). When we speak about BE without mentioning the state, it is understood that the BE and the ionization energy have the same numerical value. It is in this respect that we have said the BE of the hydrogen atom is 13.6 eV.

Elastic collision: The collision in which a moving electron collides with an atom and the electron does not lose any energy during the collision.

Inelastic collision: The collision in which a moving electron collides with an atom and the electron loses some energy during the collision.

3.13 FRANCK-HERTZ EXPERIMENT

The experiment of Franck and Hertz in 1914 provided additional confirmation of the discrete energy states predicted by the Bohr theory.

The experimental arrangement is shown in the figure. Electrons emitted from the cathode C are accelerated to the anode A by a potential V. The electrons which are able to overcome the small retarding potential between the plate P and the anode A reach P. So the current at P is noted by ammeter A.

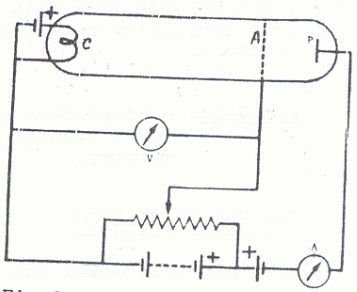


Fig.3-22: Experimental arrangement of Franck-Hertz experiment.

Franck and Hertz used a simple apparatus consisting of a mercury tube containing a cathode, a plate, and an accelerating grid which was located physically near the plate.

In this apparatus mercury vapour was bombarded with electrons of known kinetic energy. When this kinetic energy is less than the energy of the first excited state of atomic mercury, the only energy that the colliding electron loses is the small amount of kinetic energy that it can transfer to the massive mercury atom by an elastic collision. However, when the kinetic energy of the incident electron just exceeds the energy of the first excited state of mercury, than the ~~the~~ electron gives up all of its kinetic energy to the mercury atom in an inelastic collision. So by comparing the kinetic energy of the electrons before and after the collision one can determine how much energy is transferred to the target atoms.

In fig.3-22, both the plate and the grid is maintained at positive potentials with respect to the cathode, but the plate potential is slightly lower than the grid potential. This small retarding potential on the plate have the effect of preventing contributions to the plate current from electrons having negligible kinetic energy. So, if electrons are to lose most of their kinetic energy in inelastic collisions with mercury atoms, the retarding potential will prevent them from reaching the plate and a drop in plate current will result. Such a drop in plate current is found to occur at 4.9 volts as shown in fig. 3-23. At higher voltages a large drop in plate current occurs and new lines appear in the emission spectrum of mercury vapour. This behaviour is repeated at multiples of 4.9 volts.

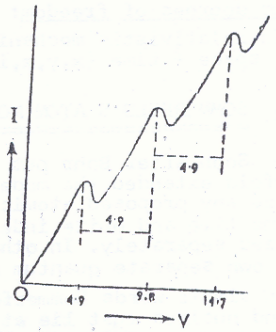


Fig.3.23: Voltage vs. Current curve for mercury vapours in the Franck Hertz experiment.

A simplified level diagram for mercury is given in fig.3-24. Lets see the critical potential as being due to discrete energy levels. If the separation between the ground state and the first excited state is actually 4.9 eV = 7.85×10^{-12} ergs, there should be a line in the mercury spectrum of wavelength λ :

$$E_2 - E_1 = h\nu = \frac{hc}{\lambda} = Ve$$

$$\text{or } \lambda = \frac{hc}{Ve} = \frac{6.625\times10^{-34}\times3\times10^{8}}{4.9\times1.6\times10^{-19}}$$

$$\text{or } \lambda = 2.531\times10^{-7}\,m = 2531\times10^{-10}\,m$$

$$\text{or } \lambda = 2531\,\text{Å}$$

This value agrees with the line 2536 Å in the mercury spectrum which is a transition from ground state to the first excited state. We conclude from this experiment that the Bohr concept of discrete energy states is correct.

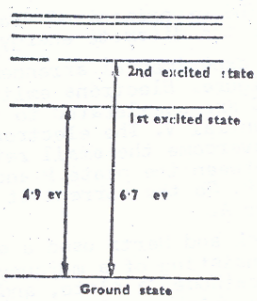


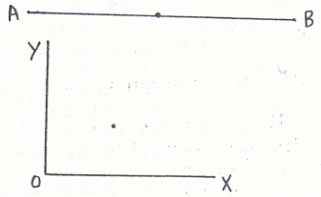
Fig.3-24: A simplified energy level diagram for mercury.

3.14 DEGREES OF FREEDOM

If a particle moves there are three kinds of degrees of freedom.

1. One degree of freedom:

   For example, when a particle moves in a straight line. It has only one axis in which it can move.



2. Two degrees of freedom:

   If a particle moves in a plane, then it has two degrees of freedom. Here there are two axis.

3. Three degrees of freedom:

   If it moves in space, then it has three degrees of freedom. As it has three axis on which it depends.

4. Four degrees of freedom:

   In relativistic mechanics, we consider time as the fourth coordinate. So space + time (x,y,z,it) is the example of four degrees of freedom.

3.15 SOMMERFELD'S ATOM MODEL (ELLIPTIC ORBITS)

Soon after Bohr put forward his famous theory of the hydrogen atom, Sommerfeld extended the model so as to include elliptic orbits. In order to quantize any proposed atomic system Wilson and Sommerfeld postulated, about the same time and quite independently, that each degree of freedom must be quantized separately. In other words each degree of freedom should be fixed by its own separate quantum number.

In simple words Sommerfeld gave the idea that electrons move in elliptical path and nucleus must lie at one of the focus. And second thing is that there are two degrees of freedom.

In case of circular orbits there is only one coordinate which varies periodically, namely the angle ϕ which the radius vector makes with the X-axis. In the case of elliptic motion, not only does the angle ϕ vary, but the lengt

of the radius vector r also varies periodically as shown in the fig.3-25. According to Wilson-Sommerfeld, the elliptic orbits are determined by the two quantum conditions:

$$\oint p_\phi \, d\phi = n_\phi h \qquad \text{.....(3.30)}$$

$$\oint p_r \, dr = n_r h \qquad \text{.....(3.31)}$$

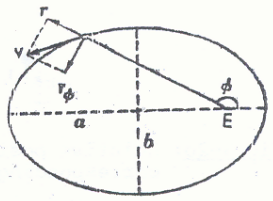


Fig.3-25: The electron moves in an elliptic orbit with the nucleus at one of the foci of the ellipse.

where $n_\phi (=k)$ is angular (or azimuthal) quantum number

$n_r (=\ell)$ is radial quantum number

r is radius

ϕ is azimuthal angle.

$p_r (= mv = m\frac{dr}{dt})$ is radial momentum corresponding to radial component of the tangential velocity

$p_\phi (= I\omega = mr^2\frac{d\phi}{dt})$ is angular momentum corresponding to transverse component.

We define

$$n = k + n_r \qquad \text{..... (3.32)}$$

as the <u>total quantum number</u>, n having the values 1,2,3, ...

According to Kepler's second law of planetary motion the angular momentum p_ϕ remains constant throughout the motion. So in eq(3.30), taking integration over a complete cycle, we have

$$\int_0^{2\pi} p_\phi \, d\phi = 2\pi p_\phi = 2\pi m r^2 \frac{d\phi}{dt} = n_\phi h \qquad \text{..... (3.33)}$$

$$\text{or} \quad p_\phi = n_\phi \frac{h}{2\pi} = n_\phi \hbar \qquad \text{..... (3.34)}$$

The quantum number n_ϕ has a simple physical significance as a measure of angular momentum in units of $h/2\pi$.

For an ellipse, the possible values of the major axis a and minor axis b as obtained from equations (3.30) and (3.31) are (The analysis is more complicated than in circular case, so details are omitted).

$$a = \frac{n^2 h^2}{4\pi^2 m e^2 Z} \qquad \text{..... (3.35)}$$

$$b = a\frac{n_\phi}{n} \qquad \text{..... (3.36)}$$

putting $n_\phi = k$, we have

$$b/a = k/n \qquad \text{..... (3.37)}$$

Also

$$\text{total energy} = W = -\frac{2\pi^2 m e^4 Z^2}{n^2 h^2} \qquad \text{..... (3.38)}$$

This value of energy is same as that for circular orbits.

We get some results from equations (3.32) to (3.38), which are the general characteristics of Sommerfeld's Elliptic orbits.

1. k cannot be zero.
2. k cannot be greater than n, since b is always less than a.

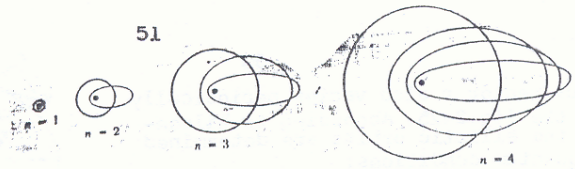


Fig.3-26: Relative positions and dimensions of the elliptical orbits corresponding to the first four values of the quantum number n.

3. for k = a, b = a, the path becomes circular.
4. corresponding to n, there are n different orbits.
5. for every path followed by the electron, the total energy is same.

3.16 RELATIVISTIC CORRECTION TO SOMMERFELD'S ATOM MODEL

(Variation of electronic mass)

The velocity of electron as it moves in an elliptic path around the nucleus varies. The velocity is maximum when the electron is nearest to the nucleus. This variation of velocity, according to special theory of relativity, results in variation of electronic mass according to the equation

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}} \qquad \text{.....(3.39)}$$

Sommerfeld admitted relativistic correction to his atom model. As a result the path of the electron is not the simple ellipse, actually the semi-major axis of the ellipse precesses around an axis passing through the focus in a plane. The path is shown in the fig.3-27, and is called rosette.

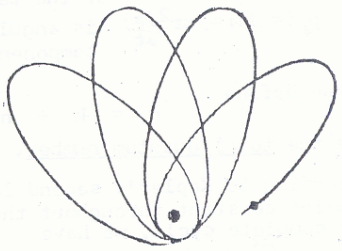


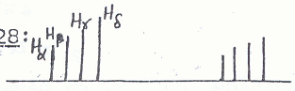
Fig.3-27: Relativistic mass variation causes precession.

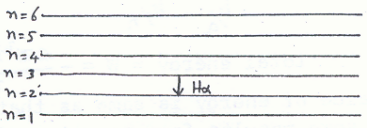
3.17 EXPLANATION (OF FINE STRUCTURE)

Consider for example, Balmer series in the spectrum of hydrogen. There are four lines in this series, which is called H_α, H_β, H_γ, & H_δ . Experimentally it has been observed that there are three lines in place of H_α line. Let us see how relativistic correction may be used to predict this experimental result. The H_α results from transition between the states n = 3 & n = 2.

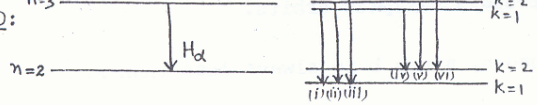
For n = 3, the various values of k is given by k = 1,2,3.

and for n = 2, k = 1,2.

Fig.3-28: 



In fig.3-29, there are six transitions which can take place.

Fig.3-30: 

3.18 SELECTION RULE

In order to decide which transitions are allowed, we apply certain rules. These rules are called selection rules. The selection rule which is applied in this case is

$$\Delta k = \pm 1$$

i.e. k, the azimuthal quantum number can change by +1 or -1 during various transitions.

Let us consider all the transitions in the fig.3-30, one by one.

(i) This transition results from the states $k = 1$ & $k = 1$

In this case $\Delta k = 0$

but selection rule is $\Delta k = \pm 1$

So this transition is not allowed.

(ii) This line results from states $k = 2$ & $k = 1$,

so $\Delta k = +1$, and selection rule is $\Delta k = \pm 1$,

this transition is allowed.

(iii) This transition results from the states $n = 3$, $k = 3$ & $n = 2$, $k = 1$

so $\Delta k = +2$, but the selection rule is $\Delta k = \pm 1$

i.e. this transition is not allowed.

(iv) This results from the states $n = 3$, $k = 1$, & $n = 2$, $k = 2$;

so $\Delta k = -1$, and selection rule is $\Delta k = \pm 1$,

i.e. this transition is allowed.

(v) This results from states $n = 3$, $k = 2$, & $n = 2$, $k = 2$

so $\Delta k = 0$,

therefore this transition is not allowed.

(vi) This line results from states $n = 3$, $k = 3$, & $n = 2$, $k = 2$;

so $\Delta k = +1$

Hence this transition is also allowed.

Application of this rule shows that each line in Balmer series consists of three component lines. Each line of Paschen series consists of five component lines.

The fine structure of Balmer lines and some of the lines of singly ionized helium have been carefully studied but the agreement with theoretical predictions is still not good. For this the spin of electron is considered, which will be discussed in later Section.

3.19 VECTOR ATOM MODEL

This model is devised to remove the limitations of previous Rutherford-Bohr-Sommerfeld atom model. As these models does not give full description of fine structure and the distribution and arrangement of electrons in atoms. The previous model help us to understand the properties of atoms. We have seen that how the three quantum numbers (n, k, n_r) tell us about the "shape" of the electron's orbit. However, there are other properties of atoms, e.g. their behaviour in magnetic fields, which are easier to understand if we

use a model in which the angular momentum is considered to behave much like an ordinary vector. The vector representation of the orbital model is called vector model.

A comet speeds up as it passes near the sun, so that the decrease in its distance r from the sun is balanced by an increase in its linear momentum p, and the product $\vec{r}\times\vec{p}$ remains constant. This is the example of gravitational orbits. In the electronic orbit, for each of the electron's possible orbits the angular momentum l remains constant.

In Bohr's theory the angular momentum was given by eq. (3.4)

$$\vec{L} = n\hbar \qquad \ldots\ldots (3.40)$$

And the orbital angular momentum was (eq.3.34)

$$p_\phi = n_\phi \hbar \qquad \ldots\ldots (3.41)$$

where n_ϕ is the angular (azimuthal) quantum number.

In the Schroedinger theory the orbital angular momentum is given by

$$|\vec{l}| = \sqrt{l(l+1)}\,\hbar \qquad \ldots\ldots (3.42)$$

where l is the orbital angular momentum quantum number, and has values

$$l = 0, 1, 2, \ldots\ldots, n-1.$$

where n is principal quantum number.

Here two points to be noted. First, the length of the vector $|\vec{l}|$ is always greater than $1\hbar$, (because $\sqrt{l(l+1)} > 1$). Secondly, the values of $|\vec{l}|$ which we say as the magnitude of the electron's angular momentum are different from Bohr's model.

For example, in Bohr's model:

for $n = 3, |\vec{l}| = 3\hbar$

In quantum-mechanical vector model

for n = 3,

have $l = 2$, with $|\vec{l}| = \sqrt{2(2+1)}\,\hbar = \sqrt{6}\,\hbar$

or $l = 1$ with $|\vec{l}| = \sqrt{1(1+1)}\,\hbar = \sqrt{2}\,\hbar$

& $l = 0$, with $|\vec{l}| = 0$

The vector $\vec{l}$ can have components along any axis in space. Selecting the z-axis, the z component of $\vec{l}$ is :

$$l_z = m_l \hbar \qquad \ldots\ldots (3.43)$$

where m_l is the magnetic quantum number, and can have the values

$$m_l = 0, \pm 1, \pm 2, \ldots\ldots \pm l.$$

The two essential features that characterise the vector atom model are the spatial quantization and spinning electron.

Example-1:

Compute the length of the angular momentum vectors that represent the motion of an electron in a state with $l = 1$ and $l = 2$, and $l = 3$.

Solution:

We have $|\vec{l}| = \sqrt{l(l+1)}\,\hbar$

putting the values for $l = 1$ & $l = 2$, & $l = 3$, we get

$$|\vec{l}| = \sqrt{1(1+1)}\,\hbar = \sqrt{2}\,\hbar$$

$$\& \quad |\vec{l}| = \sqrt{2(2+1)}\,\hbar = \sqrt{6}\,\hbar$$

$$\text{also} \quad |\vec{l}| = \sqrt{3(3+1)}\,\hbar = \sqrt{12}\,\hbar = 2\sqrt{3}\,\hbar$$

Example-2:

What are the possible z components of the vector $\vec{l}$, for $l = 2$.

Solution:

We have

for $l = 2$,

$$m_l = 0, \pm 1, \pm 2$$

so $\quad l_z = 0, \pm\hbar, \pm 2\hbar.$

3.20 SPATIAL QUANTIZATION

We have seen that in the Bohr-Wilson-Sommerfeld atom model the orbits are quantized as regards their size and form. But according to quantum theory the direction or orientation of the orbits in space must also be quantized. The introduction of such a spatial quantization makes the orbits vector quantities.

The orientation of the orbit to certain fixed direction is called spatial quantization. For angular momentum, spatial quantization means that the projection of an angular momentum vector along any single axis in space must be one of a discrete set of allowed values. At any given time there can be only one axis of quantization; if it is altered, by changing the direction of an applied magnetic field, quantized states exist only along the new field direction.

The spatial quantization condition demands that the allowed orientations of the angular momentum should be such that their projections upon the axis of measurement differ successively by one unit of $\hbar$. This behaviour represents a curious aspect of quantum physics, in which only certain orientations of angular momentum vectors are allowed. The number of these orientations is equal to $(2l + 1)$, the number of different m_l values, and the magnitudes of their successive z components always differ by $\hbar$. As an example, suppose we could prepare a collection of hydrogen atoms in a state with $l = 1$. Choosing arbitrarily a z-axis, we measure the z component of $\vec{l}$. From such measurement we expect to find $l_z = \hbar$, 0, or $-\hbar$. Choosing a completely different z-axis, we repeat the measurement and once again we find $l_z = \hbar$, 0, or $-\hbar$. This behaviour is completely different from that of classical vectors. A classical

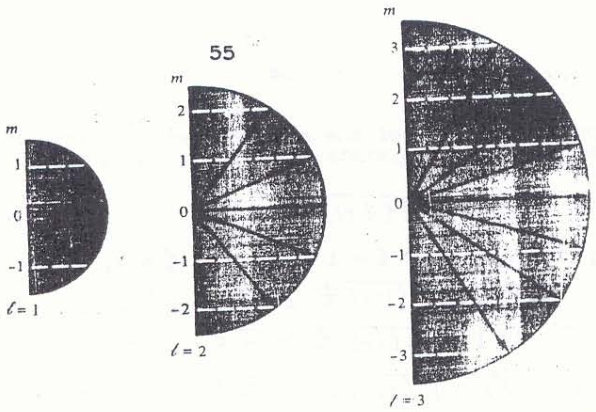


Fig.3-31: The allowed projections of the angular momentum for the cases of l = 1, 2, and 3.

vector of length l would have a z component of l if we choose our z axis along the vector, or -l if we choose the opposite direction, or 0.5 if we choose z axis at an angle of 60° to the vector, or 0.7 if we choose the z axis at 45° to the vector. A quantum mechanical vector representing l = 1 has allowed z components are +1, 0, or -1. The curious fact is that we observe this result.

3.21 MAGNETIC MOMENT AND BOHR'S MAGNETON

Previously we have studied about angular velocity $\vec{\omega}$ as

$$\vec{\omega} = \vec{r} \times \vec{v} \quad .....(3.44)$$

Also

Torque or moment of force:

$$\vec{\tau} = \vec{r} \times \vec{F} \quad .....(3.45)$$

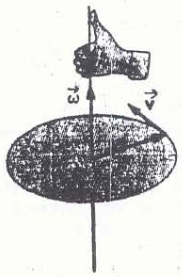


Fig.3-32: Representation of angular velocity vector $\vec{\omega}$.

Now in the study of electromagnetism we learn that a current loop has associated with it a magnetic moment, whose magnitude is equal to the product of the current and the area enclosed by the loop, and whose direction is perpendicular to the plane of the loop (fig.3-33). Then for a circular Bohr orbit of radius r, we have

$$\vec{\mu} = \hat{r} \times \vec{i}\pi r^2 \quad .....(3.46)$$

where $\vec{i}$ is in amperes and the unit vector $\hat{r}$ aids in determining the correct direction for the magnetic moment. Eq. (3.46) can be written as

$$\mu = i\pi r^2 = i A \quad .....(3.47)$$

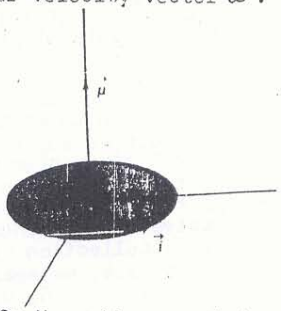


Fig.3-33: Magnetic moment due to a circular current.

For a circular orbit , we have

$$\vec{i} = \frac{-e\,\vec{v}}{2\pi r} \qquad \text{.....(3.48)} \qquad \left[\because i = \frac{q}{t},\ s = vt,\ t = \frac{s}{v} = \frac{2\pi r}{v}\right.$$

where -e is the charge of the electron.

From equations (3.46) and (3.48), we get

$$\vec{\mu} = -\frac{e}{2}\vec{r}\times\vec{v} = -\frac{e}{2m}\vec{r}\times\vec{p} \qquad \text{.....(3.49)} \quad [p = m\vec{v}$$

or $$\vec{\mu} = -\left(\frac{e}{2m}\right)\vec{L} \qquad \text{.....(3.50)} \quad [\vec{r}\times\vec{p} = \vec{L} = \text{orbital angular mom.}$$

or $$\mu = -\frac{e}{2m}L$$

According to Bohr's postulate,

In Vector Atom model $L = n\frac{h}{2\pi}$

so $L = l\,h/2\pi$

$\mu = -\frac{e}{2m}l\,h/2\pi$

or $$\mu = -\frac{eh}{2m}l = -\mu_B l \qquad \text{..... (3.51)}$$

The minus sign tells us that the magnetic moment is directed anti-parallel to the orbital angular momentum because of the negative charge of the electron. The quantity μ_B is called the Bohr magneton, and its value in mks units is approximately

$$\mu_B = \frac{eh}{2m} = 9.27 \times 10^{-24} \text{ amp-m}^2 \text{ or joule/tesla}$$

From eq. (3.51), the orbital magnetic moment is directly proportional to l.

If

l = 0, $\mu = 0$

l = 1, $\mu = \frac{eh}{2m}$

l = 2, $\mu = 2\,eh/2m$

It appears, therefore, that the orbital magnetic moment is an integral multiple of $\frac{eh}{2m}$. This quantity is the smallest constant unit in terms of which the magnitude of atomic and sub-atomic moments are measured and is known as Bohr Magneton.

3.22 EFFECT OF EXTERNAL MAGNETIC FIELD UPON MAGNETIC MOMENT

Previously, we have studied

$$\text{P.E.}_{\text{gravitational}} = \vec{w}.\vec{h} = mgh \qquad \text{....(3.52)}$$

and $$\text{P.E.}_{\text{electrical}} = \Delta U = \vec{E}.\Delta\vec{r} = -E\Delta r \qquad \text{...(3.53)}$$

Now the dipolar energy ΔE, or magnetic potential energy V_m, in a magnetic field $\vec{B}$ is defined as:

$$\Delta E = V_m = -\vec{\mu}.\vec{B} \qquad \text{..... (3.54)}$$

or $$V_m = -\mu B\cos\theta \qquad \text{..... (3.55)}$$

or $$V_m = \frac{eB}{2m} L\cos\theta \qquad \ldots\ldots (3.56)$$ $\left[\because \mu = -\frac{e}{2m}L\right.$

or $$V_m = m_l \frac{l\hbar}{2m} B \qquad \ldots\ldots (3.57)$$ $\left[\begin{matrix} L = l\hbar \\ L\cos\theta = m_l\hbar \end{matrix}\right.$

which becomes $+\mu B$ for antiparallel alignment and $-\mu B$ for parallel alignment.

So the energy of an atomic electron depends not only on 'n' and 'l' but also on m_l in the presence of magnetic field and it changes with the change in the field intensity.

Let us apply this to what we already know about the energy states of an electron in an atom. If an electron having a magnetic moment μ is in an energy state E_o in the absence of a magnetic field, then it can take on one of the additional energies $E_o \pm \mu B$ in the presence of a magnetic field B. The existence of the triplet of energy levels $\{E_o, E_o \pm \mu B\}$ for this special case has been known since 1897, and the phenomenon is referred to as the normal Zeeman effect.

So different energy states are:

$$\begin{aligned} E_1 &= E_o - \mu B \\ E_2 &= E_o \\ E_3 &= E_o + \mu B \end{aligned} \qquad \ldots\ldots (3.58)$$

Also three frequencies are:

$$\begin{aligned} \nu_1 &= \nu_o - \frac{e\hbar}{4\pi m}\frac{B}{\hbar} = \nu_o - \frac{eB}{4\pi m} \\ \nu_2 &= \nu_o \\ \nu_3 &= \nu_o + \frac{e\hbar B}{4\pi m\hbar} = \nu_o + \frac{eB}{4\pi m} \end{aligned} \qquad \ldots\ldots (3.59) \quad \left[\begin{matrix} E = h\nu \\ \nu = \frac{E}{h} \end{matrix}\right.$$

3.23 ELECTRON SPIN

The Bohr-Sommerfeld model could not explain the fine structure of H_α line in the spectrum of simplest hydrogen atom i.e. first line in the Lyman series. The strongest emission of Na^{23} is from a $3p \rightarrow 3s$ transition (Sodium D line). In a simple spectrum it appears as a single strong line. However, on close inspection it is found that it is not a simple line, but a doublet consisting of two wavelengths. This splitting of a single spectral line into two distinct but closely spaced lines is known as the fine structure of the atomic spectra. In Section 3.22 a kind of fine structure was observed when an atom was placed in an external magnetic field. This interaction of the electrons within the atom with the magnetic field is the Zeeman effect. The fine structure, like that in the sodium doublet differs from the Zeeman effect in that it does not require the presence of an external magnetic field, and the splitting between any two consecutive levels is defferent.

In 1925, Samuel Goudsmit and George Uhlenbeck, both graduate students at Leiden University, gave an hypothesis to explain the above spectral phenomenon. According to it: "Each electron performs a rotation about its own axis in addition to its motion about the nucleus.

When a body moves about an axis, it possesses angular momentum. The orbital motion of a charged body constitutes a current loop which behaves like a magnetic dipole giving rise to a magnetic moment pointing perpendicular to the plane of the loop. So the motion of a charged body about an axis produces angular momentum as well as magnetic moment. With the introduction of spinning electron two motions of the electron are considered---one is the orbital motion

and the second is due to spin. There are therefore, two angular momenta and two magnetic moments associated with the electron. The angular momentum of the electron about the nucleus can be compared with the rotation of the earth about the sun as it goes through the seasons of the year, while the spin angular momentum can be compared to the rotation of the earth on its axis as it produces day and night.

3.24 SPIN ANGULAR MOMENTUM

With the spinning electron model, Dirac relativistic quantum treatment introduced the spin angular momentum quantum number or simply spin quantum number s. The spin angular momentum is quantized according to

$$L_s = \sqrt{s(s+1)}\,\hbar \qquad \ldots\ldots (3.60)$$

in which the spin angular momentum quantum number s has only the single value $\frac{1}{2}$. The only possible value of L_s is

$$L_s = \sqrt{\tfrac{1}{2}(\tfrac{1}{2}+1)}\,\hbar = \sqrt{3}\,\hbar/2$$

The number s is called a quantum number, but it is not used along with other quantum numbers since it has only the value $\frac{1}{2}$ and does not really distinguish one state from another. The only value of the spin angular momentum can have is $\sqrt{3}\hbar/2$, and for this reason it as much a fundamental character of the electron as the mass.

If the direction along the magnetic field is chosen as z direction, then the component of L_s in the z-direction is quantized and is given by

$$L_{sz} = L_s \cos\theta = m_s \hbar \qquad \ldots.(3.61)$$

where $m_s = \pm\frac{1}{2}$ and is called magnetic spin quantum number. Usually $m_s = \frac{1}{2}$ is designated as spin-up ($\uparrow$) and $m_s = -\frac{1}{2}$ is spin-down($\downarrow$). The two possible values are shown in fig.3-35. This restriction is known as the space quantization of the spin angular momentum.

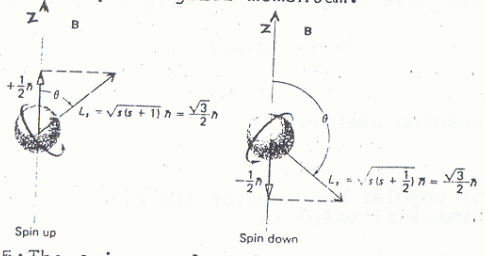


Fig.3-35: The spin angular momentum of the electron is space quantized.

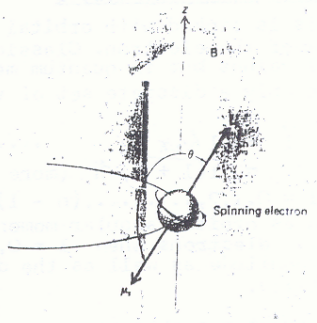


Fig.3-34: Magnetic dipole moment produced by spin of the electron.

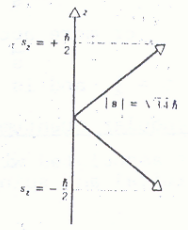


Fig.3-36: Spatial orientation of the spin angular momentum vector.

3.25 QUANTUM NUMBERS

To understand the description of the atom according to the quantum theory, it is necessary to be familiar with the idea of quantum numbers. A quantum number is a number that describes the allowable value of certain physical quantities. In vector atom model, to each of the component part is assigned a quantum number, the numerical value of which may be conveniently thought of as the length of the vector which represents it. For example, angular momentum is represented by a straight line whose direction is parallel to the axis of rotation and whose length is proportional to the magnitude of the momentum. Various quantum numbers associated with each of the electrons of a given atom are as follows:

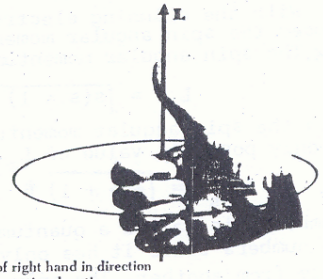


Fig.3-37: Thumb in the direction of angular-momentum vector.

1. Principal quantum number, n

This quantum number is identical to that used in Bohr-Sommerfeld model. It takes only integral values 1,2,3,....,n.

2. Orbital quantum number, l

This is linked with orbital angular momentum of the electron. Classically it can take all values but in quantum mechanics it can take only a discrete set of values given by

$$\vec{l} = l\,h/2\pi \qquad \ldots\ldots (3.62)$$
$$= \sqrt{l(l+1)}\,\hbar \quad \text{(more accurately)}$$

where $l = 0,1,2,\ldots\ldots\ldots,(n-1)$

known as orbital angular momentum quantum number. It is said to be an s,p,d,.... electrons when $l = 0,1,2,\ldots$ respectively.

The magnitude as well as the direction of the orbital angular momentum is quantized.

3. Spin quantum number, s

The spin momentum of an electron can have only discrete values,

$$\vec{s} = s\,h/2\pi \qquad \ldots\ldots (3.63)$$

or more accurately

$$\vec{s} = \sqrt{s(s+1)}\,\hbar$$

where $s = \frac{1}{2}$ and is known as spin quantum number.

4. Total angular quantum number, j

It is associated with the resultant angular momentum of the electron due to both orbital and spin motions. Its numerical value is

$$j = l + s$$

This value will be half-integer, as $s = \frac{1}{2}$. We usually write as

$$j = l \pm \tfrac{1}{2}$$

+ve for s is parallel to l and -ve for anti-parallel.

The magnitude (as well as direction) of j is quantized according to the equation

$$\vec{j} = j\, h/2\pi$$

or more accurately

$$j = \sqrt{j(j+1)}\,\hbar \qquad \ldots\ldots (3.64)$$

Three more quantum numbers are associated with the electron, when the atom is placed in a magnetic field and taking a preferential axis in space e.g. as z-axis.

According to classical theory an atom brought into a magnetic or electric field will carry out a precession such that the angular momentum $\vec{l}$ describes a cone with the field direction as axis and with a constant component $\vec{m}_l$ of the angular momentum as shown in fig.3-38.

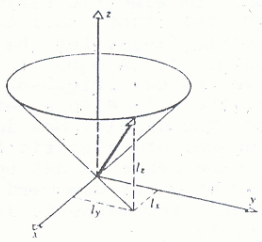


Fig.3-38: Precession of vector l with constant l_z .

5. Magnetic orbital quantum number, m_l

It is the numerical value of the projection of the orbital vector l on the field direction. m_l is an integer having values from +l to -l, i.e.

l, (l - 1), (l - 2),....1,0,-1,...-(l -1),-l.

From the arithmetical progression it is equal to (2l + 1).

According to spatial quantization, the projection ofl in the field direction must be quantized. So the angle θ can have only those permitted values for which m_l have discrete set of allowed values. Also shown in fig. 3-31.

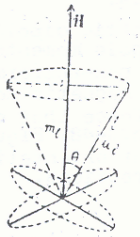


Fig.3-39: Quantization of the projection of l in the field direction.

6. Magnetic spin quantum number, m_s

It is the numerical value of the projection of the spin vector $\vec{s}$ on the field direction, m_s has (2s + 1) values from -s to +s excluding o, as s is equal to $\frac{1}{2}$. So that

$m_s = \pm\frac{1}{2}$ (only two values)

7. Magnetic total angular momentum quantum number, m_j

It is the numerical value of the projection of the total angular momentum vector $\vec{j}$ on the field direction. According to quantum theory the allowed orientations are (2j + 1) values from +j to -j excluding zero.

Now the resultant quantized vector, representing the atom as a whole can be obtained by combining individual electrons' vectors and applying the rules of spatial quantization. Small letters l,s,j are used for the state of electron and capital letters L,S,J, etc are used for the atom.

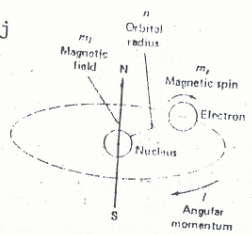


Fig.3-40: Quantum numbers describe the position and motion of an electron.

3.26 NON-UNIFORM MAGNETIC FIELD

Before discussing magnetic field, consider the behaviour of an electric dipole, which consists of two equal and opposite charges q separated by a distance r. The electric dipole moment (r x q) has direction from -ve to +ve charge. In fig.3-41, in a uniform electric field, the dipole experiences a torque that tends to rotate it in the direction of electric field. Let the field acting on positive charge is not equal to the field acting on negative charge. (fig.3-42). Now along with a net torque, there is also a net force that tends to displace the dipole. Now in fig.3-43, two dipoles with non-uniform electric field has shown. One dipole experiences a net upward force and the other experiences a net downward force. We can apply this result for magnetic dipole moment in a magnetic field.

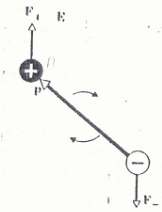


Fig.3-41: An electric dipole in a uniform electric field.

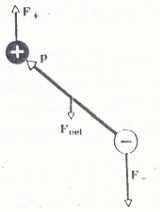


Fig.3-42: An electric dipole in a non-uniform electric field.

In the atom, magnetic moments are randomly oriented. In figures 3-44 and 3-45, there is a comparison of magnetic moments with classical magnetic moments.

A magnetic dipole moment behaves just like electric dipole moments shown in fig-ures 3-46 and 3-47. A non-uniform magnetic field not only rotates the magnetic moments, but also gives an unbalanced force that causes a displacement, as shown in fig.3-48. Figure 3-47 shows that the two different orientations give net forces in opposite directions.

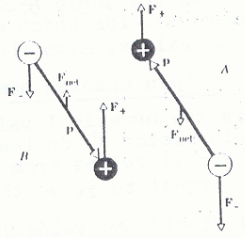


Fig.3-43: Two dipoles with oppositely directed momen-ts in a non-uniform field.

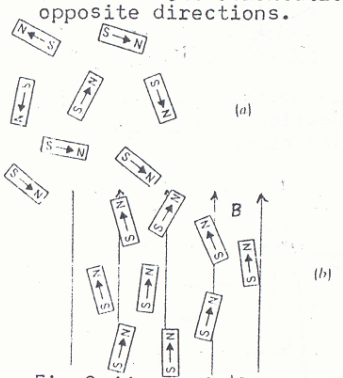


Fig.3-44: a) Collection of small magnets.b) Applied magnetic field aligned the magnetic moments.

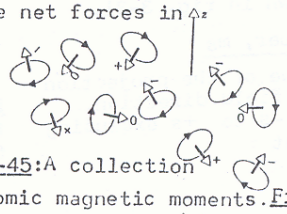


Fig.3-45:A collection of atomic magnetic moments.

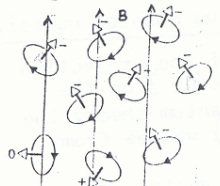


Fig.3-46: Applied field aligned the atomic magnetic moments.

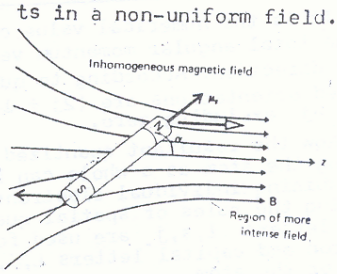


Fig.3-48: A net translational force experienced.

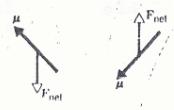


Fig.3-47: Two magnetic dipoles in nonuniform magnetic field.

3.27 THE STERN-GERLACH EXPERIMENT

In non-uniform magnetic field the behaviour of atoms were first studied by Otto Stern and Walter Gerlach in 1920. This experiment gave direct experimental verification of spatial quantization, spinning electron and quantized atomic magnetic moment.

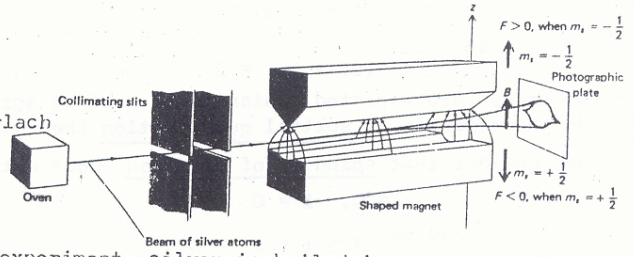


Fig.3-49: The Stern-Gerlach experiment.

In the Stern-Gerlach experiment, silver is boiled in an oven, and atoms of silver stream from an opening in the oven into an evacuated region where they travel in straight lines through slits S_1 and S_2 and this beam of atoms is allowed to pass between the poles of a magnet. The magnetic field is made as intense and non-homogeneous as possible by a suitable construction of the pole-pieces of a magnet, one piece being shaped as a knife-edge while the other had a flat face with a groove. The whole apparatus is evacuated to low pressure so that atoms move without any collision. Finally the beam fall onto a photographic plate. Classical theory predicts that a single pattern should

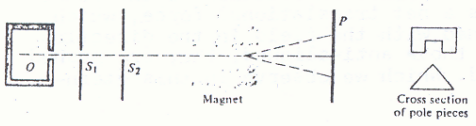


Fig.3-50: A plane arrangement of the apparatus.

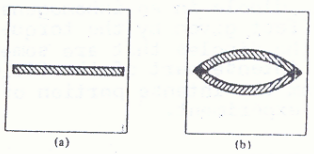


Fig.3-51: a) The pattern without magnetic field applied. b) Pattern with the magnetic field.

be produced on the photographic plate. All values from 0° to 180° for the angle θ in eq.3.61, being permissible.

$$L_{sz} = L_s \cos\theta = m_s\hbar \quad \ldots\ldots (3.61)$$

Instead, the stream of atoms splits into two distinct lines after passing through the inhomogeneous magnetic field.

A fairly sharp line was traced by the beam on the photographic plate in the absence of magnetic field. When the beam was passed through non-homogeneous magnetic field, a double trace was obtained. On entering the magnetic field, magnetic moments became oriented parallel and anti-parallel to the direction of the field as shown in figure 3-52.

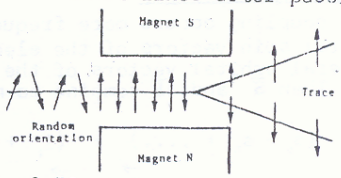


Fig.3-52: A double trace obtained.

As the shells of electrons about the nucleus are filled, the electrons are alternately aligned such that each new pair consists of one electron with spin-up and one electron with spin-down. As a result, each completed shell has a resultant spin angular momentum of zero. In silver atom, all of the inner shells are filled, but the outer shell contains a single electron. This electron has an orbital quantum number

$$l = 0, \quad s = \tfrac{1}{2},$$

$$\text{so} \quad j = l + s = \tfrac{1}{2},$$

so that

$$(2J + 1) = 2$$

So the two traces are expected, which is in perfect agreement with experimental result. So the evidence of spatial quantization theory.

If we consider that spinning of electron does not exist then

$$s = 0, \quad l = 0$$

$$\text{and so} \quad j = l + s = 0$$

there would be no splitting of atomic beam of silver into two components. As the spin exists so we see agreement of theory with experimental result.

A spinning electron creates a magnetic dipole moment (see eq. 3.54) that under the action of magnetic field B experiences a torque

$$\tau = \vec{\mu} \times \vec{B} \qquad \ldots\ldots (3.65)$$

that tends to align the dipole with the applied field B. Fig.3-48 shows a magnetic dipole in an inhomogeneous magnetic field. In addition to the rotational effect given by the torque, there is a net translational force, which pushes the dipoles that are somewhat aligned with the field in the direction of most intense part of the field; or for those anti-aligned, they are pushed away from the intense portion of the field. Which we observe in this Stern-Gerlach experiment.

3.28 SPIN-ORBIT INTERACTION

The electron has magnetic moments associated with both its orbital motion and its spin, so these magnetic moments interact with each other. This interaction is called the spin-orbit interaction. There are several ways in which the different vectors of the electrons may combine to give the vectors representing the atom as a whole. The method of combination depends on the interaction or coupling between the component vectors. Two types of combinations are to be discussed. Russell-Saunders or L-S coupling and the j-j coupling.

Russell-Saunders Coupling

This coupling occurs more frequently, so it is known as normal coupling. Here several spin vectors of the electrons combine to form a resultant vector $\vec{S}$ and several orbital vectors of the electrons combine to form a resultant vector $\vec{L}$. Then $\vec{S}$ and $\vec{L}$ combine to make $\vec{J}$. Symbolically

$$(s_1 + s_2 + \ldots.) + (l_1 + l_2 + \ldots.)$$

$$= \vec{S} + \vec{L} = \vec{J}$$

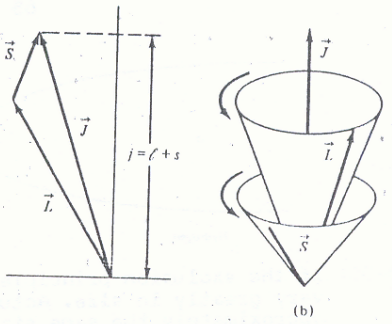


Fig.3-53: a) For a single electron the orientation of the spin and orbital moments. b) L-S coupling causes L and S to precess about J at the same angular velocity.

j-j Coupling

When the spin-orbit interaction is large, the spin and orbital momenta of each electron combine to form $\vec{J}$ for each electron, and then $\vec{J}$'s combine with each other vectorially. Symbolically

$$(s_1 + l_1) + (s_2 + l_2) + (s_3 + l_3) + \ldots\ldots$$
$$= j_1 + j_2 + j_3 + \ldots\ldots = \vec{J}$$

3.29 PAULI'S EXCLUSION PRINCIPLE

We will discuss the rule that prevents all of the electrons in an atom from falling into the 1s level. During the summer of 1922, Wolfgang Pauli went to Copenhegen to work with Bohr on the explanation of the anomalous Zeeman effect. The exclusion principle evolved from his efforts to classify the energy levels of an electron when it is placed in a strong magnetic field. To specify completely the energy state of an atom, the four quantum numbers needed are n,l,m_l, and m_s. Simply stated the Pauli exclusion principle is as follows:

Fig.3-54: Wolfgang Pauli (1900-1958) In 1945 Pauli received the Nobel Prize in Physics for his discovery of the exclusion principle.

"No two electrons in a single atom can have the same set of four quantum numbers (n,l,m_l,m_s)".

In other words, it is impossible for two electrons in an atom to be identical as regards all their quantum numbers, i.e., in such a case one of the two will be excluded from entering into the constitution of the atom. So the name "exclusion principle". For example, if two electrons have the same n,l, and m_l, they must have opposite spin.

The Pauli's principle is the most important rule governing the structure of atoms. In the classical theory, the most stable state of the electrons is when they are at their lowest energy state, the ground state. Thus, from hydrogen on up to the heaviest elements, all of the electrons should be in the first orbit, with the orbits being smaller for the heavier atoms. As a result, each individual atom should differ in size as indicated in fig.3-55, but it does not so. All of the atoms are approximately equal in size.

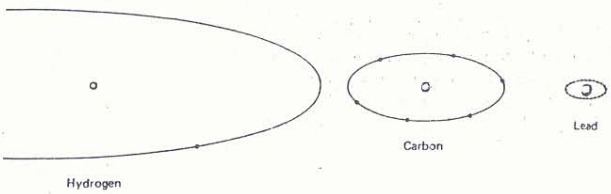


Fig. 3-55: If the exclusion principle did not apply, atoms would vary greatly in size. Actually, all atoms are of approximately the same size.

Applying the Pauli principle to the hydrogen atom, we note that the first shell n = 1, so l = 0, m_l = 0, $m_s = \pm \frac{1}{2}$, hence it has just two electron states given by the quantum numbers $(1,0,0,\pm\frac{1}{2})$.

For the second shell, n = 2, l = 0,1; $m_l = \pm 1, 0$; $m_s = \pm\frac{1}{2}$. So it has eight states given by $(2,0,0,\pm\frac{1}{2})$, $(2,1,0,\pm\frac{1}{2})$, $(2,1,1,\pm\frac{1}{2})$, and $(2,1,-1,\pm\frac{1}{2})$.

For the third shell, n = 3, l = 0,1,2; $m_l = \pm 2, \pm 1, 0$; $m_s = \pm\frac{1}{2}$. So it has eighteen states given by $(3,0,0,\pm\frac{1}{2})$, $(3,1,0,\pm\frac{1}{2})$, $(3,1,1,\pm\frac{1}{2})$, $(3,1,-1,\pm\frac{1}{2})$, $(3,2,0,\pm\frac{1}{2})$, $(3,2,1,\pm\frac{1}{2})$, $(3,2,-1,\pm\frac{1}{2})$, $(3,2,2,\pm\frac{1}{2})$, and $(3,2,-2,\pm\frac{1}{2})$.

This enumeration can be done for any shell. This process for the configuration of electrons helps for the periodic system of the elements.

3.30 FEW DEFINITIONS

Zeeman Effect:

The splitting of atomic energy levels into two or more sub-levels by means of a magnetic field.

Normal Zeeman Effect:

The affect (splitting of lines) of the light emitted from a source, giving line spectrum, when placed in a strong magnetic field.

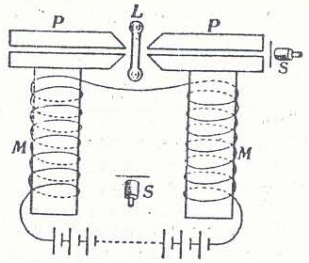


Fig.3-56: Arrangement for Zeeman effect.

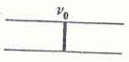


Fig.3-57:Spectrum without magnetic field

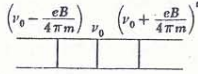


Fig.3-58:Spectrum with magnetic field present

Normal longitudinal Zeeman effect:

When the source of light is placed in a magnetic field and the emitted light is observed parallel to the field the line is seen to be broken up, into two components (only lines). The components are circularly polarized in opposite directions.

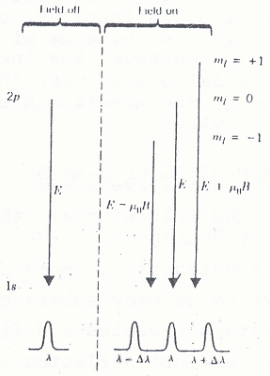


Fig.3-59: The Normal Zeeman effect.

Normal transverse Zeeman effect:

When the source of light is placed in a magnetic field and the emitted light is observed perpendicular to the magnetic field, we see three lines, including middle component of the original frequency. The components are plane polarized.

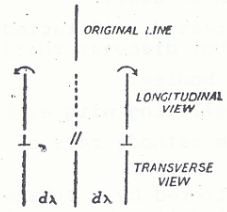


Fig.3-60: Zeeman effect.

Anomalous Zeeman effect:

The phenomenon in which spectral lines are affected by an applied weak magnetic field and split into several components. The complex pattern obtained can be explained only by quantum theory (can't be explained by the classical theory).

Stark effect:

The phenomenon in which spectral lines are affected by an applied strong electric field. This effect cannot be explained by the classical theory. It can be explained only by the quantum theory.

3.31 DISCOVERY OF X-RAYS

There is probably no subject in pure science which illustrates better than X-rays the importance to the entire world of scientific research. In the autumn of 1895, in an experiment designed to study cathode rays, Professor Wilhelm Roentgen carefully shielded a discharge tube with black cardboard. When the room was darkened and a discharge passed through the tube, Roentgen was somewhat startled to see a wavering light across the room near a work bench. He repeated the procedure and found the same. He lit a match and discovered that the source was the fluorescence of a small platinum-barium-cynide screen on the bench. It was observed that these x-rays as Roentgen called them had very great penetrating power. Within three months after Roentgen's discovery

x-rays were being put to practical use in a hospital in Vienna, Germany, for surgical operations. The use of this new aid to surgery soon spread rapidly. X-rays provide us with a kind of supermicroscope, by means of which we can 'observe' the interior of the atom. X-rays are electromagnetic radiations of very short wavelength. See appendix , for a comparison of the wavelength of x-rays with the rest of the electromagnetic spectrum.

3.32 PROPERTIES OF X-RAYS

The following are the main properties of X-rays, initially recorded by Roentgen.

1. Many substances are more or less transparent to x-rays.
2. X-rays cause many substances to fluoresce.
3. Photographic plates and films can be exposed by x-rays.
4. X-rays are not deflected by electric or magnetic fields.
5. X-rays can produce ionization in gases.
6. X-rays can be deflected, refracted, diffracted under certain conditions. (Roentgen could not discover that).
7. X-rays discharge electrified bodies.
8. X-rays are scattered after colliding with electrons.(Compton effect)
9. X-rays are generated when the cathode rays of the discharge tube strike any solid body.
10. The produced x-rays when scattered by a substance, then that substance emits new rays called its characteristic x-rays.

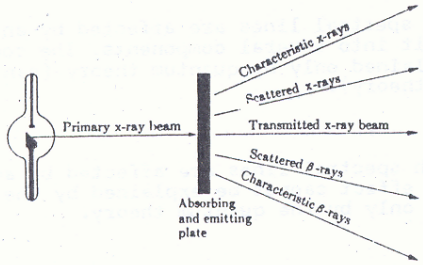


Fig. 3-61: Interaction between x-rays and matter.

3.33 PRODUCTION OF X-RAYS

When an electron is accelerated or decelrated, it radiates photons. When fast moving electrons strike a target they are slowed down and finally come to rest with the emission of photons. X-rays are produced by gas tube, the coolidge tube, and betatron. Usually the first two types are used.

i) Gas-filled tube:

Fig. 3-62 shows the x-ray tube, initially the form suggested by Roentgen. A gas pressure of the order of 10^{-3} mm of mercury provides, when voltage is applied, a few electrons and positive ions. These

positive ions, bombarding the concave surface of the cathode C, release electrons which move fastly towards anode A, give rise to X-rays. The cathode rays leaving the concave cathode surface normally are concentrated at a single point of the target anode inclined at angle 45°. The target is made of tungsten, attached with heat absorber copper support. This low-pressure gas-filled tube produce very feeble x-rays.

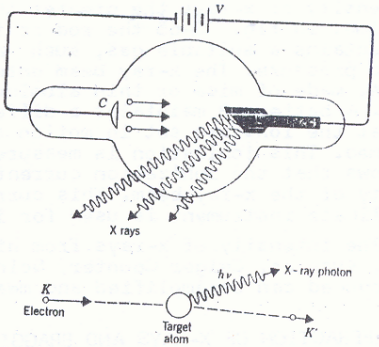


Fig.3-62: Gas-filled X-ray tube.

For typical accelerating voltages in the range of 10,000 volts, x-rays are produced. This continuous distribution of x-rays is called bremsstrahlung (German for braking, or decelerating radiation). It is different from discrete x-ray energies discussed in Section 3.31.

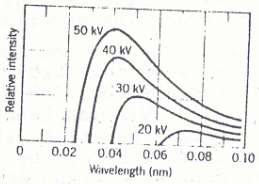


Fig.3-63:Bremsstrahlung spectra.

ii) Coolidge tube:

In 1913 an important improvement in the gas-filled tube was introduced by Coolidge. It is a thermionic tube, evacuated to the highest attainable vacuum, incorporated in the cathode a hot spiral filament of tungsten to serve as a source of electrons. The filament is heated by an adjustable current from a battery, so that the current of electrons is controlled independently of the applied voltage. The anode A is made of tungsten or molybdenum is attached with copper rod whose end outside the tube is kept cool by means of copper discs.

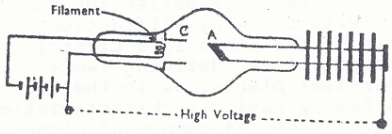


Fig.3-64: Coolidge tube.

The usual potential difference is from 50,000 to 100,000 volts for the current of 50 to 100 milliamperes. Major power received by anode is converted into heat. Only 1 % is converted into x-rays. The Coolidge tube is much more stable in operation than the gas tube.

3.34 INTENSITY MEASUREMENT OF X-RAYS

The intensity of a beam of x-rays may be measured by any one of its effects, e.g., effect on photographic plate, penetration through matter, rise of temperature of the target, ionization of gases. The ionization chamber is commonly used for measuring the intensity of x-rays. When a beam of x-rays is passed through a gas, it get ionized. The greater

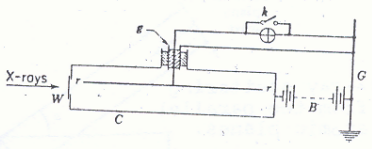


Fig.3-65: Ionization chamber for measuring X-rays.

the intensity of x-rays the greater the number of ions produced in a given time. In fig. 3-65 the rod rr is set in the ionization chamber, which contains a suitable gas, such as, methyl bromide at about atmospheric pressure. The x-ray beam enters the chamber through a thin window W, made of mica or thin aluminum, and ionizes the gas in the chamber. A battery B maintains a difference of potential between rr and C so that the ions are set in motion toward C and rr as soon as they are formed. This ionization is measured by the electrometer E. Experiment shows that the ionization current is directly proportional to the intensity of the x-ray beam. This current is of the order of 10^{-13} amperes so a delicate instrument is used for its measurement.

The intensity of x-rays from high voltages is measured by new devices, such as, Geiger Counter, Scintillation Counter. The current thus produced can be amplified and measured with appropriate circuits.

3.35 DIFFRACTION OF X-RAYS AND BRAGG'S LAW

The short wavelength of x-rays made diffraction effect difficult to observe. The experiments indicated the wavelength of the order of 10^{-10} m. Von Laue found that in some crystals the distance between the atoms were of this order. He suggested that the regular array of atoms in crystals could be used as a kind of diffraction grating for x-rays, and in 1914 Von Laue received the Nobel Prize for establishing the wave character of x-rays by diffraction from crystals. W.H. Bragg and his son, Lawrence Bragg, were awarded the Nobel Prize the next year for perfecting Von Laue's concepts of x-ray diffraction from crystals. The atoms in a crystal may be thought of as defining families of parallel planes, as in the fig. with each family having a characteristic separation between its component planes. It was suggested by W.L. Bragg in honour of whom these planes are called Bragg planes.

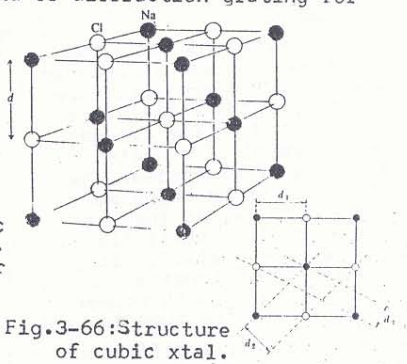


Fig.3-66:Structure of cubic xtal.

The conditions that must be fulfilled for radiation scattered by crystal atoms to undergo constructive

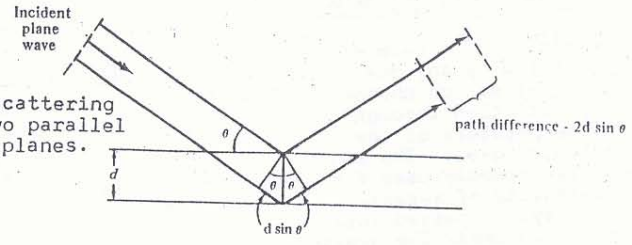


Fig.3-67:X-ray scattering from two parallel atomic planes.

interference may be obtained from a diagram like that in fig. 3-67. A beam containing x-rays of wavelength λ is incident upon a crystal at an angle θ with a family of Bragg planes whose spacing is d. The beam goes past atom A in the first plane and atom B in the next, and each of them scatters part of the beam in random directions. Constructive interference takes place only between those scattered rays that are parallel and whose paths differ by exactly λ, 2λ, 3λ, ... That is, the path difference must be $n\lambda$. The only rays scattered by A and B for which this is true are shown in fig.3-67. The first condition upon these rays is that their common scattering angle be equal to the angle of incidence θ of the original beam. This condition is same as for ordinary optical reflection. i.e.

'The angle of incidence must equal the angle of reflection'.

For this reason x-ray scattering from atomic planes in a crystal is sometimes called Bragg reflection.

In fig.3-67, ray II travels a longer path than ray I, this difference in path is equal to (AB + BC), which is a whole wavelength λ or a whole multiple of the wavelength $n\lambda$.

The condition for constructive interference is

$$(AB + BC) = n\lambda$$

from the fig. 3-67,

$$AB = d\sin\theta$$

$$BC = d\sin\theta$$

So

$$n\lambda = 2d\sin\theta \qquad \ldots\ (3.66)$$

where

n is an integer,
d is interplaner spacing,
or crystal grating space,
θ is the glancing angle

And eq. 3.66 is called the Bragg equation. It is directly verified by experiments. Bragg undertook this task and devised an apparatus known as crystal x-ray Bragg's spectrometer. A schematic diagram of this instrument is shown in the fig. 3-68. The x-rays from a tube are collimated into a narrow beam by the slits S_1 and S_2 cut in lead plates, and then strike on the crystal C, which acts like a diffraction grating. The angular position of C is read by means of a vernier. After diffraction, the x-rays enter an ionization chamber D, filled with methyl iodide, which absorbs x-rays strongly. The electrometer E records the intensity of ionization in the chamber.

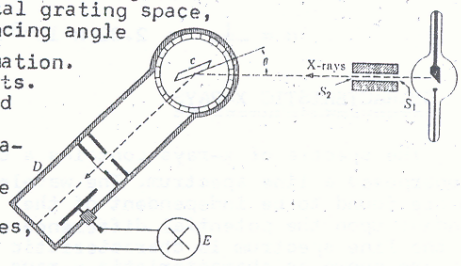


Fig.3-68: Bragg x-ray spectrometer.

In eq.3.66, if we take n = 1, then different values of θ will correspond to different values of λ. Thus the spectrometer can be used to find different wavelengths and their relative intensities.

EXAMPLE:

Compute the separation of adjacent atoms, d, in a crystal of rock salt (NaCl). The molecular weight is 58.46 and density is 2.165gm/cm^3

Solution:

We have

$$\text{mass} = m = \frac{M}{N_o} = \frac{58.45 \text{ gm/gm-mole}}{6.023 \times 10^{23} \text{ molecules/gm-mole}}$$

$$= 9.705 \times 10^{-23} \text{ gm/molecule}$$

Now the number of molecules per unit volume, N is

$$N = \frac{\rho}{m} = \frac{2.165 \text{ gm/cm}^3}{9.705 \times 10^{-23} \text{ gm/molecules}}$$

$$= 2.24 \times 10^{22} \text{ molecules/cm}^3$$

Since each molecule of NaCl contains two atoms, so

$$2N = 4.48 \times 10^{22} \text{ atoms/cm}^3$$

If there are n atoms along 1-cm edge of a cubic centimeter, then

$$n^3 = 4.48 \times 10^{22} \text{ atoms/cm}^3$$

and

$$n = 3.55 \times 10^7 \text{ atoms/cm}$$

Thus, the distance between atoms is

$$d = \frac{1}{n} = 2.82 \times 10^{-8} \text{ cm} = 2.82 \text{ Å}$$

3.36 CHARACTERISTIC X-RAYS

The spectra of x-rays contains a continuous spectrum upon which is superposed a line spectrum. The wavelengths of the continuous spectrum are found to be independent of the material of the target and dependent upon the potential difference across the tube. On the other hand the line spectrum is characteristic of the element used as target. These are known as characteristic x-rays. These can be produced by two metnods.

i) In x-ray tube, using the material as the target, whose characteristic x-rays are required.

ii) From the x-ray tube, the x-rays are made to fall on the material which later on emits its characteristic x-rays.

In the first method the velocity of the cathode rays must be high. And in second method the incident primary x-rays should be harder than the characteristic rays to be produced.

When a beam of x-rays from a platinum target incident on the face of a rack salt crystal, W.H. Bragg rotated the crystal in steps of θ and the ionization chamber in steps of 2θ. The curve of ionization current against glancing angle θ was plotted and found that the x-rays intensity did not vary uniformly with angle but rose certain angles to a sharp maximum. As shown in fig.3-69. A group of three maxima (A,B,& C) was observed. A second group of three maxima was observed at approximately double these angles. Bragg interpreted the first group of maxima as three monochromatic lines, and the second group as second order reflections.

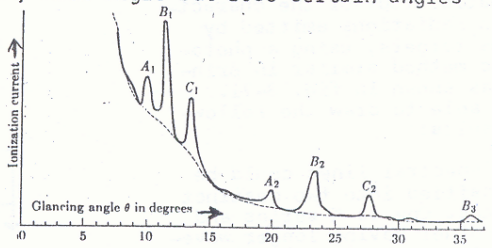


Fig.3-69: The first x-ray spectrum.

The curves similar to this fig. were obtained with other crystals, the only difference being that the maxima occured at different glancing angles, indicating that each crystal had a characteristic grating space d. The peaks of the curve in fig.3-69 represent spectral lines the wavelengths of which are characteristic of the target emitting the rays. These lines are superimposed on a continuous spectrum represented by the dotted line in the figure. These curves show the distribution of energy in the x-ray spectrum.

The Bohr theory accounts for the discreteness and the order of magnitude of the photon energies of the characteristic spectrum by assuming that these photons are emitted as a result of transitions involving inner electron shells. If, for example, a high energy electron knocks out an electron from the first shell of a target atom, the vacancy may be filled by a transition from the second shell, or the third, or higher shells. Since the electrons of the first shell have been called K-electrons, the photon emitted by a transition from the second shell to the K shell is called K_α radiation, that due to a transition from the third shell to the K shell is called K_β radiation, and so on. The second shell is designated as the L shell, the third M, and so forth, so that L_α, L_β, ... , M_α, M_β, ... radiation is defined in a similar fashion. K series is observed e.g. in the spectra of sodium and potassium. L series is observed in elements having atomic number greater than 28. M series is observed for elements having atomic number greater than 65. The N series is observed in trans uranic elements.

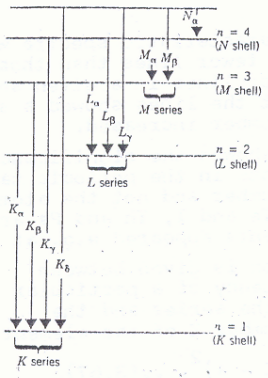


Fig.3-70: X-rays series.

3.37 MOSELEY'S LAW

In two classic papers Moseley in 1913, presented a systematic study of the characteristic radiations emitted by various targets, using a photographic method similar in principle as shown in fig. 3-71. He was able to draw the following results:

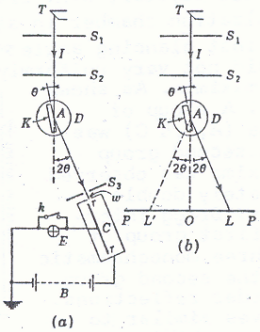
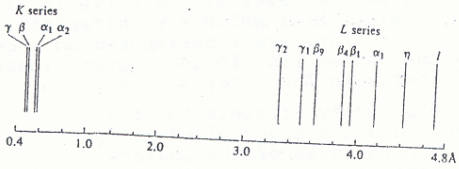


Fig.3-71: X-ray spectrometer using
a) the ionization method
b) the photographic method.

1- The spectral lines could be classified into two destinct groups. One is K series and the other having longer wavelengths as L series. It should be noted that K line is doublet, which was not known at Moseley's time.

Fig.3-72: Relative positions of the K and L x-ray series spectral lines of silver.



2- X-ray characteristic spectra was much simpler consisting of comparatively fewer lines than those observed in optical spectra. The pattern of lines in a given series was similar for most of the elements but the lines showed a steady decrease in wavelength as the atomic number increased.

3- Moseley showed the significance of atomic number. The arrangement of elements in the periodic table was thus made according to the atomic number and not the atomic weight. Four pairs Ar and K , Co and Ni, Te and I, Th and Pa are reversed in this way. Also some new elements appeared e.g. at Z = 43, 61, 72, 75, and 85.

4- A relation is given between the frequency of a particular line of the series and the atomic number Z of the element.

$$\nu = C(Z - a)^2 \quad \ldots\ldots (3.67)$$

where C and 'a' are constants. For K_α line C was found to be equal to $\frac{3}{4}Rc$, where R is the Rydberg constant, c is the speed of light, and 'a' was found to be nearly 1. The equation for the frequency

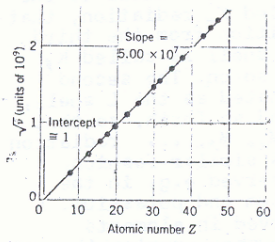


Fig.3-73: $\sqrt{\nu}$ verses Z for K_α X-rays.

of the K_α line of any element can therefore be written as

$$\nu = \tfrac{3}{4} R c (Z-1)^2 \qquad \ldots\ldots (3.68)$$

$$\text{or } \sqrt{\nu} \propto (Z-1) \qquad \ldots\ldots (3.69)$$

If we plot $\sqrt{\nu}$ verses Z, we would obtain a straight line with slope $(\frac{3}{4}Rc)^{1/2}$. As shown in fig. 3-73. The graph is known as Moseley diagram and the eq. (3.67) or (3.68) is called Moseley law, which states that <u>the frequency of a spectral line in an X-ray spectrum varies as the square of the atomic number of the element emitting it</u>.

This method gives us a simple way to determine the atomic number Z of an atom. Moseley was the first to demonstrate the type of linear relationship shown in fig. 3-73. His discovery provided the first direct means of measuring the atomic numbers of the elements. Previously, the elements had been ordered in the periodic table according to increasing mass. Moseley found certain elements listed out of order. (See previous point 3).

Moseley's work followed closely on the introduction of Rutherford's and Bohr's theory. The relationship between Bohr's theory and Moseley's work is shown in the next Section.

3.38 DERIVATION OF MOSELEY'S LAW FROM BOHR'S POSTULATES

From Bohr's Fourth assumption, we have

$$\nu = \frac{E_i - E_f}{h}$$

$$\alpha \; h\nu = E_{n_2} - E_{n_1}$$

$$\left[E_n = \frac{-2\pi^2 m Z^2 k^2 e^4}{n^2 h^2} \quad -(3.23)\right.$$

$$\frac{1}{\lambda} = \frac{\nu}{c} = \frac{h\nu}{hc} = \frac{1}{hc}\left(\frac{-2\pi^2 m Z^2 k^2 e^4}{n_2^2 h^2} - \frac{-2\pi^2 m Z^2 k^2 e^4}{n_1^2 h^2}\right)$$

$$= \frac{2\pi^2 k^2 Z^2 e^4 m}{h^3 c}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

put $\frac{2\pi^2 m k^2 e^4}{h^3 c} = R$ = Rydberg constant

$$\frac{1}{\lambda} = \frac{\nu}{c} = R Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

For R_α line

$$n_1 = 1 \text{ & } n_2 = 2$$

So $$\nu = Rc Z^2 \left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

$$\left[c = \lambda\nu \right. \quad \alpha \; \nu = \frac{c}{\lambda}$$

Taking the effective value of nuclear charge as $(Z-1)$, (as an electron goes to excited state).

$$\nu = Rc (Z-1)^2 \left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

$$\alpha \; \nu = \frac{3}{4} Rc (Z-1)^2$$

which is Moseley's Law.

3.39 THE ORIGIN OF X-RAYS

i) Emission Spectra:

When an element is bombarded with high speed electrons, it emits all its characteristic x-rays. The spectrum produced by these rays is called the X-ray emission spectrum of the element.

Figures 3-69, 3-70, and 3-72 shows the emission spectrum. Now suppose an atom in the target of an x-ray tube is bombarded by high speed electrons and that one of the two K electrons is knocked out of the atom, so that the atom is converted into an ion. Now an L electron may take its place, that is, it changes state from n = 2 to n = 1. The ion's excess energy being emitted as a quantum of radiant energy. This process is called to emit K_α line. In the similar way we obtain K_β, L_α etc. lines.

This gives an explanation of the factor $(Z - 1)^2$ instead of Z^2 in Moseley's equation. When high speed electrons are bombarded one of the K electrons is removed from the atom. It makes its 'effective' nuclear charge about 1 unit less.

X-ray lines are classified into series known as the K, L, M, N, O, etc. series. Each series of lines contains several strong lines and numerous faint ones.

ii) Absorption Spectra:

Absorption being the reverse of emission, it should be possible for a photon of the right frequency to be absorbed by an atom while one of the K electrons is raised into an L-shell, provided there is a vacancy in the L shell. Usually L shell is full. So K lines cannot be observed as absorption lines. The same is true for all of the x-ray emission lines that are commonly observed. A photon can be absorbed, however, if it has enough energy to remove an inner electron from the atom entirely.

The curve for the absorption coefficient verses wavelength, show a sudden rise at certain frequency. The critical wavelength or frequency at which absorption in the K shell begins is called K absorption limit. Similar process occurs for other series, but here more than one absorption limit exists. So for L series, three absorption limits are found, as shown in fig. 3-74, as L_I, L_{II} and L_{III} . There are five M limits seven N limits so on.

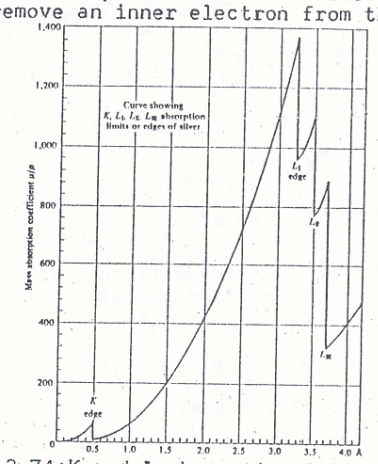


Fig.3-74:K and L absorption limits.

Lets suppose that x-rays pass through a substance.

Suppose

I_o= initial intensity of the beam

I = intensity after travesing distance x

dx = additional small thickness

dI = decrease in I after passing through dx

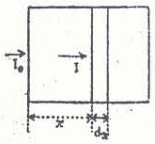


Fig.3-75: X-rays pass through a substance.

Then it is found that

$$dI \propto I$$

also $$dI \propto dx$$

so $$dI \propto Idx$$

or $$dI = -\mu Idx \quad \ldots\ldots (3.71)$$

where μ is proportionality constant called absorption coefficient. It represents the fraction of the energy removed from the beam per centimeter of the path. And negative sign shows the intensity I decreases as x increases.

The absorption coefficient μ is different for different materials. However, we can define a quantity μ_m, known as mass absorption coefficient, which is the same for all the materials. It is defined as:

$$\mu_m = \frac{\mu}{\rho}$$

where ρ is the density of the substance.

A method for studying the x-ray absorption spectrum of an element is illustrated in fig.3-76. The continuous spectrum from some suitable element T is used. A beam of x-rays coming through the slits S_1 and S_2 is incident on the absorbing material A under study. The transmitted beam is then analyzed after passing through the crystal C. The intensity of each wavelength is measured by the ionization chamber. The result of experiment, using silver as absorbing element is shown in fig. 3-74.

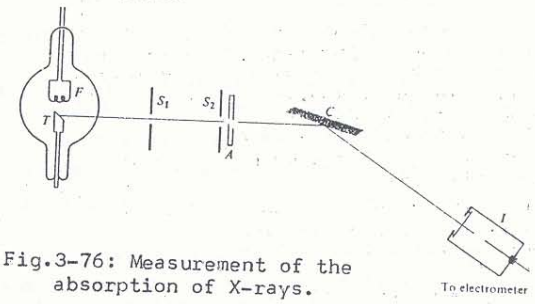


Fig.3-76: Measurement of the absorption of X-rays.

X-ray photons can only ionize the shell (e.g. L shell) electron, but it cannot excite it so as to raise it to higher shell, since all shells are normally occupied by electrons. The value of μ_m attains a peak value when absorption occurs. In this way we get peaks in the absorption curve.

iii) Fluorescent Spectra:

When an electron is removed from K shell by absorption of a photon, an electron from any outer shell may drop into the K shell with the emission of fluorescent radiation. The frequency of it will be less than the original incident photon. This phenomenon is usual in x-rays.

QUESTIONS

3.1 Does the Thomson atom model fail at large scattering angles or at small scattering angles ? Why?

3.2 Why didn't Bohr use the concept of de Broglie waves in his theory ?

3.3 Point out the serious defect in the Rutherford nuclear atom model. How was this defect removed in Bohr atom model.

3.4 List the assumptions made in deriving the Bohr theory. Which of these are a result of neglecting small quantities? Which of these violate basic principles of relativity or quantum physics?

3.5 Show how Bohr's theory of the hydrogen atom accounts for the observed spectrum of hydrogen.

3.6 We observe line spectra from moncatomic gases only. Why?

3.7 Describe Frank-Hertz experiment showing that the atomic energy states are quantized.

3.8 How did Sommerfeld modify Bohr's theory to fit it into the experimental data? What conclusions were drawn by him?

3.9 What is meant by space quantization? Summarise the quantum numbers associated with vector atom model.

3.10 Describe and explain Stern-Gerlach experiment and discuss its importance.

3.11 State and explain the Pauli's exclusion principle. Write down the electron configuration in the normal state of krypton.

3.12 Describe the two types of X-ray tubes used for the production of X-rays. Which tube is better and why?

3.13 What is Moseley's Law? Can we derive this law from Bohr's theory?

3.14 Describe X-rays emission and absoption spectra.

3.15 Record theoretical treatment of Rutherford's scattering of α -particles.

3.16 What is magnetic moment? Derive an expression for Bohr magneton.

3.17 Deduce Bragg's relation. How is it used to measure the X-rays wavelength with Bragg's spectrometer.

3.18 What are characteristic X-rays? How are they produced?

3.19 What are the chief properties of X-rays.

3.20 How does a quantized angular momentum vector differ from a classical angular momentum vector?

PROBLEMS

3-1 The series limit of the Paschen series is 8201.4 Å. What are the two longest wavelengths of the Paschen series?

Soln:

We have $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$, $n = 4, 5, 6, \ldots\ldots$

The two longest wavelengths are

$n = 4$, $R = 1.097 \times 10^{-3}$ Å so $\frac{1}{\lambda} = R\left(\frac{1}{9} - \frac{1}{16}\right)$

or $\lambda = \frac{1}{1.097 \times 10^{-3}}\left(\frac{144}{16-9}\right) = 18750$ Å

And $n = 5$, $R = 1.097 \times 10^{-3}$ Å,

$\frac{1}{\lambda} = R\left(\frac{1}{9} - \frac{1}{25}\right)$

or $\lambda = \frac{1}{1.097 \times 10^{-3}}\left(\frac{25 \times 9}{25-9}\right) = 12810$ Å

3-2 Calculate the shortest wavelength in the Lyman series.

Soln: For shortest wavelength or series limit,

$n = \infty$ so $\frac{1}{\lambda} = R(1-0)$

or $\lambda = \frac{1}{1.097 \times 10^{-3} \times 1} = 912$ Å

3-3 Find the wavelength of the spectral line corresponding to a transition in the hydrogen atom from an orbit with n = 6 to the orbit with n = 3.

Soln:

We have $\frac{1}{\lambda} = R\left(\frac{1}{p^2} - \frac{1}{n^2}\right)$

or $1/\lambda = 1.097 \times 10^{-3}\left(\frac{1}{(3)^2} - \frac{1}{(6)^2}\right)$

or $\lambda = \frac{1}{1.097 \times 10^{-3}}\left(\frac{36 \times 9}{36-9}\right) = 10940$ Å

3-4 How much energy is required to remove an electron in the n = 3 state of a hydrogen atom?

Soln:

We have $E = -\frac{2\pi^2 m k^2 e^4}{h^2}\left(\frac{1}{n^2}\right)$

for n = 3 and putting other values (as in section 3.9),

$E = -\frac{13.6}{(3)^2} = 1.51$ eV

3-5 Calculate the shortest wavelength emitted by singly-ionized helium atom.

Soln: We have for hydrogen atom (from eq. 3.29),

$\frac{1}{\lambda} = R\left(\frac{1}{p^2} - \frac{1}{n^2}\right)$ .... (3.29)

for shortest wavelength in the Lyman series,

$p = 1$, & $n = \infty$, so

$\frac{1}{\lambda} = R(1-0) = R$

so $\lambda = \frac{1}{R} = \frac{1}{1.097 \times 10^{-3}} = 912$ Å

Now for singly-ionized helium atom,

charge = +2e

So in eq. (3.28), we must supply the factor 2 to e^2 in the formula, hence the factor 2x2 or 4 to e^4,

then eq.(3.29) changes to

$$\frac{1}{\lambda} = 4R\left(\frac{1}{p^2} - \frac{1}{n^2}\right) \quad \ldots\ldots (3.29\text{-b})$$

then for shortest wavelength

$$\frac{1}{\lambda} = 4R(1-0) = 4R$$

$$\text{or } \lambda = \frac{1}{4R} = \frac{9/2}{4} = 228\ \text{Å}$$

3-6 Calculate the ionization energy of the singly-ionized helium atom.

Soln: The required ionization energy is equal to the energy corresponding to shortest wavelength, i.e., 228 Å (see prob.3-5)

Its frequency is

$$\nu = \frac{c}{\lambda} = \frac{3\times10^8}{228\times10^{-10}}$$

$$\text{And energy} = h\nu = 6.63\times10^{-34}\times\frac{3\times10^8}{228\times10^{-10}}\ \text{Joules}$$

$$\text{or energy} = \frac{6.63\times10^{-34}\times3\times10^8}{228\times10^{-10}}\times\frac{1}{1.6}\times10^{19} = 54.4\ eV = 4\times13.6\ eV$$

$$\left[\begin{array}{l} 1\ eV = 1.6\times10^{-19}\ \text{Joules} \\ \text{or } 1\ J = \frac{1}{1.6}\times10^{19}\ eV \end{array}\right.$$

i.e. it is four times greater than the hydrogen atom (Sect.3.9)

3-7 Calculate the angular speed of the electron in the normal state of the hydrogen atom.

Soln: We have from eq. (3.8),

$$\omega_n = \frac{v_n}{r_n}$$

and from eq. (3.9),

from eq. (3.14), $\frac{v_n}{r_n} = \frac{nh}{2\pi m}\left(\frac{1}{r_n^2}\right)$

$$r_n = \frac{n^2h^2}{4\pi^2mke^2}$$

$$\text{So } \omega_n = \frac{nh}{2\pi m}\left(\frac{4\pi^2mke^2}{n^2h^2}\right)^2$$

for normal state, n = 1, so

$$\omega_1 = \frac{8\pi^3mk^2e^4}{h^3}$$

UNSOLVED PROBLEMS

3:1 Calculate the longest wavelength in the Balmer series.

3:2 Determine the value of the longest wavelength in the Bracket series.
(Ans. 40751.5 Å)

3:3 Find the wavelength of the photon emitted when a hydrogen atom goes from $n = 4$ state to its ground state.
(Ans. 978.035 Å)

3:4 Find the wavelengths of the transitions in the hydrogen atom from $n = 3$ to $n = 2$ and from $n = 4$ to $n = 2$.
(Ans. 6561 Å and 4860 Å)

3:5 In the $n = 3$ state of hydrogen, find the electron's velocity, kinetic energy, and potential energy.

3:6 A beam of X-rays is observed with $\theta = 30°$ in Bragg's spectrometer. If the crystal grating space is 2.40 Å, find the wavelength of x-rays used.
(Ans. 2.40 Å)

Chapter 4

Particles and Waves

4.1 INTRODUCTION

In this chapter we discuss the experimental evidence in support of the wave behaviour of particles such as electrons. In classical physics, projectiles obey particle-type laws, such as Newtonian mechanics. Waves undergo interference and diffraction. The energy carried by a particle is confined to a small region of space. A wave, on the other hand, distributes its energy throughout all space in ever-expanding wavefronts. In contrast to this clear distinction found in classical physics, the quantum theory requires that, in the microscopic form, particles sometimes obey the rules that we have previously established for waves ! We will ignore classically clear distinction between particles and waves. In Comptom effect (see section 2.11) electrons behave like billiard balls, so we are led to believe that, we should be able to pick up an electron by a fine pair of forceps. If, however, an electron is a wave, we will certainly not be able to do so---imagine trying to grasp a sound wave. In classical physics, we use fixed trajectories and certainty of outcome but in quantum physics, we will study the terms, the probability, statistical and average measurements.

4.2 De Broglie's concept of matter waves

Prince Louis Victor de Broglie (France) was motivated by the following points.

1. Motion of particles and light rays have like character

According to the principle of least action, a moving particle chooses that path for which the action is minimum. And according to the principle of least time, a ray of light follows always a path for which the time of transit is minimum. These show the similarity between mechanics and optics.

Fig. 4.1:

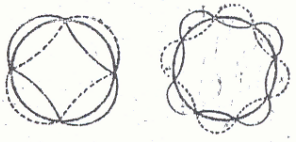
In 1929 de Broglie received the Nobel prize in physics for his discovery of the wave nature of electrons.

2. Nature loves symmetry

According to it, the two fundamental forms, matter and radiation, in which Nature manifests herself, must be mutually symmetrical. Since radiation has been shown to possess a dual nature, wave and particle, matter also might possess the same dual nature.

3. Bohr Orbits

Bohr's theory gives "integer" rules for electrons in orbits. The only phenomena invoving integers in physics were those of interference and modes of vibration of stretched strings, both involves wave motion, so de Broglie attempted to explain by regarding the electron as a standing wave around the circumference of an orbit, as shown in figure 4.2.



(a) $2\pi r = 2\lambda$ (b) $2\pi r = 4\lambda$

Fig. 4.2: Two possible modes of vibrations of a wire loop.

4. Planck's Quantum theory of radiation

If we regard radiation as a "shower" of photons, we cannot understand why all radiation should be propagated with a constant speed. Also energy is given by $h\nu$. Here frequency ν plays an important part. What can be the meaning of frequency in a shower of photons. There is nothing periodic about bullets shot from a gun or falling drops of rain. These show the short-

4.3 De Broglie Wavelength

Two decades after the explanation of photoelectric effect, in 1924 Louis de Broglie delivered his thesis for his doctorate degree, in which he suggested that since light had much of the character of particles, that particles, in particular electrons, could by the symmetry of nature be expected to exhibit wavelike properties.

Planck had related the energy of light corpuscles (photon) and the frequency of light by

$$E = h\nu \qquad \ldots\ldots (4.1)$$

Compton had shown that light waves have a momentum, usually associated only with particles. The energy expression for a photon could also be expressed as

$$E = mc^2 = pc \qquad \ldots\ldots (4.2)$$

where p is the momentum associated with the photon.

Combining eqs. (4.1) and (4.2), we get

$$pc = h\nu$$

$$\text{or} \quad p = \frac{h\nu}{c} \qquad \left[c = \lambda\nu \;\; \text{or} \;\; \frac{1}{\lambda} = \frac{\nu}{c} \right]$$

$$\text{or} \quad p = \frac{h}{\lambda} \qquad \ldots\ldots (4.3)$$

eq.(4.3) shows the wave character of photon to the particle character, having the momentum p. So a particle can have a momentum

$$p = mv = \frac{h}{\lambda} \qquad \ldots\ldots (4.4)$$

where m is the relativistic mass.

from eq.(4.4), we have

$$\lambda = \frac{h}{mv} \qquad \ldots\ldots (4.5)$$

this shows, as the momentum of the particle increases, the wavelength associated with the particle decreases.

The Planck and de Broglie relations may be expressed in the useful forms,

$$E = \hbar\omega \qquad \ldots\ldots (4.6)$$

$$p = \hbar k \qquad \ldots\ldots (4.7)$$

where $\hbar$ (= 1.054×10^{-27} erg-sec)* is a constant of dimensions energyxtime. It is sometimes called Dirac's constant, while $h = 2\pi\hbar$, is called Planck's constant.

$\omega (= 2\pi\nu)$ is angular frequency

$k \left(= \frac{\omega}{c} = \frac{2\pi}{\lambda}\right)$ is propagation vector.

*[or $\hbar = 1.054 \times 10^{-34}$ J.s]

Although de Broglie's initial work considered the problem of an electron particle, the equation is valid for all material bodies. An official BCCP's cricket ball of mass 0.16 kg thrown as a fast ball has a velocity of about 40 m/sec. The de Broglie wavelength associated with this cricket ball is :

$$\lambda = \frac{h}{m_0 v} = \frac{6.63 \times 10^{-34}}{0.16 \times 40} \times \left(10^{10}\ \frac{\text{Å}}{\text{m}}\right) = 1.04 \times 10^{-24}\ \text{Å} \qquad \ldots\ldots (4.8)$$

which is an extremely small wavelength that could not be detected. An electron travelling with the same velocity will have a wavelength

$$\lambda = \frac{h}{m_0 v} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-30} \times 40} \times \left(10^{10}\ \frac{\text{Å}}{\text{m}}\right) = 1.8 \times 10^{5}\ \text{Å} \qquad \ldots\ldots (4.9)$$

which can be readily measured in the laboratory.

The interference and diffraction effects with the inner structure of the crystals (e.g. X-ray diffraction) was unobserved before 1920. Today this wave nature is used in atomic physics, nuclear physics, to study the properties, by solid-state physicists, physical chemists, by biologists and biochemists in studying microscopic life with the electron microscope, and by astrophysicists in explaining many curious objects found in our universe.

4.4 DIFFRACTION OF WAVES FROM CRYSTALS

We know from our study of light waves, sound waves, and water waves that, for wavelengths that are much smaller than the dimensions of aperatures and obstacles, diffraction is not observed. In such cases we can assume rectilinear propagation. So, we do not expect to see around doors, trees, or other obstacles in our daily lives because the dimensions of such objects are very large as the wavelength of light. On the other hand, for wavelength that are equal or greater the dimensions of objects , diffraction effects become quite important. For example, our speech produces wavelengths of the order of meters, and we are accustomed to the diffraction of these waves around obstacles. Our world would seem quite strange if audible sound would be rectilinear propagation.

Relation (4.8) show that the cricket ball has a de Broglie wavelength is beyond the limit of detection. We note in relation (4.9) that electrons have de Broglie wavelengths that corresponds to the atomic spacings of some crystals.

We observe the diffraction of X-rays from crystals, such as NaCl crystal, in which the atoms are uniformly spaced in planes and are separated by a distance of the order of 2-5 Å. Fig. 4-3, shows two parallel rays incident

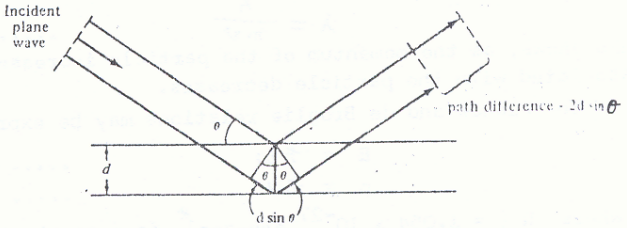


Fig. 4-3: X-ray scattering from two parallel atomic planes.

on two successive atomic planes of a cubic crystal. Note that the path difference between the two rays is $2d\sin\theta$. If the path difference is an integral number of wavelengths, the rays are in phase and the reflection will be strong. If the path difference is equal to an odd number of half wavelengths, the reflection will be weak. We have the Bragg equation:

$$2\,d\,\sin\theta = n\lambda \qquad \ldots\ldots (4.10)$$

where 'n' is an integer, called the order of the spectrum. Since the path difference varies with the angle θ , the order will depend upon both the wavelength and the angle of incidence. Fig. 4-3 considers reflections from the planes parallel to the surface of the crystal. A more realistic case is illustrated in fig.4-4.

Fig.4-4: Detail of scattering from crystal planes.

4.5 DAVISSON-GERMER EXPERIMENT

The first experimental confirmation of the wave nature of electrons followed soon after de Broglie's hypothesis ($\lambda = h/p$). In 1926, at the Bell Telephone Laboratories, Clinton Davisson and Lester Germer were investigating the reflection of electron beams from the surface of nickel crystals. Fig. 4-5, show the apparatus. A beam of electrons from a heated filament is accelerated through a potential difference V. Passing through a small aperature, the beam strikes a single crystal of nickel. Although some electrons were scattered at nearly all angles (fig.4-7(c)), it was noted that a distinct peak in the intensity occurred at an angle of 50° (fig.4-6). Further, the size of this peak varied with the energy of the incident electrons, and was found to reach a maximum of intensity for 54-volt electrons. As shown in fig.4-6. The following examples will help you to understand the significance of this experiment.

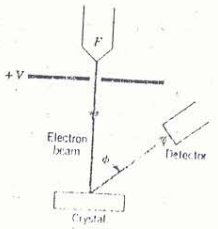


Fig.4-5: Apparatus used by Davisson and Germer to study electron diffraction.

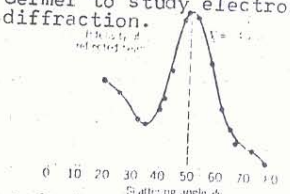


Fig.4-6: Results of Davisson and Germer.

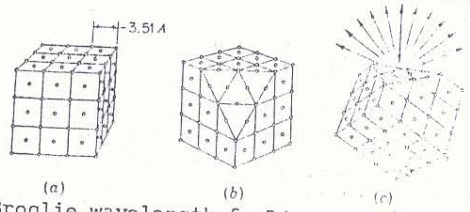


Fig.4-7: A nickel crystal showing face-centered cubic structure.

Example- 1:

Calculate the de Broglie wavelength for 54-volt electrons.

Solution:

We have

$$\tfrac{1}{2}mv^2 = Ve$$

$$\text{or } m^2v^2 = 2mVe$$

$$\text{or } mv = \sqrt{2mVe}$$

from eq. (4.5)

$$\lambda = \frac{h}{mv}$$

$$\text{so } \lambda = \frac{h}{\sqrt{2mVe}} \qquad \ldots\ldots (4.11)$$

We have

$h = 6.63 \times 10^{-34}$ Joules-sec

$m = 9.1 \times 10^{-31}$ kg

$V = 54$ volts

$e = 1.6 \times 10^{-19}$ Joules

putting the values in eq. (4.11), we get

$$\lambda = \frac{6.63 \times 10^{-34}\ J.S}{\sqrt{2 \times 9.1 \times 10^{-31} \ldots}} = 1.67 \times 10^{-10}\ m = 1.67\ \text{Å}$$

Example- 2:

If we assume an interatomic spacing of 2.15 Å for the nickel crystal. What wavelength corresponds to a first-order spectrum in the diffraction pattern for a scattering angle of 50°?

Solution:

According to Bragg equation

$$2\,d\sin\theta = n\lambda \qquad \ldots\ldots (4.12)$$

In fig.4-4, the interplanar distance is related to the atomic spacing 'a' by the relation

$$d = a\sin\phi$$

since $\sin\theta = \cos\phi$ $\quad\left[\theta = 90-\phi \text{ or } \sin(90-\phi) = \cos\phi\right.$

so eq.(4.12) becomes

$$n\lambda = 2a\sin\phi\cos\phi \qquad \left[2\sin\theta\cos\theta = \sin 2\theta\right.$$

$$\text{or}\quad n\lambda = a\sin 2\phi \qquad \ldots\ldots (4.13)$$

since the angle between the incident and the scattered beam is $2\phi = 50°$

and $n = 1$

$a = 2.15$ Å

substituting the values in eq.(4.13), we get

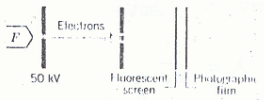
$$\lambda = 2.15\sin 50° = 1.65 \text{ Å}$$

The excellent agreement between the calculated de Broglie wavelength for 54 volt electrons and the Bragg condition illustrated by the above examples was immediately accepted as dramatic evidence for the wave nature of electrons. Hence, Davisson and Germer's experiment verified de Broglie's hypothesis, expressed as:

$$\lambda = \frac{h}{mv}$$

For this experimental work, Davisson received the Nobel prize in 1937, with G.P. Thomson, who obtained simultaneous proof as explained in section 4.6.

The wave nature of electrons can also be demonstrated by Young's double-slit experiment. It was done in 1961 by Claus Jonsson, who accelerated a beam of electrons through 50,000 V and passed it through a double slit separation 2.0×10^{-6} m and width 0.5×10^{-6} m. As shown in fig.



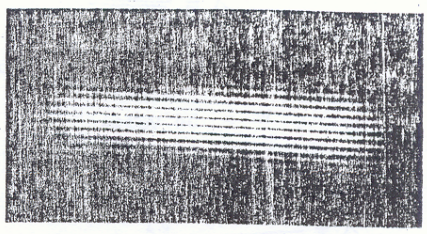


Fig.4-8: Double-slit diffraction with electrons. They produce a visible pattern when they strike a fluorescent screen (like a TV screen) and the resulting visual pattern is photographed.

4.6 ELECTRON DIFFRACTION EXPERIMENT OF G.P. THOMSON

In 1928 G.P. Thomson in Scotland extended the research on electron waves to high speed electrons. Davisson and Germer used slow electrons, but G.P. Thomson used fast electrons having energies up to 60,000 electron volts. He used a fine beam of electrons. This beam was transmitted through a thin film of gold. The gold foil was of the order of 10^{-8} m. Gold consists of a large number of micro crystals oriented at random. The electrons, diffracted by crystals making the same glancing angle, will move in a cone and produce a circular ring on the photographic plate.

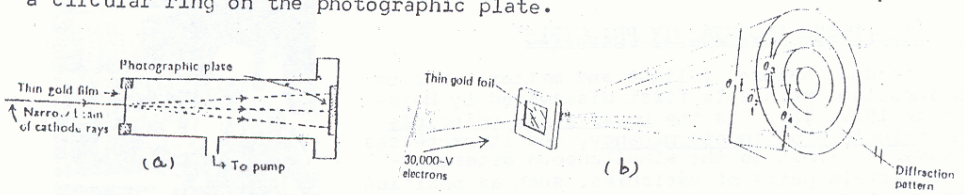


Fig.4-9: Diffraction pattern produced by passing high-speed electrons through a thin gold foil.

Several rings were formed on the plate for different values of n and θ in the Bragg equation

$$2\,d\,\sin\theta = n\lambda$$

G.P. Thomson calculated λ from the de Broglie formula

$$\lambda = \frac{h}{mv}$$

He measured θ for different values of n, and then calculated d from the Bragg equation.

This value of d , the inter-planar spacing was found to agree with the experimental value found by means of X-rays. De Broglie's law was verified again by G.P. Thomson's experiment.

4.7 PRINCIPLE OF COMPLEMENTARITY

Since photons and electrons both exhibit particle and wave characteristics, it would seem that there is really no distinction to be made between particles and waves. However, the wave nature of photons still does not give a satisfactory description of such experiments as Compton scattering or the photoelectric effect, and particles such as electrons can never travel at the speed of light.

To resolve the apparent wave-particle contradictions, Niels Bohr suggested the principle of complementarity. This states that neither waves nor particles will ever exhibit <u>both</u> their wave and particle characters in any single experiment. A complete description of either radiation or electrons requires in either case that both the wave and particle features be used--but each in its own sphere of application.

4.8 INTERPRETATION OF BOHR'S QUANTIZATION RULES

We can relate de Broglie's hypothesis with Bohr's theory.

Imagine standing waves in a stretched string. Similarly, an electron revolving in the nth orbit of radius r_n can be regarded as n "standing waves" (see fig.4-2), so that we have the relation

$$2\pi r_n = \text{length of orbit} = n\lambda \qquad .....(4.14)$$

from eq. (4.5), we have $\lambda = \frac{h}{mv}$

so eq.(4.14) becomes $2\pi r_n = n \frac{h}{mv}$ or $mvr_n = n \cdot \frac{h}{2\pi}$

which is Bohr's quantum condition. So we see that de Broglie's hypothesis agrees with Bohr's theory. De Broglie's hypothesis was experimentally verified by Davisson and Germer in 1927 and by G.P. Thomson in 1928, by studing the diffraction of electrons.

4.9 HEISENBERG UNCERTAINTY PRINCIPLE

The dual nature of light and matter leads us to an important principle first discovered by Heisenberg in 1927. This is the uncertainty principle or the principle of indeterminancy, and it expresses a fundamental limit to the simultaneous determination of certain pairs of variables, such as position and momentum.

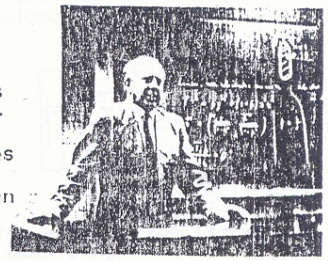


Fig.4-10: In 1932 Heisenberg received the Nobel Prize in Physics for his work in quantum mechanics.

Suppose some one starts thinking of designing super microscope to see an electron and take observations to know its position and momentum at a particular instant. The question is whether it is possible to make such observations. We shall see below that it is fundamentally impossible to make such observations even if one succeeds to construct an ideal instrument for this purpose.

To see an electron, we use a light of wavelength λ. The light consists of photons---each having a momentum h/λ. When one of these photons hits the electron, the photon will be scattered and the original momentum of the electron will be changed.

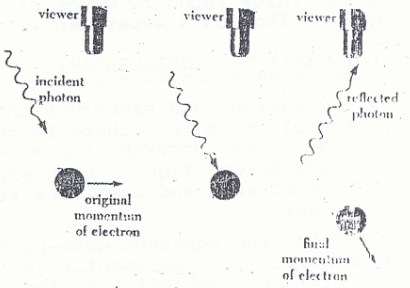


Fig. 4-11: An electron cannot be observed without changing its momentum.

One cannot calculate the exact change in momentum (Δmv) of the electron. But one can say that change of momentum of the electron will be of the same order as the momentum of the photon itself (h/λ)

so

$$\Delta p = \Delta(mv) \approx \frac{h}{\lambda} \qquad \ldots\ldots (4.15)$$

This equation gives the uncertainty in the momentum. In order to reduce the uncertainty in momentum, one must use light of large wavelength.

The position of the electron is given by the co-ordinate x, along which p is measured. There is also some error or uncertainty in the measurement of x. This error is denoted by Δx. The boundary of an object becomes blurred or indistinct when the size of the object is small and of the order of the wavelength of light used. Thus uncertainty Δx will be of the order of λ, so

$$\Delta x \approx \lambda \qquad \ldots\ldots (4.16)$$

So to reduce the uncertainty in position, one must use light of short wavelength. If one uses light of short wavelength, the accuracy in position measurement increases but the accuracy in the measurement of momentum decreases. On the other hand if one uses light of large wavelength, the accuracy in momentum measurement increases but the the accuracy in position measurement decreases. Multiplying the two equations,

$$\Delta p \cdot \Delta x \approx h \qquad \ldots\ldots (4.17)$$

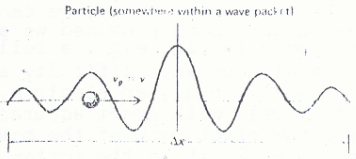


Fig.4-12: A moving particle is represented in a wave packet.

Eq.(4.17) is one form of the uncertainty principle. According to this principle, the product of the uncertainty Δp in the momentum of a body at some instant and the uncertainty in its position Δx at the same instant is approximately equal to Planck's constant. This means that it is impossible to measure the position and momentum of the electron simultaneously with perfect accuracy even with an ideal instrument.

A more sophisticated approach will show that

$$\Delta p \cdot \Delta x \geqq \hbar \qquad \ldots\ldots (4.18)$$

where $\hbar = \frac{h}{2\pi}$ and is called "h-bar".

4.10 ANOTHER FORM OF THE UNCERTAINTY PRINCIPLE

The energy E of a photon of frequency ν is given by

$$E = h\nu$$

$$\text{or } \Delta E \approx h\Delta\nu \qquad \ldots\ldots (4.19)$$

If the frequency of the wave is to be measured, the least time of measurement will be the time required for one complete wavelength to pass a reference point. This time, the time of passage of one complete cycle, related to the frequency is

$$\Delta t \approx \frac{1}{\Delta\nu} \qquad \ldots\ldots (4.20) \qquad \left[T = \frac{1}{\nu}\right.$$

combining the relations (4.19) and (4.20), we get

$$\Delta E \cdot \Delta t \approx h \qquad \ldots\ldots (4.21)$$

which is another form of the uncertainty principle. Eq. (4.21) states that the product of the uncertainty in a measured quantity of energy and the interval of time during which it is measured is approximately equal to Planck's constant. This means that the more precise we are in our measurement of time (i.e. the smaller Δt), the less precise we will be in the determination of energy.

The uncertainty principle also tells us why it is possible for both light and matter to have a dual nature (wave and particle). If one devises an experiment to force an electron to reveal its wave nature, its particle character will become fuzzy. And if we want to see its particle character, the wave nature becomes blurred. Its like a coin. One can see one face of the coin at a time.

In Newtonian mechanics, we can predict the position and momentum of a body at any future time, provided we are given its present position and momentum. If the body is big, e.g., a rolling ball, we can measure its position and momentum without any difficulty and also with a great accuracy. But if the body is small, e.g., an electron, we cannot measure its position and momentum simultaneously with great accuracy. Our errors become so large that we cannot observe the exact path of the moving body. So Newtonian mechanics fails when we try to apply it to the motion of an electron in its atom.

Example:
Estimate the minimum velocity of a billiard ball (m = 100 gm) confined to a billiard table of dimension 1 m.

Solution:

We have

$$\Delta x \sim 1 \text{ m}$$

$$h = 6.63 \times 10^{-34} \text{ J.s}$$

$$\Delta p \cdot \Delta x \approx h$$

$$\Delta p \approx \frac{h}{\Delta x}$$

$$= \frac{6.63 \times 10^{-34}}{1} = 6.63 \times 10^{-34} \text{ kg.m/s}$$

$$\text{so} \quad \Delta v = \frac{\Delta p}{m} = \frac{6.63 \times 10^{-34} \text{ kg.m/s}}{0.1 \text{ kg}} = 6.63 \times 10^{-33} \text{ m/s}$$

So quantum effects cause motion of the billiard ball with a speed the order of 6.6×10^{-33} m/s. At this speed, the ball would move a distance of 1/100 of the diameter of an atomic nucleus in a time equal to the age of the universe! Once again, we see that quantum effects are not observable with macroscopic objects.

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*4.11 BOHR'S THEORY AND THE UNCERTAINTY PRINCIPLE

Bohr assumed that an electron moves round the nucleus of an atom just like a planet moves round the sun.

Lets apply the uncertainty principle to observe the motion of the electron in the atom.

Let

Δx = uncertainty in position

Δp = uncertainty in momentum

We have

$$\Delta p \cdot \Delta x \approx h \qquad \ldots\ldots (4.22)$$

We can decrease Δx by increasing Δp.

The maximum value of Δp cannot exceed p.

$$\text{put} \quad \Delta p = p \qquad \ldots\ldots (4.23)$$

from equations (4.22) and (4.23), we get

$$p \cdot \Delta x \approx h$$

$$\text{or} \quad \Delta x \approx \frac{h}{p}$$

but $\frac{h}{p} = \lambda$ (de Broglie's wavelength)

so $\Delta x \approx \lambda$ .....(4.24)

If the electron is moving in the first orbit of radius r, then

$\lambda = 2\pi r$ ..... (4.25)

from equations (4.24) and (4.25), we get

$\Delta x \approx 2\pi r$ ..... (4.26)

It means the minimum value of Δx is larger than the radius of the orbit.

So it is impossible to know the exact position of the electron in its orbit. This means that we cannot verify experimentally Bohr's theory.

4.12 ELECTRONS CANNOT EXIST IN THE NUCLEUS

Thomson originally pictured the atom as one big positive mass embedded throughout with small electrons. Here it had no nucleus. Then from the efforts of Rutherford and Bohr, it was seen to have a massive, positively charged nucleus about which revolved electrons. Until 1931 it was generally believed that electrons did exist in the nucleus. With the discovery of neutron by Chadwick in 1932, a new theory was proposed in which the presence of nuclear electrons was denied.

Rutherford's scattering experiments (see Section 3.4) showed the size of the nucleus to be of the order of 10^{-14}m. The uncertainty in position of an electron in the nucleus is

$\Delta x = 10^{-14}$ m

$h = 6.63 \times 10^{-34}$ J.s

from the uncertainty principle

$\Delta p \cdot \Delta x \approx h$

the uncertainty in momentum is

$$\Delta p \approx \frac{h}{\Delta x} = \frac{6.63 \times 10^{-34}\ \text{J.s}}{10^{-14}\ \text{m}} = 6.63 \times 10^{-20}\ \text{kg.m/s}$$

The momentum of the electron must be at least this large.

Now $E = mc^2 = mc.c = p.c$

so kinetic energy will be

$$\Delta p \cdot c = (6.63 \times 10^{-20}\ \text{kg-m/s})(3 \times 10^{8}\ \text{m/s}) = 1.9 \times 10^{-11}\ \text{J} = \frac{1.9 \times 10^{-11}}{1.6 \times 10^{-19}} = 1.1 \times 10^{8}\ \text{eV}$$
$$= 110 \times 10^{6}\ \text{eV} = 110\ \text{MeV}$$

The energies of electrons associated with unstable atoms actually observed in β-decay, are only of the order of several electron volts. So we conclude that electrons cannot exist in the nucleus of an atom.

QUESTIONS

4.1 What is de Broglie wavelength? Obtain the equation for the wavelength of such waves.

4.2 Describe Davisson and Germer experiment for the study of the diffraction of electrons. Explain the procedure and the results obtained.

4.3 Describe the experiment of G.P. Thomson on the diffraction of electrons.

4.4 Write an essay on the wave nature of matter.

4.5 Describe the facts leading to wave particle duality.

4.6 When an electron moves with a certain de Broglie wavelength, does the electron shake back and forth or up and down at that wavelength?

4.7 What difficulties does the uncertainty principle cause in trying to pick up an electron with a pair of forceps?

4.8 Does the Uncertainty principle apply to nature itself or only to the results of experiments?

PROBLEMS

4-1 Compute the de Broglie wavelength of the following:

a) A 1000 kg automobile travelling at 100 m/s,
b) A 10 gm bullet travelling at 500 m/s,
c) An electron with a kinetic energy of 1 eV.

Soln: a) We have $\lambda = \frac{\hbar}{p} = \frac{h}{mv} = \frac{6.63\times10^{-34}}{10^3\times100} = 6.6\times10^{-39}\ m$

b) $\lambda = \frac{h}{mv} = \frac{6.63\times10^{-34}}{10^{-2}\times500} = 1.3\times10^{-34}\ m$

c) We have $K.E. = K_e = \frac{1}{2}mv^2$

or $mK_e = \frac{1}{2}m^2v^2$

or $\sqrt{2mK_e} = mv = p$

[$1eV = 1.6\times10^{-19}\ J$

so $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK_e}} = \frac{6.63\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1\times1.6\times10^{-19}}} = 12\ \text{Å}$

4-2 A charged pi meson has a rest energy of 140 MeV and a lifetime of 26 ns. Find the energy uncertainty of the pi meson.

Soln: We have

$\Delta E \sim \hbar/\Delta t$

or $\Delta E = \frac{\hbar}{\Delta t} = \frac{6.58\times10^{-16}}{26\times10^{-9}} = 2.5\times10^{-14}\ MeV.$

4-3 What is the uncertainty in the location of a photon if its wavelength of 5000 Å is known to an accuracy of one part in 10^7?

Soln: We have $p = \frac{h}{\lambda}$

$$\therefore |\Delta p| = \left| -\frac{h}{\lambda^2} \Delta\lambda \right|$$

If this momentum is in the x-direction, then the uncertainty in the x-position is

$$\Delta x \geq \frac{h}{|\Delta p|} = \frac{\lambda^2}{|\Delta\lambda|}$$

we have $\lambda = 5\times10^{-7}$ m & $\Delta\lambda = \frac{5\times10^{-7}}{10^7} = 5\times10^{-14}$ m

$$\text{So } \Delta x \geq \frac{(5\times10^{-7})^2}{5\times10^{-14}} = 5\text{ m}.$$

4-4 If the life-time of an electronic excited state is 1×10^{-9} s, what is the uncertainty in the energy of the state?

Soln: We have $\Delta E \geq \frac{h}{\Delta t} = \frac{6.63\times10^{-34}}{1\times10^{-9}} = 6.6\times10^{-25}$ J $= 4.1\times10^{-6}$ eV.

UNSOLVED PROBLEMS

4:1 Calculate the wavelength associated with 15,000 eV.

4:2 Find the deBroglie wavelength of

a) A 6-MeV proton, b) An electron moving with $v = 10^6$ m/s.

4:3 In the Davisson-Germer experiment, at what angle would the reflected electron beam appear if the accelerating voltage had been 105 volts instead 54 volts.

4:4 The speed of an electron is measured to within an uncertainty of 2.0×10^4 m/s. In how large a region of space is the electron likely to be found?

4:5 Find the deBroglie wavelength of a 2 kg object whose speed is 1 m/sec.

4:6 A pulsed ruby laser with an output of 2.0 GW produces a pulse of 10 p sec duration. What is the relative uncertainty in the measurement of the laser energy?

4:7 Determine the wavelength associated with an electron moving at

a) 0.80 c, b) 0.90 c.

4:8 An electron's position along the x-axis is known to within 10 Å. What is the minimum uncertainty in its momentum.

Chapter 5

INTRODUCTION

The phenomena of interference and diffraction were satisfactorily explained by the Wave Theory of light but at the same time the Wave Theory was unable to explain photoelectric effect and Comptom effect. While Quantum Theory of light explained the latter two and could not explain the former ones. From these results it was said that light possesses dual nature. Sometimes it behaves like particles and at others it behaves like waves.(Particle is an object with a definite position in space and waves are periodically repeated pattern in space.)

In 1924 a French Physicist Louis de Broglie came with the hypothesis that not only light but all matter possess this wave-particle duality.

5-1 DE BROGLIE'S RELATION

He proposed that with each particle of momentum $p = mv$, there was associated a wave of wavelength

$$\lambda = \frac{h}{p} \quad .....(5.1)$$

Many experiments showed agreement with this result. The key experimental fact about atomic spectra is, however, that they consist mainly of sharp distinct frequencies and this fact is adequately explained only when one does not consider electrons to be only particles but also to have a wave nature as well.

$S = vt$
or $\lambda = cT$ $(T = 1/\vartheta$
so $\lambda = c/\vartheta$
also $E = mc^2$ & $E = h\vartheta$
or $mc = E/c = h\vartheta/c$
or $p = h\vartheta/c$
or $p = h/\lambda$ $(\vartheta/c = 1/\lambda)$
or $\lambda = h/p$

5-2 QUANTUM MECHANICS

Although the simple relationship $\lambda = h/p$ quantitatively explains the scattering experiments and Bohr atom model accounts for certain features of H_2 spectrum, these theories are completely inadequate to account for a host of other observations. What is needed is a general theory---one which can be systematically applied to many different types of problems.

In short period from 1925-1928, Heisenberg, Shrodinger, Born, Dirac and many others laid foundation of what is one of the greatest theory of all times, the theory of quantum mechanics. In generality and in range of application it is unsurpassed.

5-3 MATTER WAVES

Just as in electricity one is accustomed to the idea of a field surrounding a charged particle. The electric field of a charged pith ball is not completely localized but spreads throughout space. We think of the field and the charged particle as being insaparable, i.e., as two aspects of the same entity.

Similarly, the basic experiments in Physics can be explained quantitatively by assuming that with each bit of matter there is associated a new type of field represented by the symbol Ψ and that this field has a wavelike character. The wavelength of these waves in free space is given by the de Broglie's relation

$$\lambda = h/mv \qquad [\lambda = h/p = h/mv \; ; \; p = mv]$$

These Ψ waves are intimately associated with the particle to which they belong, and the behaviour of the particles is found to be predictable only with the knowledge of what the Ψ -waves are doing at any instant.

First, the basic postulates of quantum mechanics give rules for calculating for any system the complete wave function $\Psi(x,y,z,t)$. Second, they tell us how to calculate the expected value of observable quantities with the aid of the Ψ-function. The theory can not predict a detailed behaviour of individual systems but only the average behaviour of a large number of systems.

Like the E and B fields of electricity, the Ψ-function is not itself directly observable. It is a tool for calculation. Since it gives results that are in agreement with experiments it has a certain degree of reality.

The Ψ (or $\Psi^*\Psi$) function is the only contact the macroscopic world has with the microscopic world.

5-4 HARMONIC PLANE WAVE

A harmonic plane wave travelling in the positive x-direction may be represented mathematically by either

$$\left.\begin{aligned}\Psi(0,t) &= A\cos\omega t\\ \text{or}\quad \Psi(0,t) &= A\sin\omega t\end{aligned}\right\} \qquad .....(5.2)$$

where A is a constant and denotes the amplitude.

The wave-function at the point x will be given by

$$\left.\begin{aligned}\Psi(x,t) &= A\cos(kx-\omega t)\\ \text{or}\quad \Psi(x,t) &= A\sin(kx-\omega t)\end{aligned}\right\}^{*} \qquad .....(5.3)$$

* See eq. (2.4) in the book 'Waves & Oscillations'

However, it is more convenient to use the exponential form,

$$\Psi(x,t) = A\,e^{i(kx-\omega t)} \qquad .....(5.4)$$

which is equivalent to

$$\Psi(x,t) = A\cos(kx-\omega t) + iA\sin(kx-\omega t) \qquad .....(5.5)$$

Note that either the real part or the imaginary part of eq.(5.5) should be used to represent a wave since eq.(5.5) corresponds to both functions in eq.(5.3)

5-5 DYNAMICAL VARIABLES

In classical mechanics (non-relativistic, non-quantum) one is accustomed to working with the distance x, momentum p, the total energy E etc. These are examples of quantities called dynamical variables (i.e. the quantities which change with the applied force).

In quantum mechanics the dynamical variables play a completely new role. They are converted by a set of rules into mathematical operators, which then operate on the wave function .

5-6 OPERATORS

An operator, in general, is any expression that acts on a function within some domain to produce new values occupying a given range.

For example, the number 2, acting as a multiplier, transforms every value in the domain of a function into twice that value in the range. It is then correct to say that 2 is an arithmetic operator.

The differential operator d/dx applied to a function $f(x)$ in the usual sense transforms every value of $f(x)$ into the value of its derivative

$$(d/dx)f(x) = f'(x)$$

The time-independent Shrodinger Equation can be written as:(see eq.5.21)

$$(-\hbar^2/2m\ d^2/dx^2 + V)\psi = E\psi$$

The full expression in parentheses on the left hand side can now be defined as an operator,

$$H = -\hbar^2/2m\ d^2/dx^2 + V \qquad .....(5.6)$$

called the Hamiltonian operator.

5-7 EIGENFUNCTIONS

If an operator when operated upon a function reproduces the function multiplied with a constant, then the function is called an eigen-function and the constant is called eigen-value.

Examples:

i) $d^2/dx^2(\cos 5x) = d/dx(-5 \sin 5x) = -25 \cos 5x$

here d^2/dx^2 is operator
$\cos 5x$ " eigen-function
-25 " eigen-value

ii) $d/dx\ (e^{7x}) = 7\ e^{7x}$

Here, d/dx is operator
e^{7x} " eigen-function
7 " eigen-value

iii) In general:

Let A be an operator operating upon Ψ ,
and if

$$A\ \Psi = \lambda\Psi$$

then Ψ is the eigen funtion
λ " " eigen value

and the equation is called an eigen value equation.

5-8 THE SCHRODINGER EQUATION

According to Planck's hypothesis, if a particle is oscillating with a frequency ν , its energy can have only the values

$$E = nh\nu\ ,\quad n = 0,1,2,3,4,.....$$

for a quantum of energy, we have

$$E = h\nu \qquad .....(5.7)$$

Erwin Schrodinger approached the wave-particle duality of nature by adopting the de Broglie and Planck relationships, (equations (5.1) & (5.7)) and from classical expression, defining the total energy of a particle by

$$E = T + V = p^2/2m + V$$

$$\text{or } E = p^2/2m + V \qquad .....(5.8)$$

$$\left[\lambda = \frac{h}{p}\right.$$
$$\left.\nu = E/h\right]$$

where E is total energy; V is potential energy ; p is linear momentum;
$T = \frac{1}{2}mv^2 = p^2/2m$ is (classical) kinetic energy, $(p = mv)$

Equation (5.8) is converted into a wave equation by the following substitution of operators for dynamical variables.(see article 5-5)

Dynamical variable (classical)	$\longrightarrow$	Operator (Quantum)
E	$\longrightarrow$	$i\hbar\frac{\partial}{\partial t}$ $\left[\hbar = h/2\pi\right]$*
p	$\longrightarrow$	$-i\hbar\vec{\nabla}\left[\vec{\nabla} = \frac{\partial}{\partial x}\underline{i}+\frac{\partial}{\partial y}\underline{j}+\frac{\partial}{\partial z}\underline{k}\right]$
V	$\longrightarrow$	V ...(5.9)
x	$\longrightarrow$	x

In classical mechanics x is a variable but in quantum mechanics it is a multiplication operator. And V, potential energy of a system in both the mechanics is same, and it never involves derivative.

Now $$p^2 = \underline{p}\cdot\underline{p} = (-i\hbar\vec{\nabla})(-i\hbar\vec{\nabla})$$
$$= -\hbar^2\vec{\nabla}\vec{\nabla}$$
$$= -\hbar^2\nabla^2$$

so eq.(5.8) becomes,
$$i\hbar\frac{\partial}{\partial t} = \frac{-\hbar^2\nabla^2}{2m} + V$$

Let this operator equation operates on the wave function Ψ (see article5-11)

So $$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2\nabla^2}{2m}\Psi + V\Psi \qquad .....(5.10)$$

or in one dimension:
$$i\hbar\frac{\partial\Psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V\Psi(x,t) \quad ...(5.11)$$

is known as the Schrodinger equation.

Although Schrodinger developed this eq.(5.10) from an insight into the wave character of matter, it is not derivable from first principles. This equation, like Newton's second law, is itself a first principle.

[$F = ma = m\,dv/dt = m\,d^2r/dt^2$ (a second-order, linear differential equation)

This particular form of the equation(5.10) is known as the time-dependent Schrodinger equation, because the potential energy is general enough to be a function of both position and time.

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*The combination $h/2\pi$ occurs frequently in wave mechanics and is given the special symbol $\hbar$ ("h-bar").

+ ∇ is called 'del' .

5-9 REQUIREMENTS FOR THE WAVE FUNCTION

There are certain requirements that must be set for the wave function to be a useful tool in describing the physical world. The wave function as a solution of the Schrodinger equation must meet the following requirements.

1. It must be consistent with the following relationships:

$\lambda = h/p$ ...(5.1) , $\quad \nu = E/h$ ...(5.7)

$E = p^2/2m + V$ ...(5.8)

2. It must be linear in $\Psi(x,t)$, so that the waves have the superposition property we expect of well-behaved waves. That is, if

$$\Psi_1(x,t),\ \Psi_2(x,t),\ \Psi_3(x,t),\ \ldots\ldots\ \Psi_n(x,t)$$

are solutions of the Schrodinger equation, then

$$\Psi(x,t) = a_1\Psi_1 + a_2\Psi_2 + \cdots + a_n\Psi_n = \sum_{i=1}^{n} a_i\Psi_i$$

where $a_1, a_2, \ldots\ldots a_n$ are constants, must also be a solution.

3. The function $\partial\Psi(x,t)/\partial x$ must also be linear.

4. The function $\Psi(x,t)$, as well as its derivative $\partial\Psi(x,t)/\partial x$, must be well behaved; that is, it satisfies

i) Standard boundary condition i.e. $U(x)$ finite as $x \to \pm\infty$

We expect our solution to tell us something about the probability of finding the particle.

ii) Standard continuity conditions i.e. $U(x)$ and $d/dx\ U(x)$ are both continuous for all real x. We should be very surprised to find, for example, the probability changing discontinuously, with the particle suddenly disappearing at one point in space and reappearing at another.

iii) Not identically zero, i.e., single valued, there ought not to be two different probabilities for finding the particle at the same point in space.

5. When $x \to \pm\infty$, then $\Psi(x,t)$ must approach zero:

$$\lim_{x\to\pm\infty} \Psi(x,t) \to 0$$

5-10 THE TIME-INDEPENDENT SCHRODINGER EQUATION

To find a wave equation in which the potential energy of the system does not depend upon time, we proceed as follows:

Consider Time-dependent Schrodinger wave equation in one dimension.

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V(x,t)\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t} \qquad \ldots\ldots(5.11)$$

since V does not depend upon time, therefore, instead of $V(x,t)$, we will have $V(x)$, so eq.(5.11) becomes

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V(x)\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t} \qquad \ldots\ldots(5.12)$$

To solve this equation, we assume that the solution $\Psi(x,t)$ can be expressed as a product of the functions of two independent variables of space and time co-ordinates, i.e.

Let $\Psi(x,t) = \psi(x)\phi(t)$ $\ldots\ldots(5.13)$

taking partial derivatives

$$\frac{\partial\Psi}{\partial t}(x,t) = \psi(x)\, d\phi/dt \qquad \ldots\ldots(5.14)$$

$$\frac{\partial^2\Psi}{\partial x^2}(x,t) = \phi(t)\, d^2\psi/dx^2(x) \qquad \ldots\ldots(5.15)$$

In eq.(5.12), making the substitution from eqs.(5.13), (5.14), & (5.15),

$$-\frac{\hbar^2}{2m}\phi(t)\frac{d^2\psi(x)}{dx^2} + V(x)\phi(t)\psi(x) = i\hbar\psi(x)\frac{d\phi(t)}{dt} \qquad \ldots\ldots(5.16)$$

dividing throughout by $\phi(t)\psi(x)$,

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x) = i\hbar\frac{d\phi/dt}{\phi(t)} \qquad \ldots\ldots(5.17)$$

In this equation(5.17), Right Hand Side is a function of t only and Left Hand Side is a function of space coordinates only. Therefore both sides should individually be equal to some constant called constant of separation, E (as L.H.S. is sum of K.E. and P.E.)

So we will have

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x) = E \qquad \text{.....(5.18)}$$

and

$$i\hbar \frac{d\phi/dt}{\phi(t)} = E \qquad \text{.....(5.19)}$$

from eq.(5.19)

$$i\hbar \frac{d\phi}{\phi} = E\, dt$$

$$\frac{d\phi}{\phi} = E/i\hbar\, dt = -iE/\hbar\, dt$$

$$\left[\frac{E}{i\hbar}dt = \frac{i}{i}\times\frac{E}{i\hbar}dt = \frac{iE\,dt}{-1\,\hbar} = -\frac{iE}{\hbar}dt\right.$$

$$\left.\int dx = x \qquad \int \frac{1}{x}dx = \log x\right]$$

Integrating

$$\log\phi = -iE/\hbar\, t$$

taking anti-log.

$$\phi = e^{-\frac{iE}{\hbar}t} \qquad \text{.....(5.20)}$$

multiplying both sides of eq.(5.18) by $-\frac{2m}{\hbar^2}\psi$

$$\frac{d^2\psi}{dx^2} - \frac{2m}{\hbar^2}V\psi = -\frac{2m}{\hbar^2}E\psi \qquad \text{.....(5.21)}$$

or

$$\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0 \qquad \text{.....(5.22)}$$

which is the "steady state" or <u>amplitude equation</u>, or <u>time-independent Schrodinger equation</u>.

Now from eq.(5.13) $\Psi = \psi\phi$

and from eq.(5.20) $\phi = e^{-\frac{iE}{\hbar}t}$

therefore

$$\Psi = \psi e^{-\frac{iE}{\hbar}t} \qquad \text{.....(5.23)}$$

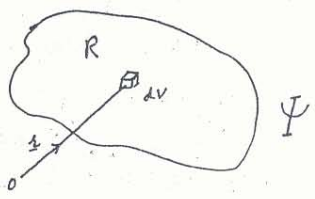
where ψ is given by eq.(5.22)

Eq.(5.23) is the solution of the Schrodinger Wave Equation when potential energy does not depend upon time.

5-11 INTERPRETATION OF Ψ

Schrodinger's own interpretation was rejected and, instead, Max Born (a German Physicist) gave an interpretation which was universally acknowledged.

If we consider a particle (in the region R) which is in motion and Ψ is the wave function associated with this particle. And $\underline{r}$ be the position vector of this point at any time t. Then, Ψ can be represented as:



$$\Psi(x,y,z,t) \quad \text{or} \quad \Psi(\underline{r},t)$$

and the complex conjugate be taken (i.e. all immaginary quantities change their signs in this process), and $\Psi^*(\underline{r},t)$ be the complex conjugate of Ψ.

Then according to Max Born $\Psi^*(\underline{r},t)\Psi(\underline{r},t)\,dV$ is the probability of finding the particle at point r and time t in volume dV. and $\Psi(r,t)$ the wave function

The intensity of a wave is proportional to the square of the amplitude of the wave. Hence, the intensity of the matter-field associated with a particle is proportional to the square of the matter-field amplitude $\Psi(\underline{r},t)$

$$|\Psi(\underline{r},t)|^2 = \Psi^*\Psi \qquad .....(5.24)$$

in which Ψ^* is the complex conjugate of Ψ.

Since the wave function can be a complex function (it can contain complex numbers of the form $a + ib$, where $i = \sqrt{-1}$)

Classical example:

$$\text{Area} = A^2$$

$(a + ib)$ is a complex number, and $(a - ib)$ is the complex conjugate of it, so

$$(a + ib)(a - ib) = a^2 + b^2$$

5-12 PROPERTIES OF WAVE FUNCTIONS

What is the physical significance of the wave function? What feature of the particle character of matter can be measured by the wave function ? The wave function will be found to be as much a physical quantity as the electric field or the magnetic field. The wave function must describe something about the location of the particle in the space-time universe, as the particle is more likely to be located in those places where intensity is large.

1. Probability Distribution

If Ψ is the wave function representing the particle, then $|\Psi|^2$ is proportional to the probability per unit length of finding the particle at a given point in space and a given time. The probability of finding the particle within an element of length dx is

$$\Psi^*\Psi\, dx \qquad .....(5.25)$$

Probability density is the probability of finding the particle in unit length $= \Psi^*\Psi$

2. Normalization

The expression (5.25) is said to be normalized if

$$\int_{-\infty}^{\infty} \Psi^*\Psi\, dx = 1 \qquad .....(5.26)$$

since the probability of finding the particle somewhere must be 1 (this represents certainty).

Example:

Show that $U = \sqrt{\frac{2}{\ell}} \sin\frac{\pi x}{\ell}$ is normalized in the interval $(0,\ell)$.

Now $U^* = U$ (it being a real quantity)

$$\therefore \int U^*U\, dx = \int_0^\ell U^2 dx$$

$$= \int_0^\ell \frac{2}{\ell}\sin^2\frac{\pi x}{\ell} = \frac{1}{\ell}\int_0^\ell \left(1 - \cos\frac{2\pi x}{\ell}\right) dx$$

$$= \frac{1}{\ell}\left[x - \frac{1}{2\pi/\ell}\sin\frac{2\pi x}{\ell}\right]_0^\ell = \frac{1}{\ell}\left[\ell - \frac{\ell}{2\pi}\sin\frac{2\pi\ell}{\ell}\right]$$

$$= \frac{1}{\ell}\cdot\ell = 1$$

Hence U is normalized in $(0,1)$.

$\int c\, f(x)\, dx = c\int f(x)\, dx$

$\int 1\cdot dx = x$

$2\sin^2\theta = 1 - \cos 2\theta$

$\int \cos x\, dx = \sin x$

$\int \cos(ax)\, dx = \frac{1}{a}\sin ax$

$\int [f(x) + g(x)]\, dx = \int f(x)\, dx + \int g(x)\, d$

$\int_a^b f(x)\, dx = F(b) - F(a) = \left[F(x)\right]_a^b$

3. Orthogonality

Consider two functions $U_1(x)$ and $U_2(x)$. If

$$\int_a^b U_1^*(x)\, U_2(x)\, dx = 0 \qquad .....(5.27)$$

Then $U_1(x)$ is said to be orthogonal to $U_2(x)$, in the interval (a,b).

Taking complex conjugate of the whole quantity, we have

$$\int_a^b U_1^*(x)\, U_2^*(x)\, dx = 0$$

or $$\int_a^b U_1(x)\, U_2^*(x)\, dx = 0$$

that is, $U_2(x)$ is orthogonal to $U_1(x)$, or we may say that:
$U_1(x)$ and $U_2(x)$ are orthogonal to each other in the interval (a,b).

Example:

Show that $\sin\phi$ and $\cos\phi$ are orthogonal in $(0, 2\pi)$

$$\int_0^{2\pi} \sin\phi \cos\phi\, d\phi = \tfrac{1}{2}\int_0^{2\pi} \sin 2\phi\, d\phi$$

$$= \tfrac{1}{2}\left[\frac{-\cos 2\phi}{2}\right]_0^{2\pi} = 0$$

4. Linearly Independent Functions

A set of n functions, $U_1, U_2, U_3, ..., U_n$ is said to be linearly independent if it is impossible to choose n constants $a_1, a_2, a_3, \ldots a_n$ not all of which are zero in such a way that the equation:

$$a_1u_1 + a_2u_2 + ...a_nu_n \neq 0$$

holds for all values of U's otherwise U's are said to be linearly dependent.

Examples:

i) $U_1 = 3x$, $U_2 = 7x$ be two functions

$-7U_1 + 3U_2 = 0$ $\quad\left[-7(3x) + 3(7x) = -21x + 21x = 0\right.$

so they are linearly dependent functions.

ii) sin n, cos x, 1 , be three functions

since $a_1 + a_2 \sin n + a_3 \cos x \neq 0$

they are linearly independent functions.

5-13 EXPECTATION VALUES

The expectation value is the mathematical expectation for the result of single measurement, or it is the average or mean of the results of a large number of measurements on independent systems.

Consider a container of gas. Suppose that at an instant t , n_1 number of particles are at position x_1, n_2 number of particles are at position x_2 and so on.

We may define the average value or expectation value as the arithmatic mean of all the values denoted by $\langle x \rangle$, that is,

$$\langle x \rangle = \frac{n_1x_1 + n_2x_2 + \cdots}{n_1 + n_2 + \cdots} = \sum_i \frac{n_i x_i}{n} = \sum_i P_i x_i \quad \text{where } P_i = \frac{n_i}{n}$$

for large number of particles, we can write

We have discovered that the uncertainty principle does not permit us to measure trajectories, positions, velocities, in the world of fundamental particles as is in classical mechanics. Instead, we must represent an elementary particle by a wave function, where it is understood that the wave function contains all of the information that can be known about the particle. Even then, this information is available to us only in the form of probabilities.

Considering eq.(5.25), for writing the quantum mechanical counterpart of eq.(5.28), the only choice is to place the operator between Ψ and Ψ^*.

So $$\langle x \rangle = \int \Psi^* x_{op.} \Psi \, dx$$

or in general case, the <u>expectation value</u> of any operator $A_{op.}$ is

$$\langle A_{op.} \rangle = \int \Psi^* A_{op.} \Psi dx \qquad \ldots\ldots(5.29)$$

<u>Example</u>:

The expectation value of momentum p is :

$$\langle p \rangle = \int_{-\infty}^{+\infty} \Psi^* p_{op} \Psi dx$$

$$= \int_{-\infty}^{+\infty} \Psi^* (-i\hbar \frac{\partial}{\partial x}) \Psi \, dx = -i\hbar \int_{-\infty}^{+\infty} \Psi^* \frac{\partial \Psi}{\partial x} dx$$

5-14 APPLICATIONS OF SCHRODINGER EQUATION

In classical mechanics the state of a system is defined by the position and momentum coordinates, that is, by Newton's second law of motion.

In Quantum Mechanics Ψ of the Schrodinger Wave Equation determines the <u>state</u> of a system.

If $\Psi, \frac{\partial \Psi}{\partial x}, \frac{\partial \Psi}{\partial y}, \frac{\partial \Psi}{\partial z}$ are <u>continuous</u>, <u>finite</u>, and <u>single-valued</u> throughout the configuration of space, the function is said to be <u>well behaved</u> which is a necessary condition for all the solutions of Schrodinger wave equation.

<u>Systems in One Dimension</u>

In order to illustrate some of the characteristics results of quantum mechanics without encountering too many of its characteristic difficulties, we shall first treat some one-dimensional problems, which are idealized but which neverthless have the essential features of situations encountered in practice.

The systems are, of course, somewhat artificial, but no more so than those involving an insect crawling on a massless rod or a bead sliding down a frictionless wire that one encounters in books on classical mechanics. Examples to be treated, and so the results obtained are also interesting in their own account, since, they throw some light on an analogous situation that exists in the real world.

We have the Schrodinger's time-independent equation in one dimension as given below:

$$\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2}(E - V)\Psi = 0 \qquad \ldots\ldots(5.22)$$

This equation is a linear, second order differential equation. Its exact solution can be found only for some form of potential energy operator, which the coordinate representation is a function of the coordinates. We will solve this for some one-dimensional (say x-only) systems for the following potentials: (As explained in articles 5-15, 5-16 & 5-17).

1- $V = 0$ for $-\infty < x < \infty$ } ----- The Free Particle

2- $V = 0$ for $x < 0$ }
$V = V$ for $x > 0$ } --------- The potential step

$V = 0$ for $0 < x < L$] --------- The potential well

5-15 THE MOTION OF FREE PARTICLES

We shall consider the motion of a free particle in one dimension only.

Starting from Schrodinger's Equation:

$$\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0 \qquad \text{.....(5.22)}$$

As the particle is moving freely,

$$V(x) = 0$$

So $$\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2}E\psi(x) = 0 \qquad \text{.....(5.30)}$$

since m, $\hbar^2$, and E (= T + V) are positive, so

put $$\frac{2mE}{\hbar^2} = k^2 \qquad \text{.....(5.31)}$$

now eq.(5.30) becomes:

$$\frac{d^2\psi(x)}{dx^2} + k^2\psi(x) = 0 \qquad \text{.....(5.32)}$$

or $$\left(\frac{d^2}{dx^2} + k^2\right)\psi(x) = 0$$

writing D for the symbol d/dx,

$$(D^2 + k^2)\psi(x) = 0$$

The auxiliary equation is

$$D^2 + k^2 = 0$$

or $$D = \pm ik$$

so solution of the differential equation is:

$$\psi(x) = A e^{ikx} + B e^{-ikx} \qquad \text{.....(5.33)}$$

Example:

To solve the equation, $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$

The auxiliary equation is

$D^2 - 5D + 6 = 0$

or $(D - 2)(D - 3) = 0$

therefore $D = 2$ & 3

Hence solution is

$y = c_1 e^{2x} + c_2 e^{3x}$

To determine the physical meaning of each term in eq.(5.33), we write the complete time-dependent function (from eq.(5.23))

$$\Psi(x,t) = \psi(x)\, e^{\frac{iE}{\hbar}t}$$

or $$\Psi(x,t) = e^{i\omega t}\psi(x) \qquad \text{.....(5.34)}$$

$E = \hbar\omega$ or $\omega = \frac{E}{\hbar}$

or $2\pi\nu = \frac{2\pi E}{\hbar} = \frac{E}{\hbar/2\pi} = \frac{E}{\hbar}$

or $\omega = E/\hbar$

$e^{i\theta} = \cos\theta + i\sin\theta$

where θ is phase angle.

from equations (5.33) and (5.34), we have

$$\Psi(x,t) = (A e^{ikx} + B e^{-ikx})\, e^{i\omega t}$$

or $$\Psi(x,t) = A e^{i(\omega t - kx)} + B e^{-i(\omega t + kx)}$$

or $$\Psi(x,t) = A e^{i(kx - \omega t)} + B e^{-i(kx + \omega t)} \qquad \text{.....(5.35)}$$

From the knowledge of article 5-4, it is seen that the first term $A e^{+i(kx-\omega t)}$ describes the motion of the particle in the positive direction of the x-axis, since with phase $(kx - \omega t)$, x is always positive, and the second term $B e^{i(kx+\omega t)}$ the motion in the negative direction, because the phase $-(kx + \omega t)$ gives negative x, or the wave is being reflected from ∞ where there may be a potential barrier, but it is assumed that there is no such barrier existing at infinity. Thus we have

$$\psi(x) = A e^{ikx} \qquad \text{.....(5.36)}$$

It is readily seen that the integral $\int_{-\infty}^{\infty}\psi^*\psi\, dx$ diverges.

$\psi^*\psi = A^* e^{-ikx} A e^{ikx} = A^* A = |A|^2$ and $\int_{-\infty}^{\infty}\psi^*\psi\, dx = \int_{-\infty}^{\infty}|A|^2 dx = |A|^2\int_{-\infty}^{\infty}dx = \infty$

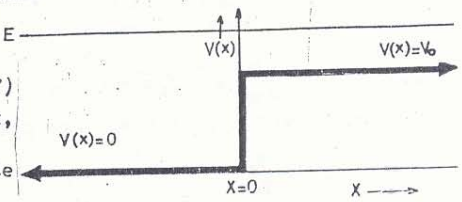
it shows that we cannot normalize a free particle, i.e., the probability of finding the particle somewhere is not certain.

5-16 THE POTENTIAL STEP (SIMPLE BARRIER)

Let us suppose that it is possible to produce a truly monoenergetic beam of electrons of energy E. The time dependence of ψ will now be simply periodic and we shall neglect it. The space dependence is given by (from eq.(5.22))

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = [E - V(x)]\psi \qquad .....(5.37)$$

(We note that if V were independent of x, this equation would be easy to solve).



Assume that it is possible to create a discontinuous potential as shown in fig.5.2.

Let a beam of electrons be incident from the left, of the form (as from eq.5.36)

$$\psi_{inc.} = A e^{ikx} \qquad .....(5.38)$$

where k is given in eq.(5.31).

We are going to find the effect of the discontinuity in potential upon this beam. It is easy to write down wave equations valid within each of the two regions of the fig.(5.2).

region 1 $$-\frac{\hbar^2}{2m}\frac{d^2\psi_1}{dx^2} = E\psi_1 \qquad .....(5.39)$$

region 2 $$-\frac{\hbar^2}{2m}\frac{d^2\psi_2}{dx^2} = (E-V)\psi_2 \qquad .....(5.40)$$

We must now distinguish between two cases, according as (E - V) is positive or negative. In the former case, classical physics asserts that the particle will penetrate region 2 and travel freely there, as though with reduced velocity; whereas if $E < V$, the particle cannot get in.

Case 1 $E > V$

The terms E and (E - V) represent the kinetic energies of electrons in the two regions, and it will be convenient to express them in terms of their associated momenta $\hbar k_1$ and $\hbar k_2$ respectively.

region 1 $$E = \frac{\hbar^2 k_1^2}{2m}, \quad \frac{d^2\psi_1}{dx^2} = -k_1^2\psi_1 \qquad .....(5.41)$$

region 2 $$E - V = \frac{\hbar^2 k_2^2}{2m}, \quad \frac{d^2\psi_2}{dx^2} = -k_2^2\psi_2 \qquad .....(5.42)$$

the solutions of which are, as done for eq.(5.32),

region 1 $$\psi_1 = A e^{ik_1x} + B e^{-ik_1x} \qquad .....(5.43)$$

region 2 $$\psi_2 = C e^{ik_2x} + D e^{-ik_2x} \qquad .....(5.44)$$

where the constants of integration, from the mathematical point of view, are arbitrary. From the physical point of view they are not arbitrary, however, since two of them can be assigned values in terms of the imagined conditions of the experiment.

We have already assumed in eq.(5.38), that

now

$$\left.\begin{aligned} \psi &= A e^{ik_1 x} && \text{is the incident wave in region 1} \\ \psi &= B e^{-ik_1 x} && \text{is the reflected wave in region 1} \\ \psi &= C e^{ik_2 x} && \text{is the transmitted wave in region 2} \\ \psi &= D e^{-ik_2 x} && \text{is the reflected wave from infinity in region 2} \end{aligned}\right\} \quad \ldots(5.45)$$

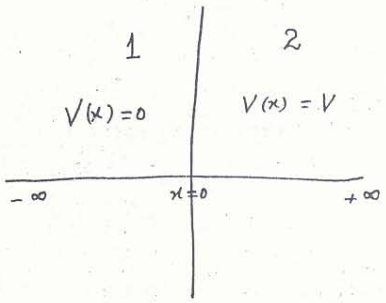
Since the last wave has no physical significance. We assume that there are none electrons incident on the apparatus from right, so we set

$D = 0$, so eq.(5.44) becomes

$$\psi_2 = C e^{ik_2 x} \qquad \ldots\ldots(5.46)$$

Case 2: $E < V$

In this case the kinetic energy to the right of the barrier would be negative, so that in classical physics the particle could not enter at all. In order to bring out the similarities with case 1, we introduce $\hbar k_1$ and a "pseudo momentum" $\hbar k_2$ analogous to the previous $\hbar k_2$ so that we have:



region 1 $E = \frac{\hbar^2 k_1^2}{2m}$, $\frac{d^2\psi_1}{dx^2} = -k_1^2 \psi_1$ ...(5.47)

region 2 $V - E = \frac{\hbar^2 k_2^2}{2m}$, $\frac{d^2\psi_2}{dx^2} = k_2^2 \psi_2$ ...(5.48)

whence

region 1 $\psi_1 = A e^{ik_1 x} + B e^{-ik_1 x}$ ...(5.49)

region 2 $\psi_2 = C e^{-k_2 x} + D e^{k_2 x}$ ...(5.50)

Here again we set $D = 0$, but on different grounds. Previously it was done so because there were no particles incident from the right, but now we do so because the wavefunction increasing towards the right is not a possible one, since it represents a divergently increasing possibility of finding a particle as one goes farther and farther into the forbidden region.

Applying the boundary conditions (to equations (5.51) & (5.55))

then to equations (5.53) & (5.57) $\psi_1(x=0) = \psi_2(x=0)$

$\psi_1'(x=0) = \psi_2'(x=0)$

[See foot note on next page for eqs (5.51) & (5.55)]

We have from equations (5.52) and (5.56)

$$A + B = C \qquad \ldots\ldots(5.59)$$

from equations (5.54) and (5.58), we have

$$ik_1(A - B) = -k_2 C$$

$$\text{or} \quad A - B = \frac{+ik_2}{k_1} C \qquad \ldots\ldots(5.60)$$

By adding equations (5.59) and (5.60), we get the solutions, in terms of the given A as:

$$2A = C + \frac{ik_2}{k_1} C = \left(1 + \frac{ik_2}{k_1}\right) C = \left(\frac{k_1 + ik_2}{k_1}\right) C$$

$$\text{or} \quad 2.k.A$$

$$\text{or} \quad C = \frac{2k_1}{k_1 + ik_2} A \qquad \text{.....(5.61)}$$

from equations (5.59) and (5.61), we have

$$A + B = \frac{2k_1}{k_1 + ik_2} A \quad \text{or} \quad B = \frac{2k_1}{k_1 + ik_2} A - A$$

$$\text{or} \quad B = \left(\frac{2k_1}{k_1 + ik_2} - 1\right) A \quad \text{or} \quad B = \left(\frac{2k_1 - k_1 - ik_2}{k_1 + ik_2}\right) A$$

$$\text{or} \quad B = \frac{k_1 - ik_2}{k_1 + ik_2} A \qquad \text{.....(5.62)}$$

From relations (5.45), we see that

A is coefficient (or amplitude) of incident wave

B is coefficient (or amplitude) of reflected wave

We define reflection coefficient, R, as the ratio of the number of reflected particles to the number of incident particles, it follows from the ratio of the reflected to the incident intensity. (see eq.5.24).

From eq.(5.62), we get

$$R = \left|\frac{B}{A}\right|^2 = \left(\frac{B}{A}\right)^* \frac{B}{A} = \frac{k_1 + ik_2}{k_1 - ik_2} \cdot \frac{k_1 - ik_2}{k_1 + ik_2} = 1$$

This shows that the reflection is total. It is natural, since the wave entering the forbidden region 2 is exponentially damped, so that everything incident is ultimately reflected.

$$\psi_1 = A e^{ik_1 x} + B e^{-ik_1 x} \qquad \text{——— (5.49)}$$

$$\psi_2 = C e^{-k_2 x} + D e^{k_2 x} \qquad \text{——— (5.50)}$$

$$\left[\begin{array}{l} e^{i\theta} = \cos\theta + i\sin\theta \\ e^{-i\theta} = \cos\theta - i\sin\theta \end{array}\right.$$

from eq.(5.49), we have

$$\psi_1 = \underline{A\cos k_1 x} + iA\sin k_1 x + \underline{B\cos k_1 x} - iB\sin k_1 x$$

$$\text{or} \quad \psi_1 = (A+B)\cos k_1 x + i(A-B)\sin k_1 x \qquad \text{——— (5.51)}$$

$$\text{At } x = 0, \quad \psi_1 = (A+B)\cos(k_1 x 0) + i(A-B)\sin(k_1 x 0) \qquad \left[\begin{array}{l} \cos(0) = 1 \\ \sin(0) = 0 \end{array}\right.$$

$$= (A+B)\cos(0) + i(A-B)\sin(0)$$

$$\text{or} \quad \psi_1 = A + B \qquad \text{——— (5.52)}$$

Now from eq.(5.49), we have

$$\frac{d\psi_1}{dx} = \psi_1' = ik_1 A e^{ik_1 x} - ik_1 B e^{-ik_1 x} \qquad \left[\frac{d}{dx} e^{ax} = a \cdot e^{ax}\right.$$

$$\text{or} \quad \psi_1' = ik_1 \left[A e^{ik_1 x} - B e^{-ik_1 x}\right]$$

$$= ik_1 \left[\underline{A\cos k_1 x} + iA\sin k_1 x - \underline{B\cos k_1 x} + iB\sin k_1 x\right]$$

$$\text{or} \quad \psi' = ik_1 \left[(A-B)\cos k_1 x + i(A+B)\sin k_1 x\right] \qquad \text{——— (5.53)}$$

$$\text{At } x = 0, \quad \psi_1' = ik_1 \left[(A-B)\cos(k_1 x 0) + i(A+B)\sin(k_1 x 0)\right]$$

$$\text{or} \quad \psi_1' = ik_1 (A-B) \qquad \text{——— (5.54)}$$

Now from eq.(5.50), we have

$$\psi_2 = C e^{i(ik_2 x)} \qquad \left[\begin{array}{l} i^2 = -1 \\ \& \ D = 0 \end{array}\right.$$

$$\text{or} \quad \psi_2 = C(\cos ik_2 x + i\sin ik_2 x) \qquad \text{——— (5.55)}$$

$$\text{At } x = 0, \quad \psi_2 = C(\cos(ik_2 x 0) + i\sin(ik_2 x 0))$$

$$\text{or} \quad \psi_2 = C \qquad \text{——— (5.56)}$$

from eq.(5.50), we have $\frac{d\psi_2}{dx} = -k_2 C e^{-k_2 x} = -k_2 C e^{i(ik_2 x)}$

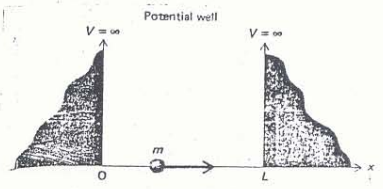
$$\text{or} \quad \psi_2' = -k_2 C(\cos ik_2 x + i\sin ik_2 x) \qquad \text{——— (5.57)}$$

At $x = 0$

5-17 SOLUTION OF ONE-DIMENSIONAL BOX (POTENTIAL WELL)

As a simple application of the steady-state Schrodinger Equation.

Consider a particle moving along a straight line OL of length l with impenetrating walls (i.e., height of the walls goes to infinity). The point O is taken as the origin and the line OL as x-axis. Now the particle of mass m is moving freely and cannot jump over the walls, that is, the motion is confined to the straight line of length l and this line is called one dimensional box because it confines the motion of the particle. We may visualize this situation as describing a particle trapped in an infinitely deep potential well, which has infinitely hard, perfectly rigid walls from which the particle rebounds elastically.



A particle of mass m is constrained to move in one dimension in a potential well. The particle rebounds perfectly elastically from the walls of the infinite potential.

Since the particle is moving freely and no external force is acting on it, therefore

$$\vec{F} = 0$$

As $F = -dV/dx$, so $dV/dx = 0$,

or $V =$ constant.

that is, when particle is in free motion its potential energy is constant. We choose such a scale of energy that

$$V = 0.$$

Since V does not depend upon time, we consider the time-independent Schrodinger equation.

$$\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0 \qquad \text{.....(5.22)}$$

$$\Psi = \psi e^{-\frac{iE}{\hbar}t} \qquad \text{.....(5.23)}$$

since $V = 0$, so eq.(5.22) is

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}E\psi = 0$$

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0$$

where $$k^2 = \frac{2m}{\hbar^2}E \qquad \text{.....(5.31)}$$

here m, $\hbar^2$, and E are positive, because V = 0, and T is always positive, since (E = T + V).

therefore k^2 is a positive quantity.

$$\begin{cases} i = \sqrt{-1} \\ e^{i\theta} = \cos\theta + i\sin\theta \\ e^{-i\theta} = \cos\theta - i\sin\theta \end{cases}$$

Now auxiliarly equation is (as done in article 5-15)

$$D^2 + k^2 = 0$$

or $$D = \pm ik$$

So the solution is $$\psi = A_1 e^{ikx} + B_1 e^{-ikx} \qquad \text{....(5.63)}$$

or $$\psi = A\cos kx + B\sin kx \qquad \text{....(5.64)}$$

$$\begin{cases} e^{ikx} = \cos kx + i\sin kx \\ e^{-ikx} = \cos kx - i\sin kx \end{cases}$$

$$\begin{aligned} \psi &= A_1(\cos kx + i\sin kx) + B_1(\cos kx - i\sin kx) \\ &= A_1\cos kx + A_1 i\sin kx + B_1\cos kx - B_1 i\sin kx \\ &= (A_1 + B_1)\cos kx + (A_1 - B_1)i\sin kx \\ &= A\cos kx + B\sin kx \end{aligned}$$

To evaluate the constants A and B, we proceed as follows:

Since the particle can not be found outside the walls, therefore, probability of finding it outside the walls is zero.

i.e. $\Psi\Psi^* = 0$ at $x = 0$ and $x = l$

or $\Psi = 0$ at $x = 0$ and $x = l$

so eq.(5.23) gives $0 = \psi e^{\frac{-iE}{\hbar}t}$

Now $e^{\frac{-iE}{\hbar}t}$ is zero only when $t \to \infty$, but that case is not considered.

Therefore $\psi = 0$

So we have two boundary conditions:

i) $x = 0$ $\quad \psi = 0$

ii) $x = l$ $\quad \psi = 0$

Imposing the Ist boundary condition on eq.(5.64), we have

$$0 = A \cdot 1 + B \cdot 0 \quad \text{or} \quad A = 0$$

so the solution reduces to

$$\psi = B \sin kx \qquad \text{.....(5.65)}$$

Imposing the IInd boundary condition on eq.(5.65)

$$0 = B \sin kl$$

Here either B is zero or sin kl is zero, but if $B = 0$, the equation disappears and the particle itself disappears.

Therefore, only result we can have from here is:

$$\sin kl = 0$$
$$= \sin 0,\ \sin(\pm\pi),\ \sin(\pm 2\pi)$$
$$= \sin n\pi \ , \quad n = 0, \pm 1, \pm 2, \ldots\ldots$$

or $kl = n\pi$

or $k = \frac{n\pi}{l}$

from eq.(5.31), $k = 0$ gives $E = 0$, which is not possible on physical grounds, therefore, $n = 0$ is rejected. Also -ve n gives no new values. So we have

$$k = \frac{n\pi}{l}, \quad n = 1, 2, 3, \ldots\ldots \qquad \text{.....(5.66)}$$

squaring both sides $k^2 = \frac{n^2\pi^2}{l^2}$

or $\frac{2mE}{\hbar^2} = \frac{n^2\pi^2}{l^2}$

$E = \frac{n^2\pi^2\hbar^2}{2ml^2}$

$E_n = \frac{n^2\hbar^2}{8ml^2}$; $n = 1, 2, 3, \ldots$ $\qquad$ .....(5.67)

$k^2 = \frac{2mE}{\hbar^2}$

$\hbar = \frac{h}{2\pi}$; $\hbar^2 = \frac{h^2}{4\pi^2}$

$k^2 = \frac{2mE}{\hbar^2}$ --- (5.31)

or $E = \frac{k^2\hbar^2}{2m}$

m and $\hbar^2$ are constants

so $k = 0$, gives

$E = \frac{0 \times \hbar^2}{2m} = 0$

The particle can have only those values of energy given by eq.(5.67). We express this by saying the energy is quantized into discrete values or levels.

Comparing eq.(5.67) with the case of a free particle, it shows that energy is being quantized, when a particle is bound to be in a finite region of space.

In eq.(5.67), giving different values to n , we get different energy values;

$$E_1 = \frac{\hbar^2}{8ml^2}$$

$$E_2 = \frac{4\hbar^2}{8ml^2}$$

$$E_3 = \frac{9\hbar^2}{8ml^2}$$

.....

.....

$$E_n = \frac{n^2\hbar^2}{8ml^2}$$

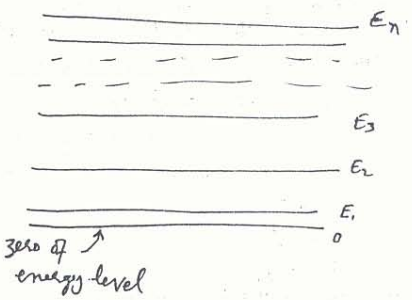
These relations show that the particle moving in one dimensional box can possess a minimum energy of $\hbar^2/8ml^2$ and not less than that.

E_1 is called <u>normal energy</u>, <u>zero point energy</u>, or <u>ground state energy level</u>. In other words, the particle cannot have a zero energy. This conclusion contradicts classical mechanics because it is a result of the uncertainty principle.

E_2, E_3, ...E_n are called <u>excited state energy levels</u>.

We can easily see that particles can not have energy between E_1, E_2, etc. because no other value of n is permissible. This is expressed by saying that energy values are <u>discrete</u> and the number 'n' is called quantum number.

<u>Energy level diagram</u>



QUESTIONS

5.1 What is the significance of wavefunction in Quantum mechanics? What should be its characteristics?

5.2 What is the significance of operators in Quantum mechanics.

5.3 Write down time-dependent Schrodinger's wave equation and separate out its spatial and time-dependent parts.

5.4 Obtain time-independent Schrodinger's equation for a bound particle in three dimensions.

5.5 Solve Schrodinger's equation for motion of a particle in one dimensional box.

5.6 Newton's laws can be solved to give the future behaviour of a particle. In what sense does the Schrodinger equation also do this? In what sense does it not?

5.7 Why is it important for a wave function to be normalized? Is an unnormalized wave function a solution to the Schrodinger equation?

5.8 What happens to the probability density in the infinite well when $n \to \infty$? Is this consistent with classical physics?

5.9 What is a wave function? Why we need to define it? Summarize the properties of wave functions.

5.10 Suppose particles were incident on the step potential from the positive x direction. Which of the four coefficients of eqs. (5.49) and (5.50) would be set to zero? Why?

PROBLEMS

5-1 Which of the following are eigen-functions of the operator d^2/dx^2? Give eigen-value when appropriate.

a) $\sin x$ b) $\sin^2 x$ c) e^{3x} d) $\sin nx$

Soln:

a) $\frac{d^2}{dx^2} \sin x = -\sin x$; the eigen-value is -1

b) $\frac{d^2}{dx^2} \sin^2 x = 2 - 4\sin^2 x$; Not an eigenfunction

c) $\frac{d^2}{dx^2} e^{3x} = 9\, e^{3x}$; the eigenvalue is $+9$

d) $\frac{d^2}{dx^2} \sin nx = -n^2 \sin nx$; the eigenvalue is $-n^2$.

5-2 Normalize the wave functions for the potential well given in eq. (5.65) i.e. $\psi = B \sin kx$

Soln: $1 = \int_0^\ell \psi\psi^* dx = \int_0^\ell |\psi|^2 dx = |B|^2 \int_0^\ell \sin kx\, dx = |B|^2 \ell/2$

$\therefore B = \sqrt{2/\ell}$ and $\psi = \sqrt{2/\ell} \sin kx$ ----- (5.68)

5-3 A classical particle can rest indefinitely in a well. (i.e.K.E. and P.E. = 0 w.r.t. bottom). But the uncertainty principle prohibits this for a quantum particle. Show that the kinetic energy required by the uncertainty principle provides an estimate of the ground state energy.

Soln: Let

$$\Delta x \cdot \Delta p = \hbar/2$$

then for $\Delta x = \ell$, $\Delta p = \hbar/2\ell$

then $E_{min} = \frac{p^2}{2m} = \frac{\hbar^2}{8m\ell^2} = \frac{8\pi^2\hbar^2}{8m\ell^2} = \frac{\pi^2\hbar^2}{2m\ell^2}$

$E = \frac{1}{2}mv^2 = \frac{1}{2}\frac{m^2v^2}{m} = \frac{p^2}{2m}$

$\hbar = 2\pi\hbar$ see §4.3

which is in agreement with the result obtained. (See eq.5.67)

5-4 An electron is trapped in a one-dimensional region of length 1.0×10^{-10} m.

a) How much energy must be supplied to excite the electron from the ground state to the first excited state?

b) In the ground state, what is the probability of finding the electron in the region from $x = 0.09 \times 10^{-10}$ to 0.11×10^{-10} m?

c) In the first excited state, what is the probability of finding the electron between $x = 0$ and $x = 0.25 \times 10^{-10}$ m?

Soln: a) We have (from eq. 5.67)

$$E_o = \frac{\pi^2\hbar^2}{2m\ell^2} = \frac{(3.14)^2(1.05\times10^{-34})^2}{2\times9.1\times10^{-31}\times(10^{-10})^2} = 6\times10^{-18}\,J = 37\,eV$$

In the ground state the energy is E_o,

in the first excited state, the energy is $4E_o$ (see eq.5.67)

the difference which must be supplied is

$$4E_o - E_o = 3E_o = 111 \text{ eV}$$

b) We have (from eq.5.25) (see eqs 5.66 & 5.68)

$$\text{probability} = \int_{x_1}^{x_2} \psi\psi^* dx = \int_{x_1}^{x_2} |\psi|^2 dx = \frac{2}{\ell}\int_{x_1}^{x_2} \sin^2\frac{\pi x}{\ell}\,dx$$

$$= \left|\frac{x}{\ell} - \frac{1}{2\pi}\sin\frac{2\pi x}{\ell}\right|_{x_1}^{x_2} = 0.0038 = 38\%$$

c) We have

$$\text{probability} = \int_{x_1}^{x_2} \left(\frac{2}{\ell}\right)\sin^2\frac{2\pi x}{\ell}\,dx$$

$$= \left|\frac{x}{\ell} - \frac{1}{4\pi}\sin\frac{4\pi x}{\ell}\right|_{x_1}^{x_2} = 0.25$$

5-5 Show that the average value of x is $\ell/2$, independent of the quantum state.

Soln: We have from eq. 5.29,

$$\langle x\rangle = \int \psi^* x_{op}\psi\,dx$$

since $\psi = 0$ except for $0 \le x \le \ell$, we have 0 & ℓ as limits,

$$\langle x\rangle = \frac{2}{\ell}\int_0^{\ell} \left(\sin^2\frac{n\pi x}{\ell}\right)x\,dx = \ell/2.$$

UNSOLVED PROBLEMS

5:1 Show that the Hamiltonian and the energy operator commute and also determine whether the Hamiltonian and momentum operator commute.

5:2 Prove that $\psi(x) = \exp\left[-\frac{1}{2}x^2\right]$ is an eigen function of the operator $\left(\frac{\partial^2}{\partial x^2} - x^2\right)$ and hence determine the corresponding eigen value.

5:3 A particle which is confined in an infinite square well potential i.e.

$$V(n) = 0, \quad (n) \leq a$$
$$V(n) \geq \infty, \quad (n) \geq a$$

has an uncertainty in the position : $\Delta x = 2a$. If the magnitude of the momentum is at least as large as the uncertainty in momentum, prove that the ground state energy is about

$$E_o = \frac{\hbar^2}{8ma^2}$$

5:4 What will be the uncertainty in momentum and velocity for an electron which is confined in a range of 10×10^{-8} cm.

5:5 A wave function $\psi(x) = A_x \sin(2n\pi x/\ell)$ is defined only within a region $0 \leq x \leq \ell$. Use this normalization condition to evaluate the constant A_x.

5:6 Show whether $\Psi(x,t) = A \sin(kx - \omega t)$ is a solution to the Schrodinger equation.

5:7 Show that for a free particle

$$\Psi(x,t) = A \cos\left(kx - \frac{E}{\hbar}t\right) + iA \sin\left(kx - \frac{E}{\hbar}t\right)$$

where $k = \sqrt{2mE/\hbar}$ is a solution to the Schrodinger equation.

5:8 Suppose that a particle in the one-dimensional box emitted photons in jumping from one state to a lower one, with no restrictions on the change of n. In terms of E_o, list all possible photon energies that could be emitted when a particle goes from $n = 4$ to ground state.

Chapter-6
Properties of the Nucleus

Introduction

When we go deep inside the atom, we find at its center the nucleus, occupying only 10^{-15} of the volume of the atom, but providing the electrical force that holds the atom together. If there would be only Coulomb forces, the mutual repulsion would cause the atom to fly apart.

One of the goals of nuclear physics is to understand the internal structure of the nucleus. Another goal that is even more challenging is to understand the "glue" that holds the nucleus together.

There are many similarities between atomic structure and nuclear structure. Nuclei are subject to the laws of quantum physics. They have ground and excited states and emit photons (γ-rays).

There are two major differences between atomic and nuclear properties. In atomic physics, the electrons experience the force provided by the nucleus in nuclear physics, there is no such external agent. The second problem is that we cannot write the nuclear force in a simple form like the Coulomb force or the gravitational force.

Nuclear physicists study collisions and scattering interactions between the elementary particles and nuclei. Particles extremely high energy are required for this purpose. A variety of accelerating devices have been built. Some monstrous machines require many acres of ground. For example, the Stanford Linear Accelerator, which produces 20 GeV electrons, is 2 miles long. At Fermilab, the main accelerator is 4 miles in circumference.

There is a practical upper limit to the energies achieved in man-made devices. Perhaps the next high-energy physics laboratory will be in space, for observing nuclear events that cannot be taken in the laboratory.

6-1 NUCLEAR STRUCTURE

Definitions

Nucleus: The positively charged dense central part of an atom.

Proton: A fundamental unit of positive electric charge having a mass of 1.6725×10^{-27} Kg.

Neutron: A neutral atomic particle having a mass of 1.67482×10^{-27} kg.

Nucleon: A proton or neutron in the nucleus of an atom.

Atomic Number or Charge Number, Z: The number of protons in the nucleus of an atom.

Mass Number or Nucleon Number, A: The sum of the number of protons and neutrons in the nucleus of an atom.

Neutron Number, N (=A - Z): The number of neutrons in the nucleus of an atom.

Nuclide: The chemical symbol used for a nuclear species, corresponding to the value of Z, with A-value written as a superscript and the Z-value written as a subscript. (e.g. ${}_2He^4$)

Isotope: Nuclei having the same atomic number but different mass number.

Isobar: Nuclei of different elements that have the same mass number.

Isotone: Nuclei with same neutron number but different charge number.

Isomer: Longlived exicited state of an nucleus.

Mirror Nuclei: The pairs of nuclei having odd mass number such that atomic

Magic Numbers: Certain values of Z and N that produce unusual stability. These are:
Z or N = 2, 8, 20, 28, 50, 82, 126.
(e.g. for Z = 20; six stable isotopes,
for Z = 50; ten stable isotopes)

Stellar matter: Constituent matter of the stars.

Electron-Volt (eV): The energy required to remove an electron between two points which have a potential difference of one volt.

Fermi,F: A unit of length in nuclear physics, equal to 10^{-15} meter.

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From the work of Rutherford, Bohr, and their contemporaries, it was learned that the positive charge of the atom is confined in a very small nuclear region at the center of the atom.

The particles of the nucleus, the proton and neutron are, except for their electric charges, so similar to one another, they are classified together as nucleons.

The chemical properties of a certain element depend on the atomic number Z, but not on the mass number A.

Approximately 1400 different nuclides are now known to exist but only about one-fifth of them are stable.

If atomic matter could be squeezed sufficiently to remove all of the empty space in atoms, the density would increase by a factor of about 10^{13}. This is thought to account for the enormous densities of stellar matter, such as in white dwarfs and neutron stars, where the gravitational forces can be sufficiently great to approach nuclear densities.

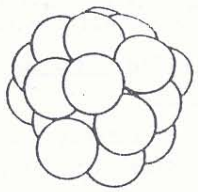


Figure 6.1:
A nucleus visualised as a cluster of tightly packed spherical nucleons.

6-2 NUCLEAR CHARGE

All nuclei are positively charged and the charges on nuclei are integral multiple of charge on an electron. Nuclear charge is the total number of positive charges carried by a nucleus. If Z be the atomic number of a nucleus, and e the elementary charge, the charge on that nucleus is Ze, since the number of protons in it is Z and the neutrons have no charge. Nuclear charge determines;

1. The number of electrons in the atom.
2. The energy levels of the atom.
3. The chemical and physical properties of the atoms.

6-3 NUCLEAR MASS

The nucleus of an atom contains nearly all of its mass. Since the nucleus consists wholly of protons and neutrons, the mass of nucleus is equal to the sum of the masses of the protons and the neutrons. Nuclear mass strictly means the mass (or weight) of the nucleus proper; but it is often called atomic mass (or atomic weight) which would rather mean the mass or weight of the whole atom.

The knowledge of nuclear mass gives us information on nuclear properties.

Nuclear masses can be measured with the help of a device known as mass spectrograph. To measure accurately, one should first of all, separate the different isotopes contained in one and the same element and then determine their masses individually. The weight of the individual isotopes thus measured is ordinarily termed the atomic mass. To get at the nuclear mass from the quantity, one must pay attention to the atomic number of the element and the degree of ionization. Because positive rays containing atoms singly or multiply ionized are used in mass spectrographs. If Z be the atomic number of the element and singly ionized beam is used, the mass of the (Z - 1) electrons must be deducted from the experimentally measured isotopic mass to obtain the actual nuclear mass.

Atomic masses are expressed in atomic mass units (a.m.u.). An atomic mass unit is equal to 1/12 of C^{12}, an isotope of Carbon.

6-4 SIZE OF THE NUCLEUS

The Rutherford scattering experiment revealed that nuclei are of finite size.

If we plot the nuclear density against the distance from the center of the nucleus (fig. 6.2), we see the variation of nuclear density $\rho(r)$ with the distance r from the centre of the nucleus.

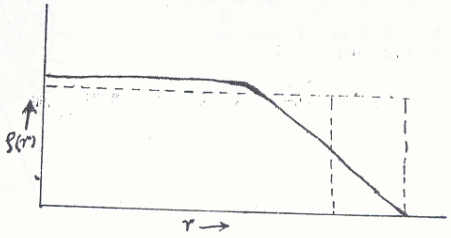


Figure 6.2: Approximate variation of $\rho(r)$ with r.

This figure shows that the nuclear density stays constant upto a distance we call the nuclear radeus.

To determine the size of the nucleus i.e. of the order of 10^{-13} cm, needs great refined methods. The different methods can be divided into two groups.

1. In nuclear methods we take light nuclear particle e.g. proton, neutron, or α-particle.
2. In electric method we take electrically charged non-nuclear particle e.g. electron or meson.

The results obtained by them show that:

The value of R is found to be smaller when deduced from electron scattering method than from neutron scattering. This indicates that the distribution of nuclear matter and nuclear charge is not identical throughout a nucleus. Also these results show that the nuclear radius varies from 2×10^{-13} cm for the lightest proton nucleus upto about 10×10^{-13} cm for the heaviest Uranium nucleus.

It is found that <u>volume of a nucleus is directly proportional to its mass number A</u>.

For a nucleus of radius R, the volume of the nucleus is $4/3\pi R^3$. Thus

$$4/3\pi R^3 \propto A$$

or $$R^3 \propto 3A/4\pi$$

or $$R^3 = \text{constant } A$$

or $$R = (\text{constant})^{\frac{1}{3}} A^{\frac{1}{3}}$$

$$R = R_o A^{\frac{1}{3}}$$

where $R_o = 1.3 \times 10^{-15}$ m $= 1.3$ F (1 F = 1 fermi $= 10^{-15}$ m)

6-5 ANGULAR MAGNETIC MOMENTS

With any charged particle with the angular momentum it is associated a magnetic moment. The orbital motion of the charged particles within the nucleus produces a certain electric current density which in turn gives rise to magnetic effects. Also the rotationof the nucleus produces magnetic moment parallel to its spin. The representation of the nucleon spins and associated magnetic moments is shown in fig. 6.3.

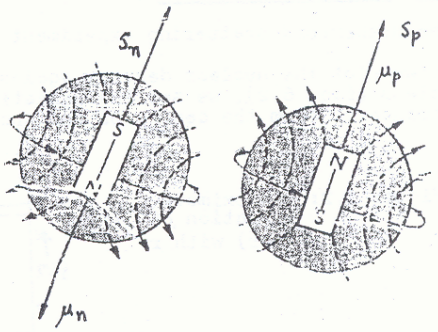


Figure 6.3: Nucleon spins and associated magnetic moments. S_p and S_n are the spins of proton and neutron.

There are a number of ways in which the magnetic moments of nuclei can be measured, and the results are useful in explaining nuclear structure. These give us information about the quantum numbers of the particles in the nucleus and nuclear energy levels etc.

There is an effect in atomic spectra called hyperfine structure. It is a very fine splitting of spectrum lines, or energy levels on a scale much smaller than the ordinary fine structure of lines, and not explainable by any of the theories we have discussed so far. It was suggested by Pauli that the origin of hyperfine splitting lay in a magnetic moment and angular momentum of the nucleus.

In article 3-21, it was stated that the spin of the electron is accompanied by a magnetic moment. It should not be surprising, then, to learn that other elementary particles having spin also possess magnetic moments. The unit used for nuclear moments is called the <u>Nuclear magneton</u>, and its symbol is μ_N. It is defined in a manner similar to the Bohr magneton, $[\mu_B = \frac{e\hbar}{2m}]$, except that the mass of the proton replaces the mass of the electron in the denominator. Thus nuclear magneton is

Nuclear magneton is smaller than the Bohr magneton by a factor of 1836. The magnetic moment of the proton is not simply μ_N, but is

$$\mu_p = 2.7928\ \mu_N \approx 1.41 \times 10^{-26} \text{ joules/tesla}$$

Surprisingly enough, the neutron also has a magnetic moment. Since the moment is directed opposite to the neutron's spin direction, it is written with a minus sign, that is,

$$\mu_n = -1.9135\ \mu_N$$

Also it gives a indication that the neutron is a complex particle containing negative and positive charges in equal amounts and the negative charge is, on the average, farther from its axis of rotation.

It is worth noting that the magnetic moments of the elementary particles have provided the strongest argument against the existence of electrons in the nucleus. The electron, having a magnetic moment roughly 600 times greater than any known nuclear moment, could hardly remain undetected within the nucleus.

6-6 QUADRUPOLE MOMENT

For easy explaination it is generally assumed that nuclei exhibit a spherically symmetrical character. Certain nuclei are known to possess an electric quadrupole moment. The behaviour of electric quadrupole is rather different from that of a magnetic dipole qualitatively and quantitatively. The electric quadrupole moment is actually a measure of the deviation of the charge distribution in a nucleus from the spherical symmetry. If a nucleus having uniform charge distribution is imagined to be an ellipsoid of revolution whose semi axes are 'a' along the symmetry axis, and 'b' along a direction perpendicular to it, then the quadrupole moment Q is given by:

$$Q = \frac{2}{5} Z(a^2 - b^2)$$

This shows that a positive value of Q corresponds to prolate spheroids ($a > b$) and negative to oblate spheroids ($a < b$). The magnitude of quadrupole moment also depends on the size of the nucleus, the extent of deviation from spherical symmetry and the magnitude of the charge.

The nuclei with proton numbers 2, 8, 20, 50, and 82 have the quadrupole moment Q equal to zero or very small. When a new shell begins to form, the quadrupole moment is negative. As the number of protons in the unfilled shell is increased, Q becomes positive and increases until it reaches a maximum, when the shell is about two-third filled. After this Q decreases to zero at the magic proton numbers, and then finally it becomes negative. The large values of the quadrupole moments indicate that in some casesthe nucleus is far from spherical. These large values can be explained if it is assumed that the part of the nucleus consisting of the filled shells forms a core which can be deformed by the nucleons in the unfilled shell.

6-7 BINDING ENERGY AND MASS DEFECT

The study of nuclear mass determinations reveals that the nuclear mass is less than the sum of the mass its constituents in the free state. The question arises as to where this mass goes when the nucleus is formed by assembling its constituents?

The clue lies in the mass energy relationship, which is in special theory of relativity

$$E = (\Delta M)c^2$$

where ΔM is the decfease in mass which is converted into energy E at the time of formation of the nucleus.

The difference in mass ΔM is called the mass defect. It is defined as: The arithmetic difference between the mass of a nucleus and the larger combined mass of its constituent particles.

Also binding energy is defined as: the energy needed to break up a nucleus into its constituent particles.

Stable nuclei have smaller masses than the combined masses of their constituent particles. The nucleus, helium ($_2He^4$), for example, has a mass of 4.00281 amu, while the two protons and two neutrons of which it is composed have a total mass (as free particles) of 4.03315 amu. The 'missing' mass ΔM is therefore 0.03034 amu, which is equivalent to 28.2 MeV.

$$E = \Delta Mc^2 = .03034 \times 1.66 \times 10^{-27} \times 9 \times 10^{16} \text{ joules} = x$$

$$= \frac{x}{1.6 \times 10^{-19}} \text{ eV} = y \qquad [1 \text{ amu} = 1.66 \times 10^{-27} \text{ Kg}$$

$$= \frac{y}{10^{-6}} \text{ MeV} = 28.2 \text{ MeV}$$

This means that when two protons and two neutrons combine to form a helium nucleus, 28.2 MeV of energy will be released. Conversely if we supply 28.2 MeV energy to a helium nucleus, it will break up into two protons and two neutrons.

This energy equivalent of the mass discfepancy in a nucleus is its binding energy, and is a measure of the stability of the nucleus. Binding energies arise from the action of the forces that hold nucleons together to form nuclei, just as ionization energies of atoms, which must be provided to remove electrons from them, arise from the action of electrostatic forces. As a general rule, the outermost electrons of atoms are bound by energies of 1 to 10 eV whereas each nucleon is bound to the nucleus by an energy of 1 to 10 MeV.

The binding energy per nucleon, arrived at by dividing the total binding energy of a nucleus by the number of nucleons it contains, is a most interesting quantity. The binding energy per nucleon is plotted as a function of A in figure 6.4. The curve rises steeply at first (a sharp rise for A = 4), and then more gradually until it reaches a maximum in the vicinity of A = 56, corresponding to iron nuclei, and then drops slowly. Evidently nuclei of intermediate mass are the most stable, since the greatest amount of energy must be supplied to liberate their nucleons.

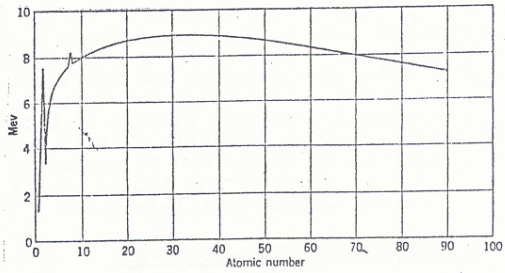


Figure: 6.4: Binding energy per nucleon, as a function of atomic number. The two peaks at low atomic number are $_2He^4$ and $_8O^{16}$.

From the above figure, note that the avergage binding energy per nucleon is roughly 7 or 8 MeV for most nuclides except for the lighter elements. This behaviour is evidence that the nuclear force 'saturates'. That is, each nucleon can interact only with a limited number of other nucleons, namely, its adjacent neighbours (see fig. 6.1). Saturation is further proof of the extremely short range of the nuclear force. By way of contrast, the Coulomb and gravitational forces are infinite in range, so that a given charge or mass interacts with every other charge or mass in an aggregate, regardless of the dis-

The Binding energy of the nucleus is given by

$$B = (ZM_p + NM_n - M)\, c^2$$

where

Z = charge number

M_p = mass of the proton

N = Neutron number

M_n = mass of the neutron

M = mass of the nucleus

c = velocity of light.

To measure the degree of stability, we use the term binding fraction f, which gives us the binding energy per nucleon. Thus

$$f = \frac{B}{A} = \frac{c^2}{A}(ZM_p + NM_n - M)$$

The binding fraction is closely related to the packing fraction $\frac{(M - A)}{A}$ and is not identical to it.

QUESTIONS

6.1 Give a brief account of the general properties of nuclei.

6.2 State briefly the experimental evidence to show that the nucleus is made up of protons and neutrons.

6.3 What is the general relationship between the atomic number of an atom and the number of its isotopes?

6.4 What is meant by nuclear mass defect? What is nuclear binding energy? How are they related?

6.5 What do the isotopes of an element have in common? How do they differ?

6.6 Distinguish between electric dipole and electric quadrupole. How does the concept of electric Quadrupole moment account for the shape of the nucleus?

6.7 In how many ways the nuclear radii can be measured?

6.8 Why does the electric dipole moment in an atomic nucleus vanish?

PROBLEMS

6-1 Calculate the binding energy of the alpha particle. What is the binding energy per nucleon?

Soln:

$$2(m_p + m_n) = 2(1.6725\times10^{-27} + 1.6748\times10^{-27}) = 6.6946\times10^{-27}\ kg$$

Mass of α-particle $= 6.643 \times 10^{-27}$ kg

$$\Delta M = 0.0503 \times 10^{-27} = 5.03 \times 10^{-29}\ kg$$

so Binding energy, $E_B = \Delta Mc^2$

$$= 5.63\times10^{-29}\times9\times10^{16}$$

$$= 4.5\times10^{-12}\ Joules$$

$$= \frac{4.5\times10^{-12}}{1.6\times10^{-19}} = 28.2\ MeV$$

Binding energy/nucleon $= \frac{E_B}{4} = \frac{28.30}{4} = 7.075\ MeV/nucleon$

6-2 How much energy is required to remove one neutron from a deutron?

Soln:

$$\Delta M = (m_H + m_N) - m_D$$

$$= 1.6729\times10^{-27} + 1.6748\times10^{-27} - 3.343\times10^{-27}$$

$$= .005\times10^{-27}$$

$$E = \Delta Mc^2$$

$$= \frac{.005\times10^{-27}\times9\times10^{16}}{1.6\times10^{-19}} = 2.223\ MeV$$

6-3 Calculate the density of nuclear matter for $_6C^{12}$. Radius of the carbon nucleus is 3×10^{-15} m and its mass is 19.92×10^{-27} kg.

Soln: We have

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

$$\text{or} \quad \rho = \frac{m}{4/3\,\pi r^3} = \frac{19.92 \times 10^{-27}}{\frac{4}{3} \times 3.14 \times 3 \times (10^{-15})^3} = 2 \times 10^{17} \text{ kg/m}^3$$

6-4 Calculate the minimum energy required to remove a proton from $_{20}Ca^{42}$.

Soln: $_{19}K^{41}$ would be formed after removal of one proton.

$$E = (m_{_{19}K^{41}} + m_p - m_{_{20}Ca^{42}})\, 931.5$$

$$= (40.962 + 1.008 - 41.959) \times 931.5 \qquad [1 \text{ amu} = 931.5 \text{ MeV}$$

$$= 10.28 \text{ MeV}.$$

UNSOLVED PROBLEMS

6:1 What will be the kinetic energy of a proton with a momentum of 1.1×10^{-20} kg-m/sec. The mass of the proton is 1.67×10^{-27} kg. (Ans. 0.23 MeV)

6:2 Calculate the momentum of a particle whose wavelength is 10^{-14} m. (Ans. 6.63×10^{-20} kg-m/sec)

6:3 The mass of $_{10}Ne^{20}$ is 19.9924 amu. Find its binding energy.

6:4 The binding energy of $_{17}Cl^{35}$ is 298 MeV. Find its mass. (Ans. 34.97 amu)

6:5 How much energy is required to remove one neutron from $_8O^{16}$.

6:6 Calculate the binding energy per nucleon in MeV for $_{16}S^{32}$.

6:7 Calculate the minimum energy required to remove a neutron from $_{20}Ca^{44}$.

6:8 Find the difference in binding energies for the mirror nuclei $_{11}Na^{23}$ and $_{11}Mg^{23}$.

Chapter 7

Natural Radioactivity

7.1 DISCOVERY

Henri Becquerel, a french physicist, was the first to discover in 1896 radioactivity. The discovery was quite accidental, as Becquerel was actually investigating an interesting point about X-rays which was in no way connected with radioactivity, Experimenting on the double sulphate of U and K, as no sunlight was available due to cloudy weather, he left the substance wrapped up in a paper in a drawer where there were also some photographic plates packed in black paper. A few days later he discovered that the photographic plates were effected though they were well wrapped up in a black paper and the salt itself has not been exposed to sunlight. He therefore surprised that the salt under study might be emitting, even in the dark some powerful radiations which could penetrate the paper wrappers and affect the photographic plates. The results obtained from a series of experiments with uranium clearly showed that the active radiations came from the salt itself and that exposure to sunlight had no influence on it.

Fig.7.1: Antoine Henry Becquerel was awarded the Nobel prize in 1903 with Curies for the study of the phenomenon of radioactivity.

The name of "Becquerel rays" was first given to the radiations emitted by uranium. It was later changed into "radioactivity" to designate all such rays given out not only by uranium but by several other substances which were soon discovered.

One important discovery was radium made by Madame Curie and her husband Pierre Curie in 1898. Some other scientists discovered that many other heavy elements such as thorium, actinium, radio-thorium, etc. showed a similar activity.

7.2 General properties:

1. The radiations emitted by radioactive substances are found to posses the following general properties:
   - i) High penetrating power
   - ii) affect photographic plates,
   - iii) ionise gases,
   - iv) cause scintillations on fluorescent screens,
   - v) develop heat,
   - vi) produce chemical effects.
2. As the radiations are given out, new elements are formed in an irreversible process, each element having its own characteristic radiations.
3. The emission of radiation is spontaneous, uneffected by any external agent.
4. The emission is not instantaneous but prolonged.
5. The process of separation of the radioactive substances from the original mineral ores is a difficult task.
6. There is nothing abnormal about the radioactive elements as regards their physical and chemical properties; all of them have their places in the periodic table.

7.3 Types of Radiations from Radioactive Substances

It can be shown by means of experiments that there are three kinds of radiations from naturally occuring radioactive substances. In these experiments a thin beam of rays is supplied by a small piece of radioactive material at the bottom of a small hole drilled in a lead block so that a fairly parallel beam of radiations is obtained. The whole arrangement is placed in air tight chamber which is kept evacuated by a vacuum pump in order to avoid the

angles to the plane of the diagram, and directed away from the reader. When the plate is developed two distinct spots are found, one in the direct line of the hole in the lead block and one deflected to the right. The undeviated spot is caused by the neutral rays called γ-rays, the spot to the right is caused by rays of negatively charged particles called β-rays.

If the weak magnetic field is replaced by a strong one in the same direction the undeviated spot is again found together with a spot deflected to the left, which is caused by positively charged particles called α-rays. In the strong field the path of β-rays is deflected so much that it does not reach the plate.

After some time, it was found that the three types of rays were not simultaneously emitted by all radioactive elements. Some elements emit α-rays, some β-rays, and γ-rays may accompany either process.

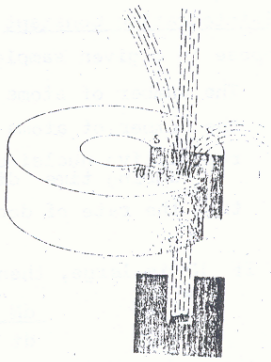


Fig.7.2:
Types of radioactivity.

α-Particles

1. Deflection experiments showed that α-rays are positively charged particles.
2. Rutherford's experiments gave the velocity of α-rays from RaC as 2×10^9 cm/sec, which is approximately 1/15 of velocity of light.
3. The α-particles have:
   charge = 2
   mass = 4 a.m.u. = 4x931 Mev
4. α-particles are helium ions.
5. They produce fluorescence in certain substances.
6. They have less penetrating power as compared with β-particles and γ-rays.
7. When α-particles pass through matter they produce ionization.

β-Particles

1. β-particles are negatively charged particles.
2. They contain swiftly moving electrons.
3. They have greater velocity than the cathode rays electrons.
4. The value of e/m decrease by increasing velocity.
5. These have greater penetrating power. They can pass one mm of Al, as compared with α-particles, which cannot pass greater than 0.006 cm of Al.
6. The ionisation density produced in a gas by β-rays is much less intense than that produced by α-rays.
7. Like α-rays they also produce fluorescence.

γ-Rays

1. γ-rays are the electromagnetic waves.
2. They are nearly 10 to 100 times as penetrating as the β-rays.
3. They produce less ionization than the β-rays. (The ionisation produced by α, β & γ-rays is roughly in the order of 10,000, 100, and 1.
4. The fluorescence produce by γ-rays are similar to that of the characteristic X-rays depending upon the nature of the substance emitting them.

7.4 Rutherford-Soddy Transformation Theory

The early experiments of Crookes, Becquerel, Rutherford and Soddy gave a simple explanation of the spontaneous production of radioactive matter observed experimentally. A simple law discovered in 1913, can be stated as follows:

In all known radioactive transformations either an α- or a β-particle is ejected by the atom. When an α-particle is given out, a new atom is formed whose mass number is less by four [illegible]

units than those of the original atom. When a β-particle is expelled, the new atom formed has the same mass number as the original atom but the atomic number is increased by one unit.

With the help of the above displacement law, we can determine mass and atomic numbers of different elements formed in the successive radioactive changes.

7.5 Disintegration Constant

Suppose in a given sample of radioactive material;

The number of atoms at $t = 0$ is N_0

the number of atoms after time t is N

radioactive nuclei decay into daughter nuclei during time dt is dN

then the rate of decay $= \frac{dN}{dt}$

If N is large, then $\frac{dN}{dt}$ will be proportional to N, i.e.

$$\frac{dN}{dt} \propto N$$

As the number N decreases as time increases, so we denote it by inserting a negative sign,

$$-\frac{dN}{dt} \propto N$$

$$\text{or} \quad -\frac{dN}{dt} = \lambda N \qquad \ldots\ldots (7.1)$$

where λ is the proportionality constant and is called the decay constant or disintegration constant.

from eq. (7.1) λ is defined as:

$$\lambda = -\frac{1}{N}\frac{dN}{dt} \qquad \ldots\ldots (7.2)$$

$$\text{or} \quad \frac{dN}{N} = -\lambda\, dt \qquad \ldots\ldots (7.3)$$

7.6 The Radioactive-decay Law

From the definition of decay constant, we have

$$\lambda = -\frac{1}{N}\frac{dN}{dt}$$

$$\text{or} \quad \frac{dN}{N} = -\lambda\, dt \qquad \ldots\ldots (7.3)$$

There is no way of predicting when a particular nucleus will decay, nor is there any known way to stimulate its decay. As far as we can tell, the decay rate is not affected by extreme temperatures or pressures, by strong electromagnetic fields, or by its past history. Regardless of how long a particular parent has survived, the probability that it will decay in the next time interval dt is given by $\lambda\, dt$.

If we say that the number of parent nuclei at time $t = 0$ was N_0, then we may integrate eq.(7.3) in order to find the population N at any time t. Thus

$$\int_{N_0}^{N} \frac{dN}{N} = -\int_0^t \lambda\, dt$$

$$\text{or } \left|\log N\right|_{N_0}^{N} = -\lambda \left|t\right|_0^t \qquad \left[\int \frac{1}{x}dx = \log x\right.$$

$$\text{or } \log N - \log N_0 = -\lambda (t-0)$$

$$\text{or } \log \frac{N}{N_0} = -\lambda t \qquad \left[\log a - \log b = \log \frac{a}{b}\right.$$

taking antilog, we get

$$\frac{N}{N_0} = e^{-\lambda t}$$

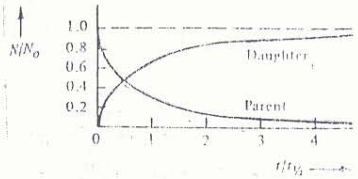
$$\text{or } N = N_0 e^{-\lambda t} \qquad \ldots\ldots (7.4)$$

Eq.(7.4) is the exponential law of radioactive decay.
Note that the population of parent nuclei decreases exponentially with time. If the daughter nuclei is stable, then the population of daughter atoms will increase exponentially according to the equation

$$N_{daughter} = N_0 - N = N_0 (1 - e^{-\lambda t}) \qquad \ldots\ldots (7.5)$$

These processes are illustrated in fig. (7.3), which can be obtained from the experimental data.

Fig. (7.3):
Relative amounts of radioactive parent and stable daughter nuclides as a function of time.



7.7 The Half Life and the Mean Life

Half Life:

It is the time interval during which the activity of any radioactive sample reduces to one half of its original value; or it is the time interval during which the number of radioactive atoms reduces to half of their initial value.

To calculate the half life of an atom consider eq. (7.4)

Half life is denoted by $T_{\frac{1}{2}}$

when $T = T_{\frac{1}{2}}$, number of undecayed atoms $N = \frac{N_0}{2}$

where N_0 is the initial Number of atoms.

eq. (7.4) becomes

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

$$\text{or } \frac{1}{2} = e^{-\lambda T_{1/2}}$$

$$\text{or } e^{+\lambda T_{1/2}} = 2$$

taking logrithm on both sides,

$$\lambda T_{1/2} = \log 2$$

$$\text{or } T_{1/2} = \frac{1}{\lambda} \log 2 = \frac{0.693}{\lambda}$$

$$\text{So } T_{1/2} = \frac{0.693}{\lambda} \qquad \ldots\ldots (7.6)$$

In eq. (7.6), if time is expressed in seconds λ is then expressed in sec^{-1}. If λ is large $T_{\frac{1}{2}}$ is small. The half life $T_{\frac{1}{2}}$ is a characteristic constant for a radioactive element. The radioactive elements can be differentiated by the value of $T_{\frac{1}{2}}$.

Now, number of undecayed atoms after time $T_{\frac{1}{2}}$ = 50 %

after a time $2T_{\frac{1}{2}}$ = 25 %

after a time $3T_{\frac{1}{2}}$ = 12.5 %

after a time $4T_{\frac{1}{2}}$ = 6.25 %

after a time $10T_{\frac{1}{2}}$ = .1 %

that is, the fraction though becomes very small but never reaches zero. After ten half lives the activity has fallen to about 0.1 % of the original amount, and is usually negligible in comparison with the initial value.

Mean Life:

The sum of lives of all the nuclei of a radioactive element divided by total number of nuclei in it is called its mean life.

If t_1 = life time of dN_1 nuclei

t_2 = " " " dN_2 "

t_3 = " " " dN_3 "

Then Mean life, $$T = \frac{t_1 dN_1 + t_2 dN_2 + t_3 dN_3 + \cdots}{dN_1 + dN_2 + dN_3 + \cdots}$$

or $$T = \frac{\int_0^{N_0} t\, dN}{\int_0^{N_0} dN}$$

where $N_0 = dN_1 + dN_2 + \cdots$ (total number of nuclei)

from the disintegration law (eq. 7.4)

$$N = N_0 e^{-\lambda t}$$

$$\text{or } dN = -\lambda N_0 e^{-\lambda t}$$

$$\therefore T = \frac{\int_{\infty}^{0} t \cdot -\lambda N_0 e^{-\lambda t}}{N_0}$$

[taking integration by parts as we have two variables]

$$= \frac{N_0}{N_0} \int_0^{\infty} \lambda\, t\, e^{-\lambda t}\, dt$$

$$= \lambda \int_0^{\infty} t\, e^{-\lambda t}\, dt$$

$$= \lambda \left| t \frac{e^{-\lambda t}}{-\lambda} \right|_0^{\infty} - \lambda \int_0^{\infty} \frac{e^{-\lambda t}}{-\lambda}\, dt$$

[as $e^{-\infty} = 0$]

$$= \frac{\lambda}{\lambda} \int_0^{\infty} e^{-\lambda t}\, dt = \left| \frac{e^{-\lambda t}}{-\lambda} \right|_0^{\infty}$$

$$\therefore T = \frac{1}{\lambda} \qquad \cdots\cdots (7.7)$$

7.8 Successive Radioactive Decay

For the sake of clarity we have assumed in the previous sections that nuclei of pure radioactive species disintegrate into stable nuclei. But in practice, a sample of a naturally occuring radioactive substance decays into another radioactive substance, the latter in turn, decays into another radioactive substance and so on. This succession of disintegration is called "radioactive chain" or "series".

Analysis of the general case of n members of the series has been carried out by Bateman in 1910. A simpler case will be considered here, namely, a radioactive species A decaying into another species B , which in turn decays into a stable product C.

The number of atoms of the three kinds at any time t are denoted by N_1, N_2, N_3, respectively; and the disintegration constants are λ_1 & λ_2

$$\text{(A)}_{N_1} \xrightarrow{\lambda_1} \text{(B)}_{N_2} \xrightarrow{\lambda_2} \text{(C)}_{N_3\ (\text{stable})}$$

The element A which decays into successive elements is called "Parent Element" and the first product obtained from the parent element (i.e. B in this case) is called "Daughter Element"; and the next product of disintegration, C is called "Grand Daughter Element".

7.9 Units of Radioactivity

In a sample of radioactive material, what is detected is the effect produced by the number of particles emitted in unit time; this is the result of the number of atoms or nuclei disintegrating in unit time. Historically it was found that in one gram of radium very nearly 3.7×10^{10} nuclei disintegrate per sec.

For radium $T_{\frac{1}{2}} = 1620$ years

$$\lambda = \frac{.693}{T_{\frac{1}{2}}}\ \text{sec}^{-1} = \frac{.693}{1620} = 13.8 \times 10^{-12}\ \text{sec}^{-1}$$

Avogadro's number (no. of atoms in one gram) $= 6.02 \times 10^{23}$

mass number of Ra $= 226$ a.m.u.

therefore N, radioactive atoms in one gram $= \frac{6.02 \times 10^{23}}{226} = 2.66 \times 10^{21}$

Rate of disintegration or activity,

$$\frac{dN}{dt} = -\lambda N$$

$$\left|\frac{dN}{dt}\right| = 13.8 \times 10^{-12} \times 2.66 \times 10^{21} = 3.7 \times 10^{10}\ \text{disinteg./sec}$$

so one gram of radium = one curie

The standard unit "curie" is the quantity of radioactive material which gives exactly 3.7×10^{10} disintegrations per second.

Accordingly millicurie gives 3.7×10^{7} disintegrations per second and microcurie gives 3.7×10^{4} disintegrations per second.

Another unit of radioactivity is "rutherford" (rd) defined as the amount of radioactive substance which gives 10^{6} disintegrations/sec. The millirutherford (mrd) and microrutherford (μrd) correspond to 10^{3} dist./sec and 1 disintegrations/sec. respectively.

7.10 Transformations in Radioactive Decay

The transformations in radioactive decay processes will be represented by the general equation

$$X^* \longrightarrow X + x \qquad (T_{\frac{1}{2}}) \qquad \ldots\ldots (7.8)$$

where X^* and X represent the symbols which are needed to show parent and daughter elements, respectively, and x represent the corresponding symbols for the ejected particle.

The eq. (7.8) is illustrated below by specific examples.

In case of alpha decay it has been established by experiment that the daughter nucleus is always 4 units less in atomic mass and 2 units less in atomic number. This difference is accounted for by the fact that the ejected alpha particle has the mass number 4 and the charge number 2. So

$$A = A^* - 4 \qquad N = N^* - 2 \qquad Z = Z^* - 2$$

and with

$$x = \alpha$$

An example is: $${}_{88}Ra^{226} \longrightarrow {}_{86}Rn^{222} + \alpha \quad (1620 \text{ yrs}) \quad \ldots\ldots (7.9)$$

$\begin{cases} A^* \text{ is mass number of parent} \\ A \text{ " " " " daughter} \end{cases}$

$\begin{cases} Z \text{ is the charge number} \\ N \text{ " " Neutron number} \end{cases}$

In the case of β^--decay a neutron in the nucleus transforms into a proton by emitting an electron. The nuclear transformation may be represented by eq. (7.8) with

$$A = A^* \qquad N = N^* - 1 \qquad Z = Z^* + 1$$

and with

$$x = \beta^-$$

An example is

$${}_{90}Th^{231} \longrightarrow {}_{91}Pa^{231} + \beta^- \quad (25.5 \text{ hr}) \quad \ldots\ldots (7.10)$$

In β^+-decay a proton transforms into a neutron by emitting a positron. The nuclear transformation may be represented by eq.(7.8) with

$$A = A^* \qquad N = N^* + 1 \qquad Z = Z^* - 1$$

and with

$$x = \beta^+$$

An example is

$${}_{29}Cu^{61} \longrightarrow {}_{28}Ni^{61} + \beta^+ \quad (3.4 \text{ hr}) \quad \ldots\ldots (7.11)$$

In decay by gamma emission, the nucleus simply emits energy in the form of a gamma ray and does not change. So

$$A = A^* \qquad N = N^* \qquad Z = Z^*$$

and with

$$x = \gamma$$

An example is

$${}_{38}Sr^{87m} \longrightarrow {}_{38}Sr^{87} + \gamma \quad (2.8 \text{ hr}) \quad \ldots\ldots (7.12)$$

A gamma-emitting nuclide which exists for a measureble half-life is said to be an isomer of the daughter nuclide. In eq.(7.12) we have denoted the isomer by a superscript m.

7.11 Radioactive Series

Most of the radioactive elements found in nature are members of four radioactive series, with each series consisting of a succession of daughter products all ultimately derived from a simple parent nuclide. There are only 14 radioactive nuclides found in nature whose half-life is of the same order or larger in magnitude than the age of the universe, which is estimated as 4.5×10^9 years. Some of these are listed in Table-I.

The radioactive series of transformations follow the displacement laws, discovered by Fajans and Soddy:

1. When an α-particle is emitted, the element moves back two places in the Periodic Table.

Table-I: Natural radioactive nuclides with half-lives of the same order or larger than the age of the universe, which is 4.5×10^9 yrs.

Nuclide	Half life(yr)
Rb^{87}	6.2×10^{10}
In^{115}	6.0×10^{14}
Ce^{142}	5.0×10^{15}
Nb^{144}	3.0×10^{15}
Pt^{192}	1.0×10^{15}
Th^{232}	1.4×10^{10}
U^{235}	7.1×10^{9}
U^{238}	4.5×10^{9}

Table-II: The four radioactive series.

Series	Parent	Half-life (years)	Stable end product
Thorium	$_{90}Th^{232}$	1.39×10^{10}	$_{82}P^{208}$
Neptunium	$_{93}Np^{237}$	2.25×10^{6}	$_{83}Bi^{209}$
Uranium	$_{92}U^{238}$	4.51×10^{9}	$_{82}Pb^{206}$
Actinium	$_{92}U^{235}$	7.07×10^{8}	$_{82}Pb^{207}$

The four possible radioactive series are listed in the Table-II and III. Because of the short half-life of neptunium, even traces of this element in nature are undetectable by ordinary methods. Now physicists are able to produce new isotopes and it is then possible to trace neptunium series.

7.12 Dating by Radioactive Decay

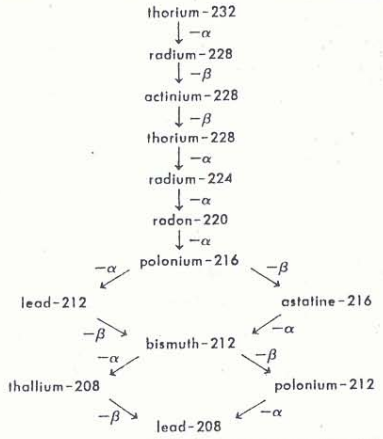
The decay of radioactive elements is independent of the physical and chemical conditions imposed on them. The decay of an individual particle from a given nucleus is a random process, but the gross decay of the many nuclei in a given sample provides a very convenient way of measuring time. By measuring the ratio of uranium to lead in rock samples, formed as a result of a prehistoric volcanic eruption, a rather exact time can be determined. Uranium dating measures times of the order of millions of years.

Since all plants use carbon dioxide from the atmosphere for growth, a portion of the carbon in plants is radioactive C^{14} and the plants are slightly radioactive. When a plant dies, no additional C^{14} is taken in, and that within the plant body begins to decay without being replaced. Measurement of the relative amounts of C^{14} and C^{12} in an organic sample gives a method of dating. Some interesting archeological events dated by radiocarbon dating are listed in Table-IV.

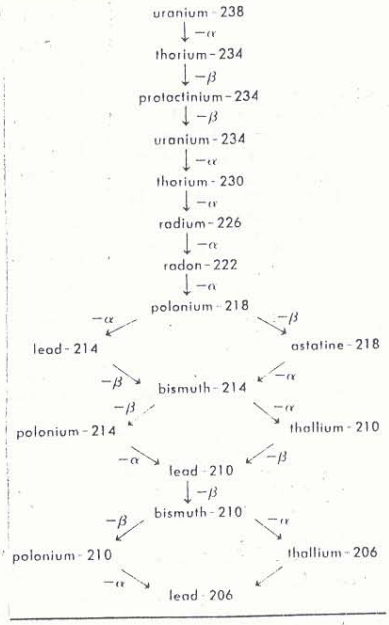
Another element that is continuously produced in the atmosphere by cosmic ray bombardment is tritium, H^3, the heavy isotope of hydrogen decays with a half-life of 12.4 years. A small percentage of water contains H^3 instead H^1, and by measuring the relative amounts of both in samples of water at various locations, some indication is given as to how long ago this water was in the form of rain.

Table-III: The four possible radioactive series.

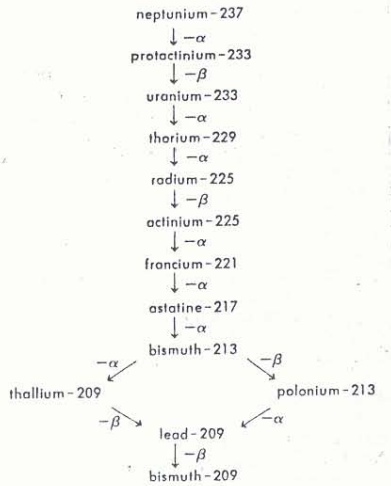
The Thorium Series



The Uranium Series



The Neptunium Series



The Actinium Series

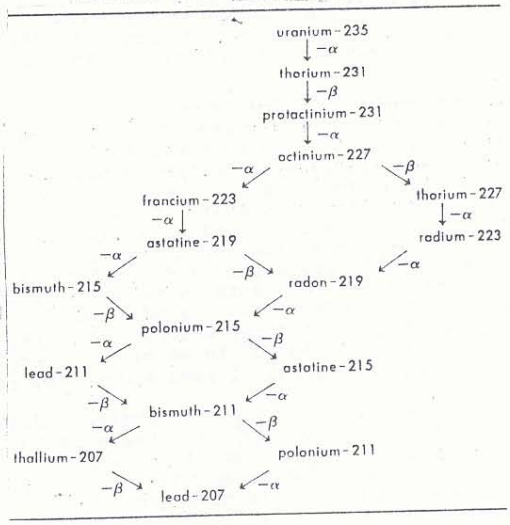


Table IV: Some typical samples dated by radio-carbon dating.

Sample	C^{14} Age (yr)
1. Charcoal from Lascaux cave (well known for wall painting)	15,516
2. Slab of wood from roof beam of tomb of Vizier Hemaka	4,883
3. Deer antler from Indian site in Butler County, Kentucky	4,333
4. Wood from the deck of the funeral ship from tomb of Pharaoh Sesostris III	3,621
5. Wood from giant redwood tree known as the Centennial Stump, felled in 1874	2,710

7.13 Radioactive Equilibrium

Radioactive equilibrium can be had only when the mean life of the parent is greater than that of the daughter. The two kinds of equilibrium can be distinguished:

1. Transient Equilibrium:

Suppose an element of decay constant λ_1, transforms into an element of decay constant λ_2.
Let N_1^0 denote the number of atoms of parent at time $t = 0$, let this number fall to N_1 after time t.

If $\lambda_1 < \lambda_2$, $T_1 > T_2$ $\quad \{\lambda_1 \simeq \lambda_2\}$

i.e. the parent is longer-lived than the daughter but the half life of the parent is not very long.

If the parent and the daughter are separated so that the parent can be assumed to be initially pure, the number of atoms of parent and daughter at any time t are :

$$N_1 = N_1^0 e^{-\lambda_1 t} \qquad \ldots\ldots (7.13)$$

As shown in eq. (7.4).

At time $t = 0$, the number of atoms of daughter is zero. Let the number of atoms of daughter rise from 0 to N in time t. As the atoms of parent transform into atoms of daughter the number of atoms of daughter increases at the rate of after time t. But this number also decreases at the rate of since daughter is being transformed into some other element and its decay constant is :
Thus the net rate of increase is

$$\lambda_1 N_1 - \lambda_2 N_2$$

and so $$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$$

putting the value of N_1 from eq.(7.13), we get

$$\frac{dN_2}{dt} = \lambda_1 N_1^0 e^{-\lambda_1 t} - \lambda_2 N_2$$

or $$\frac{dN_2}{dt} + \lambda_2 N_2 = \lambda_1 N_1^0 e^{-\lambda_1 t} \qquad \ldots\ldots (7.14)$$

Multiplying both sides of eq.(7.14), by $e^{\lambda_2 t}$

$$e^{\lambda_2 t}\frac{dN_2}{dt} + N_2(\lambda_2 e^{\lambda_2 t}) = \lambda_1 N_1^0 e^{-\lambda_1 t} e^{\lambda_2 t}$$

or $$\frac{d}{dt}(e^{\lambda_2 t} N_2) = \lambda_1 N_1^0 e^{(\lambda_2-\lambda_1)t}$$

Integrating, we get

$$e^{\lambda_2 t} N_2 = \frac{\lambda_1 N_1^0}{\lambda_2-\lambda_1} e^{(\lambda_2-\lambda_1)t} + C \qquad \ldots\ldots(7.15)$$

where C is a constant of integration.
When t = O, N = O, so from eq.(7.15), we get

$$0 = \frac{\lambda_1 N_1^0}{\lambda_2-\lambda_1} + C$$

or $$C = -\frac{\lambda_1 N_1^0}{\lambda_2-\lambda_1} \qquad \ldots\ldots (7.16)$$

from eqs.(7.15) and (7.16)

$$e^{\lambda_2 t} N_2 = \frac{\lambda_1 N_1^0}{\lambda_2-\lambda_1} e^{(\lambda_2-\lambda_1)t} - \frac{\lambda_1 N_1^0}{\lambda_2-\lambda_1}$$

Dividing both sides by $e^{\lambda_2 t}$

$$N_2 = \frac{\lambda_1 N_1^0}{\lambda_2-\lambda_1} e^{-\lambda_1 t} - \frac{\lambda_1 N_1^0}{\lambda_2-\lambda_1} e^{-\lambda_2 t}$$

or $$N_2 = \frac{\lambda_1 N_1^0}{\lambda_2-\lambda_1}(e^{-\lambda_1 t} - e^{-\lambda_2 t}) \qquad \ldots\ldots (7.17)$$

After t becomes sufficiently large $e^{-\lambda_2 t}$ becomes negligible compared with $e^{-\lambda_1 t}(\because \lambda_1 < \lambda_2)$, and the number of element of the daughter becomes:

$$N_2 = \frac{\lambda_1 N_1^0}{\lambda_2-\lambda_1} e^{-\lambda_1 t} \qquad \ldots\ldots (7.18)$$

from eqs.(7.13) & (7.18), we get

$$N_2 = \frac{\lambda_1}{\lambda_2-\lambda_1} N_1 \qquad \ldots\ldots (7.19)$$

Thus the daughter eventually decays with the same half life as the parent. From eq.(7.19),

$$\frac{N_2}{N_1} = \frac{\lambda_1}{\lambda_2-\lambda_1} = \text{Constant} \qquad \ldots\ldots (7.20)$$

This state of affairs is called a <u>transient equilibrium</u>, since N_2 and N_1 are both decreasing with time.

Here, the daughter substance first grows and reaches a maximum, when the ratio of the daughter to the parent become a constant. Both substances will eventually break down at the decay rate of the parent.

In eq.(7.20), N_1 and N_2 are the amounts of the parent and daughter at any instant after the daughter has grown to its maximum, λ_1 and λ_2 their respective decay constants. It is seen that $\frac{N_2}{N_1}$ is a constant which indicates the state of equilibrium; but the equilibrium is transient, since both will decay at the decay rate of the parent.

<u>Definition</u>: If the rate of production of a certain member of a radioactive series is not constant but changes very little during a life time of any of the subsequent members, a condition is attained called "transient equilibrium".

2. Permanent Equilibrium:

If the mean life of the parent is very great, so that the amount of the parent can be considered practically constant.

Here $\lambda_1 << \lambda_2$ or $\frac{1}{\lambda_1} >> \frac{1}{\lambda_2}$; $T_1 >> T_2$ $\quad \{T_1 \approx \infty\}$

The case is that of a long-lived parent and a short lived daughter. Again it is assumed that the daughter has been separated from the parent so that the latter is initially pure.

After some time, when $t >> \frac{1}{\lambda_2}$, the term $e^{-\lambda_2 t}$ becomes negligible in eq.(7.17), but the value of the term $e^{-\lambda_1 t}$ remains nearly equal to 1, so by eqs. (7.13) and (7.17), we have

$$N_1 = N_1^0 e^{-\lambda_1 t} \approx N_1^0$$

$$N_2 = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1^0 \left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right) \approx \frac{\lambda_1}{\lambda_2} N_1^0 \quad \text{since } \lambda_2 - \lambda_1 \approx \lambda_2$$

or $N_1 \approx N_1^0$ .....(7.21)

$$N_2 \approx \frac{\lambda_1}{\lambda_2} N_1^0 \quad .....(7.22)$$

from eqs. (7.21) & (7.22), we get

$$\frac{N_2}{N_1} = \frac{\lambda_1}{\lambda_2}$$

or $N_2 \lambda_2 = N_1 \lambda_1$ .....(7.23)

This means that the amount of the two substances are inversely proportional to their decay constants. Under these conditions the daughter breaks up as fast as it is formed. The state of equilibrium is indicated by the constancy of the ratio $\frac{N_2}{N_1}$ and the permanancy of the equilibrium by N_2 being constant.

In eq. (7.22), the number of atoms of the daughter approaches a constant value; the activity of the daughter product becomes the same as that of the parent. This condition is known as "secular" or "permanent equilibrium".

Definition: If the initial number of a given series of radioactive atoms is formed at a constant rate, the various members of the series will eventually attain equilibrium concentrations in the sample, dependent upon their respective mean lives, such that all members have equal activity. This kind of equilibrium is called "secular equilibrium".

Example:

An example of permanent equilibrium is found in the formation of radon from radium, since the mean life of Ra (2300 yr) is very great compared with that of Rn (5.5 days). As shown in fig.7.4. After a time, long compared with its mean life, Rn is in permanent equilibrium with Ra and its amount becomes constant as indicated by the point of intersection in the figure.

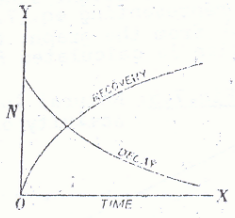


Fig. 7.4: Decay and recovery curves of radon.

7.14 Measurement of Half-life

Half life of radioactive elements are measured by different methods. In each case, we measure the decay constant λ by experiment, and then the half-life $T_{\frac{1}{2}}$ is determined usually from eq. (7.6), viz.

$$T_{\frac{1}{2}} = \frac{0.693}{\lambda} \qquad \ldots\ldots (7.6)$$

a) Measurement of extremely short half-lives

When α-particles emitted, they travel a certain distance R, called the range of α-particles. The range R and λ are related with an empirical relation, called Geiger-Nuttall law as:

$$\log \lambda = A + B \log R \qquad \ldots\ldots (7.24)$$

where A and B are constants.

In eq.(7.24), λ increases with R, this means that long ranges of α-particles have short half lives. This equation is used to measure the decay constant of elements whose half-life is very short ($\sim 10^{-7}$ sec). Knowing λ, we can determine the half-life $T_{\frac{1}{2}}$ by using eq.(7.6).

b) Measurement of moderately short half-lives (high value of λ)

Let number of atoms of a sample at t = 0 is N_0

" " " " " " after time t is N

then N is given by eq.(7.4)

$$N = N_0 e^{-\lambda t} \qquad \ldots\ldots (7.25)$$

taking logarithm on both sides

$$\log N = \log(N_0 e^{-\lambda t})$$

$$= \log N_0 + \log(e^{-\lambda t})$$

or $$\log N = \log N_0 - \lambda t \qquad \ldots\ldots (7.26)$$

putting $\log N = y$, and $\log N_0 = c$

so eq.(7.26) becomes

$$y = c - \lambda t$$

therefore $$\frac{dy}{dt} = -\lambda \qquad \left[\frac{d}{dx}(\text{Const.}) = 0\right.$$

Hence the slope of the straight line graph gives the value of λ.

The decay curve is drawn from experimental data of measured intensity with time. If the log of intensity is plotted against time a straight line graph is obtained representing eq.(7.26). After finding λ from the graph, the value of half life is calculated from the eq.(7.6)

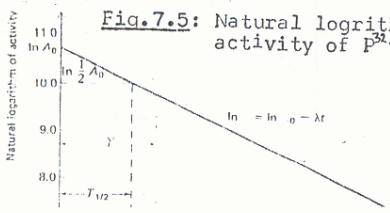


Fig.7.5: Natural logrithm of activity of P^{32} vs time.

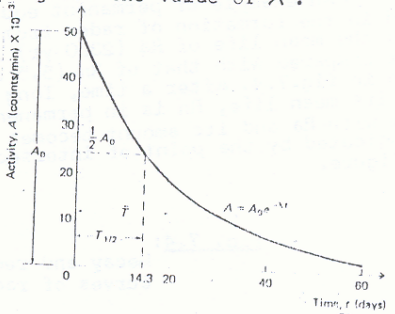


Fig.7.6: Activity of P^{32} vs decay time.

c) Measurement of very long half-lives (so small λ)

We measure long half-lives, of the order of millions of years, by the following two methods.

i) By counting α-particles:

Here we count the α-particles emitted/sec by a known quantity of the substance. Since each disintegrating atom emits only one α-particle, the number of α-particles counted must be equal to the number of atoms disintegrating. from eq.(7.25)

$$N = N_0 e^{-\lambda t}$$

$$\text{or} \quad \frac{dN}{dt} = -\lambda N_0 e^{-\lambda t} = -\lambda N$$

$$\text{so} \quad \left|\frac{dN}{dt}\right| = \lambda N$$

$$\text{or} \quad \lambda = \frac{1}{N}\left|\frac{dN}{dt}\right|$$

The decay constant λ is therefore equal to the ratio of the atoms distintegrating /unit time to the total number of atoms present.

For thorium, Geiger and Rutherford found by the method of scintillations that it emits 4500 α-particles/sec./gm.

Now 1 gm. of Th. contains 2.61×10^{21} radioactive atoms

$$\text{therefore} \quad \lambda = \frac{4500}{2.61\times10^{21}} = 1.72\times10^{-18}\ \text{sec}^{-1} = 5.44\times10^{-11}\ \text{yrs}^{-1}$$

$$\text{so} \quad T = \frac{\log 2}{\lambda} = \frac{.693}{5.44\times10^{-11}} = 1.28\times10^{10}\ \text{yrs}.$$

ii) By determining the quantities of substances in permanent equilibrium:

Conditions of such an equilibrium show that (from eq.7.23)

$$N_1\lambda_1 = N_2\lambda_2$$

Hence if N_1, λ_1 and N_2 are known λ_2 can be calculated.

Thus the period of uranium was determined from the known period(half life) of radium and was found to be 4.5×10^9 yrs.

QUESTIONS

7.1 What is transmutation? What changes take place in the nucleus of an atom when an α-particle is emitted? What changes occur with the emission of β-particle ?

7.2 Radium decays into helium and radon. Why radium is regarded as element rather than a compound?

7.3 How was the existence of three types of radiation in natural radioactivity demonstrated experimentally?

7.4 Show that radioactive decay obeys the exponential law and hence calculate the half life of a radioactive element.

7.5 What is the difference between X-rays and γ-rays. Are these like ordinary electromagnetic waves like light rays.

7.6 Can the natural radioactivity of radioactive elements be controlled by changing the environments?

7.7 Give an account of the law of radioactive disintegration. Show how the average life of a radioactive element is connected with its transformation constant.

7.8 Define half-life. How is it used to find the decay constant. Define various units of radioactivity.

7.9 When a radioactive nucleus disintegrates, the products have kinetic energy even though the original nucleus was at rest. What is the source of this energy?

7.10 Suppose we wish to do radioactive dating of a sample whose age we guess to be t. Should we choose an isotope whose half-life is i) $>> t$, ii) $\sim t$, iii) $<< t$?

7.11 It has been observed that there is an increased level of radon gas, Rn^{86} , in the air just before an earthquake. Where does the radon come from? How is it produced? How is it released? How is it detected?

7.12 Show that the average life of a radioactive element is greater than its half-life.

7.13 What is radioactive equilibrium? Distinguish between "transient equilibrium" and "secular equilibrium".

7.14 How will you determine, a) extremely short half-lives, b) moderately short half lives, c) long half lives.

PROBLEMS

7-1 Experimentally it is found that a gram of Thorium emits 4500 α-particles/sec. Find its half life.

Soln: No. of atoms in one gram = N = $\frac{\text{Avogadro's No.}}{\text{At. wt. of Th.}}$

$$= \frac{6.02 \times 10^{23}}{232.12} = 2.6 \times 10^{21}$$

We have from eq. (7.1)

$$\frac{dN}{dt} = |-\lambda N| = \lambda N$$

$$\text{or } \lambda = \frac{dN}{dt} \times \frac{1}{N} = \frac{4500}{2.6 \times 10^{21}}$$

$$\text{Now } T_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{4500} \times 2.6 \times 10^{21} = 1.28 \times 10^{10} \text{ years.}$$

7-2 Calculate the activity of 1 gm of $_{38}Sr^{90}$. Its half life is 28 years.

Soln: We have

$$T_{\frac{1}{2}} = 28 \times 365 \times 24 \times 60 \times 60$$

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{.693}{28 \times 365 \times 24 \times 60 \times 60}$$

$$N = \frac{6.02 \times 10^{23}}{90} = 6.69 \times 10^{21}$$

So activity = $\frac{dN}{dt} = \lambda N = \frac{.693}{28 \times 365 \times 24 \times 60 \times 60} \times 6.69 \times 10^{21} = 5.23 \times 10^{12} \text{ sec}^{-1}$

$$= \frac{5.23 \times 10^{12}}{3.7 \times 10^{10}} = 141 \text{ Curies.}$$

7-3 An activity of a radioactive sample decreases by a factor of 10 in 5 min. Find the decay constant. Also determine the half-life and the mean life.

Soln: We have from eq. (7.4),

$$N = N_0 e^{-\lambda t}$$

$$\text{or } \frac{N_1}{N_2} = \frac{N_0 e^{-\lambda t_1}}{N_0 e^{-\lambda t_2}} = e^{\lambda(t_2 - t_1)}$$

$$\text{or } 10 = e^{\lambda(300 \text{ sec.})}$$

$$\text{or } \ln 10 = \lambda \times 300$$

$$\text{or } \lambda = \frac{\ln 10}{300} = \frac{2.302}{300} = 7.7 \times 10^{-3} \text{ sec}^{-1}$$

$$\text{Also } T_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{7.7 \times 10^{-3}} = 90 \text{ sec.}$$

and from eq. (7.7)

$$\text{Mean life, } T = \frac{1}{\lambda} = \frac{1}{7.7 \times 10^{-3}} = 142.9 \text{ sec.}$$

7-4 The half life of $_{92}U^{238}$ is 4.51×10^9 years. What percentage of its nuclides that existed 10 billion years ago still survive?

Soln: We have $\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{4.51} = 0.154$ per 10^9 years

from eq (7.4) $\frac{N}{N_0} = e^{-\lambda t} = e^{-0.154 \times 10} = e^{-1.54} = 0.214$

So 21.4 % of the original number still survive.

7-5 a) The half-life of Au^{198} is 2.7 days. What is its decay constant?
b) Suppose we had a 1.00 μg sample of Au^{198}. What is its activity?

Soln: We have $T_{\frac{1}{2}} = \frac{0.693}{\lambda}$

$$\text{or } \lambda = \frac{0.693}{2.7 \times 24 \times 60 \times 60} = 2.97 \times 10^{-6}\ \text{sec}^{-1}.$$

b) No. of atoms in one gm. = $\frac{\text{Avogadro's No.}}{\text{At. wt.}}$ =

$$\text{or in 1 } \mu\text{gm of sample} = N = \frac{1 \times 10^{-6} \times 6.02 \times 10^{23}}{198} = 3.04 \times 10^{15}\ \text{atoms}$$

$$\text{so activity} = \frac{dN}{dt} = \lambda N = 2.97 \times 10^{-6} \times 3.04 \times 10^{15} = 9.03 \times 10^{9}\ \text{decays/sec.}$$

UNSOLVED PROBLEMS

7:1 The half-life of radium is 1590 years. In how many years will one gram of pure metal i) lose one milli gram, ii) reduced to one milli gram.

7:2 The half life of neutrons is 12 minute. Calculate their decay constant.

7:3 The half life of radium is 1590 years, find the time during which 15/16 of a given sample will decay.

7:4 A detector reading shows a radioactive sample emitting $3_2 \times 10^3$ counts/min. Ten minutes later the meter records 12×10^2 counts/min. Calculate the radioactive sample.

7:5 The decay constant of Pb^{214} is 4.3×10^{-4} sec^{-1}. Calculate the activity of 1 kg of the sample, and the weight of a sample whose activity is 4 curies.

7:6 The half life of U^{235} is 7.1×10^8 years and its daughter Th^{231} have half life of 24.6 hours. If 1 gram of U^{235} freshly separated. Find the activity of Th^{231} growing in the sample of U^{235} after 2 hours. What kind of equilibrium is reached?

7:7 Calculate the activity of 5×10^{-3} gm of $_{86}Rn^{222}$.

7:8 The half life of $_{83}Bi^{210}$ is 5 days. Calculate decay constant. What fraction of a given sample will remain after 15 days?

7:9 The half life of C^{14} is 5730 years. What percentage of C^{14} contained in a bone would disintegrate in 1000 years?

7:10 How many gram of $_6C^{14}$ would be required in order to have an activity of 5 curies?

7:11 What activity would be expected for carbon samples from bones that are said to be 2000 years old?

Chapter 8

NUCLEAR REACTIONS

8.1 Accelerators

To a great extent, the growth of nuclear physics has depended upon the development of methods for producing beams of high-energy particles. Radioactive substances emit alpha particles, beta particles (electrons), and gamma rays, which have been and still are used for nuclear projectiles to bombard matter of various sorts to investigate how they interact with the elements. Quite a number of high-voltage machines are constructed which have greatly extended the range of intensities, energies, and types of particles which are available for research.

All of the machines that can produce a high dc voltage called accelerators. Although research with very-high energy accelerators will undoubtedly shed some light on the nature of nuclear forces, these accelerators have been developed primarily for research in fundamental particle physics.

8.2 The Cyclotron

This machine was first devised in 1930 by E.O. Lawrence at Berkely Institute, California, U.S.A. It is a device used to accelerate positively charged particles (or ions) to high speeds (energies).

Principle:

Here the principle involved is known as resonance acceleration. Instead of accelerating the particles with high voltage they are subjected to periodic low voltage for gaining several million electron volts. This is achieved by subjecting the particles to an alternating tension of high frequency. In this way, if a singly charged particle falls through a potential difference of 10,000 volts one thousand times, then it would gain energy equal to 10 Mev.

Construction:

It consists of a large cylindrical box placed between the poles of an electromagnet, as shown in fig. 8-1. The box is exhausted until a very high

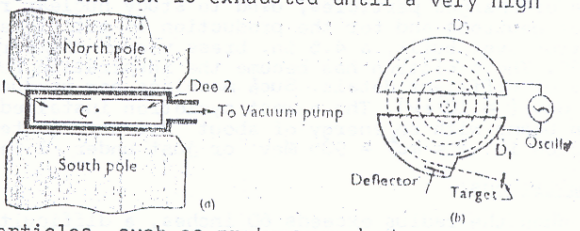


Fig.8-1:
a) side view of a cyclotron.
b) Path of a charged particle in a cyclotron.

vacuum exists inside. Charged particles, such as protons or deutrons, are fed into the centre of the box. Inside the box are two semi-circular hollow, metal electrodes called 'dees' (on account of their shape like the letter 'D') that are connected to a source of high-voltage electricity. An alternating potential of the order of 10,000 volts and of frequency $\sim 10^7$ cycles is applied between the dees. The huge electromagnet has vertical magnetic field $\sim$15,000 gauss.

Operation:

The positive ions at C will be drawn to the dee which happens to be negative at the moment. Inside the dee, it will move with constant speed, being a field-free space. Due to magnetic field acting in perpendicular direction, the ion will move in semi-circular path. Now the frequency of alternating potential and magnetic field strength be so chosen that the ion will reach the gap just when the voltage is reversed, then the particle will move with

greater velocity. After making a number of revolutions the particle approach the circumference of the dee. The particles are then deflected toward the target by a certain electrode.

Mathematical details:

Since the ion is moving in circular path under magnetic force, so we have

$$\text{Centripetal force} = \frac{mv^2}{r} \qquad \ldots\ldots (8.1)$$

$$\text{Magnetic force on a moving charge} = Bqv \qquad \ldots\ldots (8.2)$$

from equations (8.1) and (8.2), we get

$$Bqv = \frac{mv^2}{r}$$

$$\text{or} \quad r = \frac{mv}{Bq} \qquad \ldots\ldots (8.3)$$

The time T requied for completing a semi-cicular path of radius r is given by

$$T = \frac{\pi r}{v}$$

putting the value of r from eq.(8.3)

$$T = \frac{\pi \times m\cancel{v}}{\cancel{v}Bq}$$

$$\text{or} \quad T = \frac{\pi m}{Bq} \qquad \ldots\ldots (8.4)$$

$$\left\{ \begin{array}{l} S = vt \\ \text{or } t = \dfrac{S}{v} = \dfrac{2\pi r/2}{v} \\ \text{circumference of} \\ \text{a circle} = 2\pi r \end{array} \right.$$

We see that the time taken to describe a semi-circle is independent of both the radius of the path and the velocity of the ion. As the speed of the particle increases, it moves in a larger circular path but the time taken by it to cover the distance in one dee remains the same.

Cyclotron in use:

The cyclotron is extremely important to nuclear physics as an intense source of charged particles, used to study nuclear reactions, for the production of neutrons and for the production of artificial radioactivity. The first cyclotron ever built, a 4.5 in. brass vacuum chamber constructed by Lawrence in 1930. The cyclotron has become the inspiration for a wide family of accelerators of different detail. Such machines now produce beams of protons $\sim$ 1 Gev (= 10^9 ev). The newest cyclotron installed in Illinois, deliver proton beams with an energy of about 200 Gev. (Note that the rest mass energy of proton is $E = mc^2$ = 935 Mev or just under 1 Gev).

Limitations:

When the radius exceeds 60 inches, a difficulty arises.
At higher energies, the mass of the particle begins to increase according to the relativistic expression:

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

and the particle goes out of step in the cyclotron.
Electrons have relativistic speeds so they cannot be accelerated in it.
This relativistic effect of the mass can be removed by two techniques:

1. The frequency of alternating potential may be decreased as mass increases. It is adopted in frequency-modulated cyclotron or synchro-cyclotron.
2. The magnetic field strength may be increased proportionatly. It is done in synchrotron.

8.3 Linear Accelerator

In the linear accelerator, charged particles are accelerated by an oscillating electric field applied to a series of electrodes, with an applied frequency which is in resonance with the motion of the particles. In its earliest form, which illustrates the principles of this type of machine, the linear accelerator consisted of a set of cylindrical electrodes, of increasing length, arranged as shown in fig.8-2. Alternate cylinders, enclosed in a glass vacuum chamber, are connected together, the odd-numbered cylinders being joined to one terminal and the even-numbered to the second terminal of a high-frequency power supply. Ions from a discharge tube at one end move along the axis of the tubes and are accelerated on crossing the gaps between the tubes; They are not accelerated inside the tubes because the potential is constant there.

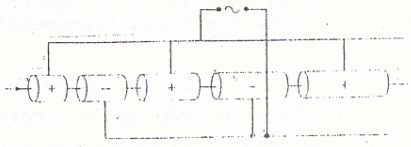


Fig.8-2: Very simplified linear accelerator.

Suppose that the ions are positively charged and moving from left to right, as in the figure. If the first cylinder is positive and the second negative, the ions are accelerated and travel through the second cylinder at a speed which is constant but greater than the speed in the first cylinder. The second cylinder is just long enough so that when the ions reach the gap between the second and third cylinders, the potentials are reversed.The second cylinder is now positive and the third negative, so that the ions are again accelerated, this time in the gap between the second and third cylinders. The ions can be kept in phase with the reversals of potential by making successive cylinders longer to allow for the increasing speed of the ions, with the result that the ions gain additional energy each time they pass from one cylinder to the next.

The early linear accelerators could be used only for heavy, slowly moving ions such as mercury ions, because the oscillators available did not have high enough frequencies. The intensive development of high-frequency methods in the field of radar during past years has made possible the design of linear accelerators with which protons and even electrons can be accelerated to energies upto several hundered Mev. These machines have both advantages and disadvantages as compared with circular machines.

1. A smaller magnet can be used than is needed for a circular machine producing particles of the same energy.
2. The size and cost of the linear machine are lesser and economical.
3. The particles automatically emerge from a linear accelerator.

The main disadvantages are:

1. Here as the particle pass through a number of alternating fields and a number of power sources, which require many pieces of high frequency apparatus, with the resulting expense and complication.
2. A very long and elaborate tube have to be built, which makes a difficult design problem.

The linear accelerator can be used to obtain energies in the range above 1 Bev. In this energy range, the betatron is impractical because of the huge magnet size and the energy loss by radiation, while the synchrotron will probably be limited by radiation loss. This loss, which is caused mainly by circular motion is not serious in the linear accelerator. The proton linear accelerator is useful at energies in the cyclotron energy range. Beams of fast electrons from a linear accelerator can be used to probe the structure of the nucleus.

8.4 Introduction of Nuclear Reactions

If energetic nuclear particles from a radioactive source, a particle accelerator, a reactor, or in the cosmic radiation are allowed to fall upon bulk matter, there is the possibility of a nuclear reaction. The first such reaction in which a transmutation of the target was identified was

$$N^{14} + \alpha \longrightarrow O^{17} + p \qquad \ldots\ldots (8.5)$$

which was studied in 1919 by Rutherford using alpha particles from a naturally radioactive source. The first transmutation reaction using artificially accelerated particles was

$$Li^{7} + p \longrightarrow He^{4} + \alpha \qquad \ldots\ldots (8.6)$$

which was observed in 1930 by Cockcroft and Walton.

We place two restrictions on the types of reaction that we choose to discuss here:

1. We assume that the projectile has a mass number A less than five and that the target nucleus has a larger mass number.
2. We assume that the energy available for the reaction is not greater than a few times the mean binding energy of single nucleons, say 50 Mev.

Exactly what happens while the target and projectile are interacting with each other is beyond the possibility of complete description at this stage of development of nuclear theory. Let us then ignore the details of the interaction and see what can be learned by simply examining the reaction products after they have removed themselves from the site of the interaction. It is only the before and after situations that are readily accessible to experimental observation in any case.

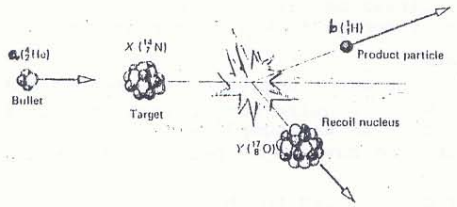
A typical reaction can be written as:

$$a + X \longrightarrow Y + b \qquad \ldots\ldots (8.7)$$

where X and Y are heavy nuclei, or for short:

$$X\,(a,b)\,Y \qquad \ldots\ldots (8.8)$$

Because of the small available energy that was postulated b will in general also have mass number $A < 5$.



8.5 Types of Nuclear Reactions

Commonly used bombarding particles are:

i) proton, ii) deutron, iii) neutron, iv) α-particle, v) γ-rays & X-rays.

The normal bombarding energy is 10 Mev.

We can classify into the follwoing simple types:

1. Elastic Scattering:

The incident particle strikes the target nucleus and leaves without energy loss, usually, with altered direction of motion. An example is:

$$He^{4} + Au^{197} \longrightarrow He^{4} + Au^{197}$$

2. Inelastic Scattering:

The incident particle reappears after the interaction with a lower energy; Some of its energy has been taken up by the target nucleus, which is thus excited to a higher quantum state. An example is:

$$Li^{7} + H^{1} \longrightarrow Li^{7*} + H^{1}$$

3. Simple Capture:

The incident particle is captured by the target nucleus, and a new nucleus is formed. Usually the residual nucleus will radiate γ-quanta. An example is:

$$C^{12} + H^{1} \longrightarrow (N^{13}) \longrightarrow N^{13} + \gamma$$

4. Disintegration:

On striking the target nucleus, the incident particle is absorbed and a different particle ejected. An example is:

$$Be^{9} + He^{4} \longrightarrow C^{12} + n^{1}$$

5. Photoexcitation and Photodisintegration:

A γ-ray is absorbed by the target nucleus, exciting it to a higher quantum state; if the energy is sufficient, a particle may be ejected. An example is:

$$H^{2} + \gamma \longrightarrow H^{1} + n^{1}$$

6. Spontaneous Decay:

β- and α-decay processes may be regarded as nuclear reactions.

8.6 Conservation Laws for Nuclear Reactions

The following are conservation laws:

1. Conservation of the total number of nucleons:

The sum of the mass numbers before and after a reaction must be the same.

2. Mass-energy conservation:

For an isolated system, since mass and energy are interchangeable the total mass-energy of the system remains constant.

3. Conservation of linear momentum:

The total linear momentum of the products must be equal to the momentum brought in by the bombarding particle.

4. Conservation of electric charge:

The total electrical charge is conserved in nuclear ...

5. Conservation of angular momentum, J:

The total angular momentum J, comprising the vector sum of the intrinsic angular momenta I(spins) and relative orbital momentum l of the particles, is conserved.

6. The Parity Rule:

The parity of the system determined by the target nucleus and bombarding particle must be preserved throughout the reaction.

8.7 Practical examples of Nuclear reactions

The following are some commonly used simple examples of nuclear reactions.

1. Proton induced reactions with the production of α-particle. An example is:

$$_4Be^9 + {}_1H^1 \longrightarrow [{}_5B^{10}] \longrightarrow {}_3Li^6 + {}_2He^4$$

2. Proton induced reactions with the production of a neutron. An example is:

$$_8O^{18} + {}_1H^1 \longrightarrow [{}_9F^{19}] \longrightarrow {}_9F^{18} + {}_0n^1$$

3. Proton induced reactions with the production of a γ-photon. An example is:

$$_6C^{12} + {}_1H^1 \longrightarrow [{}_7N^{13}] \longrightarrow {}_7N^{13} + \gamma$$

4. Deutron induced reactions with the production of α-particle. An example is:

$$_8O^{16} + {}_1H^2 \longrightarrow [{}_9F^{18}] \longrightarrow {}_7N^{14} + {}_2He^4$$

5. Deutron induced reactions with the production of proton. An example is:

$$_{11}Na^{23} + {}_1H^2 \longrightarrow [{}_{12}Mg^{25}] \longrightarrow {}_{11}Na^{24} + {}_1H^1$$

6. Deutron induced reactions with the production of neutron. An example is:

$$_3Li^7 + {}_1H^2 \longrightarrow [{}_4Be^9] \longrightarrow {}_4Be^8 + {}_0n^1$$

7. Neutron induced reactions with the production of α-particle. An example is:

$$_3Li^6 + {}_0n^1 \longrightarrow [{}_3Li^7] \longrightarrow {}_1H^3 + {}_2He^4$$

8. Neutron induced reactions with the production of proton. An example is:

$$_{13}Al^{27} + {}_0n^1 \longrightarrow [{}_{13}Al^{28}] \longrightarrow {}_{12}Mg^{27} + {}_1H^1$$

9. Neutron induced reactions with the production of neutron. An example is:

$$_{13}Al^{27} + {}_0n^1 \longrightarrow [{}_{13}Al^{28}] \longrightarrow {}_{13}Al^{26} + {}_0n^1 + {}_0n^1$$

10. Neutron induced reactions with the production of γ-ray photon. An example is

$$_1H^1 + {}_0n^1 \longrightarrow [{}_1H^2] \longrightarrow {}_1H^2 + \gamma$$

11. Photons induced reactions called photodisintegration. An example is:

$$_1H^2 + \gamma \longrightarrow [{}_1H^2] \longrightarrow {}_1H^1 + {}_0n^1$$

8.8 Q Value of Nuclear reactions

In the notation of nuclear reaction,

$$X\ (x,y)\ Y$$

x is the bullet,
X " " target
y " " product particle
Y " " recoil nucleus.

From the mass-energy conservation, we have

Rest mass energy of target + Energy(Rest mass + Kinetic) of bullet = Energy(Rest mass + kinetic) of recoil nucleus + Energy (Rest mass+kinetic) of product particle

or in symbols

$$M_x c^2 + m_x c^2 + K_x = M_y c^2 + K_y + m_y c^2 + K_y \quad \ldots\ldots (8.9)$$

The "energy change" or "Q-value" of the reaction is defined by the mass difference :

$$Q = \text{Input mass} - \text{output mass}$$

or $$= \text{Kinetic energy of output particles} - \text{Kinetic energy of Input particles}$$

or in symbols

$$Q = (M_x + m_x)c^2 - (M_y + m_y)\ c^2 \quad \ldots\ldots (8.10)$$

$$= K_Y + K_y - K_x \quad \ldots\ldots (8.11)$$

Q can be thought of as the energy which would be released in the reaction if the bombarding energy were just zero.

The energy created is the difference between the output kinetic energy and the input kinetic energy, according to the equation

$$(\Delta m)\ c^2 = \Delta E = Q$$

If Q is positive, the reaction is exoergic i.e. energy is released in the process.

If Q is negative, the reaction is endoergic, here output mass is larger than the input mass, i.e. the energy is absorbed from some outside source.

8.9 Cross-section for nuclear reaction

A convenient way to express the probability that a bombarding particle will interact in a certain way with a target employs the idea of cross section. The effective cross section for a nuclear reaction measures the target area surrounding the nucleus in which if a collision occurs a particular reaction will be produced. The reaction will take place if the bullet particle passes through this area; otherwise the reaction will not occur.

To determine a reaction cross section,

consider a slab of target, where

t = thickness of the slab

A = area of the slab

n = No. of target nuclei/unit volume

σ = cross section of each nucleus for particular interaction

N = No. of incident particles per sec.

dN = No. of interacting particles with nuclei.

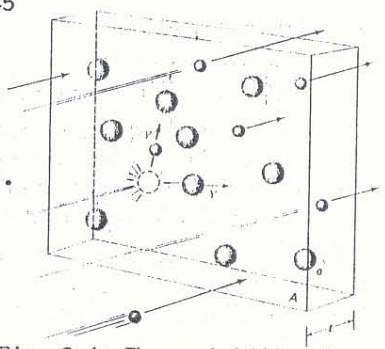


Fig. 8-4: The probability for a nuclear reaction is given by the fraction f=dN/N = $n\sigma t$

therefore

total exposed area = $n\sigma At$

so

The probability for a nuclear reaction is :

$$P = \frac{\text{total exposed area}}{\text{total area}} = \frac{n\sigma At}{A} = n\sigma t \quad \ldots. \quad (8.12)$$

this probability is also equal to

$$P = \frac{\text{interacting particles}}{\text{incident particles}} = \frac{dN}{N} = n\sigma t \quad \ldots. \quad (8.13)$$

and we see that P is directly proportional to σ.

If the reaction is endoergic, the cross-section is zero if the energy of the bullets is less than the threshold energy.

While cross sections, which are areas, should be expressed in m^2, it is customary to express them in barns,

where 1 barn = 10^{-28} m^2 .

EXAMPLE:

A thin sheet of Co^{59}, 0.02 cm thick, is irradiated with a neutron flux of 10^{16} neutrons/m^2-sec. If the radioactive capture reaction that takes place is

$$_{27}Co^{59}(n, \gamma)_{27}Co^{60}$$

how many nuclei of Co^{60} will be produced at the end of a 1 hour irradiation period?

Soln:

The density of Co^{59} is 8.9 x 10^3 kg/m^3, and the cross section for neutron capture by Co^{59} is 30 barns.

The number of target nuclei per unit volume is

$$n = \frac{\rho N_A}{M} = \frac{(8.9\times10^3\ kg/m^3)(6.02\times10^{26}\ nuclei/kg\text{-}mole)}{59\ kg/kg\text{-}mole} = 9.1\times10^{28}\ nuclei/m^3$$

The number of neutrons per square meter per second that will experience an interaction is

$$dN = Nn\sigma t = 10^{16}\times9.1\times10^{28}\times30\times10^{-28}\times2\times10^{-4} = 5.5\times10^{14}\ neutrons/m^2\text{-}sec.$$

This is also the number of Co^{60} atoms per square meter of target material that will be produced. At the end of a 1-hour period, there will be $5.5\times10^{14}\times3600\ sec. = 2\times10^{18}\ nuclei/m^2$.

8.10 NUCLEAR FISSION

Introduction:

We define nuclear fission as the splitting of a heavy nucleus into nuclei of medium mass, with the release of nuclear energy.

This word "fission" was borrowed from biology, where it denotes the division of a cell into two cells of roughly the same size.

Heavy nuclei have less binding energy per nucleon than do intermediate nuclei. This can be seen in fig. 6.4. So it is sometimes favourable for a heavy nucleus to split into two nuclei having nearly equal masses. When fission occurs, the difference in total binding energy is released as kinetic energy of the fragments and as radiation.

The lighter nuclei are about 1 MeV per nucleon more tightly bound than the heavy nucleus (see fig. 6.4), there is an energy conversion of about 200 MeV in each fission process. This should be compared with the energy released in an α-particle disintegration is only about 5 MeV and the energy released in a chemical combustion (of coal and oil) is of the order of 4 eV. The energy converted per atom is roughly 10^8 times greater in nuclear reactions than in chemical reactions.

In a nucleus, there is a tussle between the nuclear force, and the electrostatic repulsion of protons. For most nuclei, the nuclear force dominates, but for heavy nuclei there is a delicate balance between the nuclear and electrostatic forces, which can easily be shattered.

Enough data had been collected by 1934, on the disintegration of neclei bombarded by neutrons to lead Enrico Fermi, Emilio Segre, and co-workers to try to produce elements of atomic number greater than 92 by bombarding uranium with neutrons. In their experiments they found new elements assumed to be transuranic elements--that is, elements of atomic number greater than 92.

Otto Hahn and Fritz Strassman working at the Kaiser Wilhelm Institute in Berlin in 1938, showed by very careful chemical analysis that barium $_{56}Ba^{141}$ is formed in solutions of uranium compounds that irradiated $_{92}$with slow neutrons. This led them to the concept of fission as an explanation. Fission has been observed for many nuclei with mass number greater than 200. It has been induced by neutrons, photons, electrons, mesons, protons, deutrons, and alpha particles.

8.11 FISSION OF URANIUM

The element uranium has three isotopes which possess identical chemical properties. Where ever it is extracted it contains the three isotopes with $_{92}U^{238}$ at 99.3 %, $_{92}U^{235}$ at 0.7 % and $_{92}U^{234}$ less than 0.001 % . As the proportion of $_{92}U^{234}$ is very small, we can ignore its presence.

The first fission process to be discovered was that which occurs when uranium is bombarded with neutrons. With thermal (or slow) neutrons, the effect is due entirely to U^{235} . The slow neutrons have a high cross section for capture, which is about 550 barns for $_{92}U^{235}$. It is because they spend a greater length of time in the field of the

nucleus. Slow neutrons are most often produced by moderation of the neutron beam by materials rich in hydrogen, such as water or paraffin.

The schematic equation for the fission process is

$$_{92}U^{235} + {}_{0}n^{1} \longrightarrow {}_{92}U^{236*} \longrightarrow X + Y + 3{}_{0}n^{1} + Q$$

where X and Y are the <u>fission fragments</u>. The fragments are not uniquely determined, because there are various combinations of fragments possible and a number of neutrons are ejected. Typical fission reactions are following:

$$_{92}U^{235} + {}_{0}n^{1} \longrightarrow {}_{56}Ba^{144} + {}_{36}Kr^{89} + 3{}_{0}n^{1} + Q$$

$$_{92}U^{235} + {}_{0}n^{1} \longrightarrow {}_{54}Xe^{140} + {}_{38}Sr^{94} + 2{}_{0}n^{1} + Q$$

where Q is the energy released in the reaction.

When the nucleus of U^{235} is struck by a neutron this is absorbed and the new complex mass U^{236} has now such a neutron excess that it splits up into two large fragments. Almost as a large water drop does when it oscillates excessively, as shown in fig. 8-5.

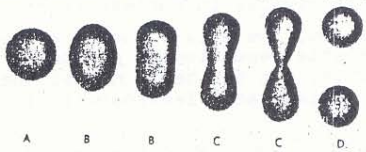


Fig. 8-5: Possible steps in the process of nuclear fission.

Although the sum of the two large fragments always adds up to (in round numbers) 234, there is a wide distribution in possible products. The yield, shown to a percentage log scale is illustrated in fig. 8-6. There is a tendency for masses to concentrate respectively round 90 and round 140. The mass range covered is considerable, from 72 to 158, but the variation is somewhat exaggerated by the fact that a log plot is used. Actually about 97 per cent of the total fission products fall within the narrow range 85-104 for the lighter fragments and 130-149 for the heavier. This asymmetry in fission process is attributed to the effect of magic numbers. (See section 6-1).

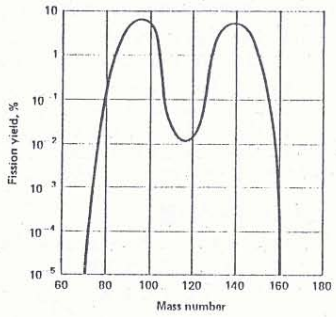


Fig. 8-6: Mass distribution of fragments from fission of U^{235}.

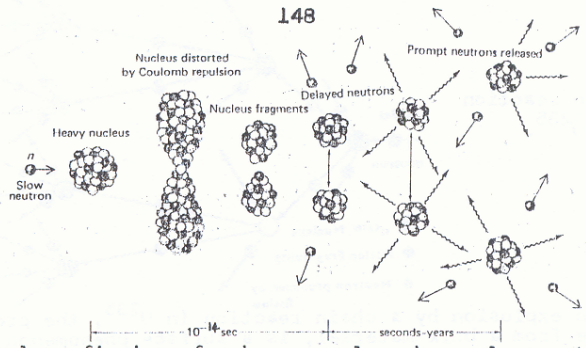


Fig. 8-7: Nuclear fission of a heavy nucleus by a slow neutron.

It is considered on strong grounds that the whole fission takes place within 10^{-14} second, during which period the two fission neutrons are ejected. Most are slow but some are fast. These two neutrons play a vital part in the utilization of the fission energy in the atomic energy projects. It is found that some 0.75 percent more neutrons are emitted for some time after the fission, up to periods of several minutes. The emitted neutrons following fission have velocities of the order of 10^9 cm/sec.

There are three features to be noted.

Firstly, in the fission reaction, most of the energy released as kinetic energy of the fragments changes into heat. This heat can be used to boil water, and the resulting steam can then be used in a certain way to derive a turbine to generate electricity.

Secondly, the average number of neutrons produced is greater than one, making possible the chain reaction.

Thirdly, the fission process is controlable(as compared with fusion).

8.12 CHAIN REACTION

We define the chain reaction as, "a continuing process of nuclear splitting in which the material or energy which starts the reaction is also one of the products".

In the fission of Th^{232}, U^{235}, U^{238}, and Pu^{239} more than one neutron is emitted. It leads to the possibility of a chain reaction in a mass of fissionable material. Whether the chain reaction remains steady, builds up, or dies down depends on the competition between the production of neutrons and their loss through some processes.

To start the process of fission one neutron is needed, but in each event of fission two to three neutrons are liberated. Such a sequence of fissions is called uncontrolled chain reaction. Few steps are shown in fig. 8-8.

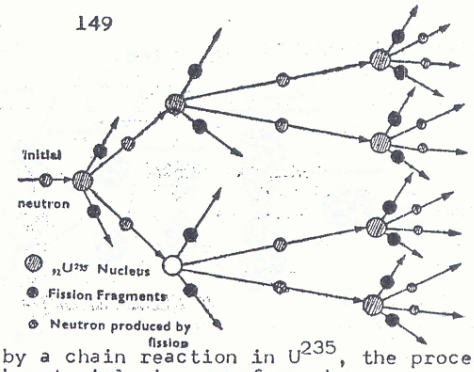


Fig. 8-8: Chain reaction in U^{235}.

To realise an explosion by a chain reaction in U^{235}, the process of neutron escape from a bulk material, is a surface phenomenon depending on the surface area, whereas the fission capture takes place with volume effect. The probability that most neutrons produced by fission would escape the mass of U^{235}, if it is too small, and the fission would be cut off before it became very large. As the mass is increased to a critical value a point is reached where a self-sustaining chain reaction sets in with rapid liberation of an anormous amount of energy. The mass of uranium required for producing a smooth chain reaction is called the critical mass and the volume occupied by this mass of uranium is called the critical volume. The first estimates made in 1939, were sufficiently encouraging that committees were set up independently in Britain and in the United States to study further the military potentials. At the same time, the out break of the war and the general recognition among physicists of the implications of these developments led to a voluntary censorship of publication which resulted in the disappearance of the subject from the open literature for next many years.

The critical mass of high purity U^{235} is perhaps 22 kilograms or in aqueous solution it is nearly 0.8 kg.

One way to describe the operating level of a reactor is to define the effective neutron reproduction factor , k, as

$$k = \frac{\text{average number of neutrons produced/unit time}}{\text{average number of neutrons disappeared/unit time}}$$

The fission chain reaction will be subcritical, when $k < 1$, and its power output level is dying down.

The fission chain reaction will be critical or steady when $k = 1$, and it will be operating at a constant power output level. As is done in a nuclear reactor.

For $k > 1$, the reactor power level will increase rapidly, and it is said to be super-critical. As the fission occurs in an atomic bomb.

8.13 NUCLEAR FISSION ENERGY

The large amount of energy is released in fission process. The following calculations give the idea of the amount of energy.

The fission of one U^{235} nucleus liberates $200 \times 1.6 \times 10^{-6}$ erg $= 3.2 \times 10^{-4}$ erg.

After multiplication with the Avogadro number, the product gives the energy released in the fission of 1 gram atom, that is, 235 gm of U^{235}, which is equal to 1.93×10^{20} ergs. The energy liberated by the complete fission of 1 kg of U^{235} would be 8.21×10^{20} ergs or about 2×10^{10} kilo-calories, which is roughly equivalent to the energy released in the explosion of 20,000 tons of TNT (trinitrotoluene).

In terms of power units the results of energy are also interesting. As

$$1 \text{ MeV} = 1.6 \times 10^{-6} \text{ erg} = 1.6 \times 10^{-13} \text{ Watt-sec.}$$

The fission of one U^{235} nucleus gives 3.2×10^{-11} Watt.sec. and 3.1×10^{10} fission/sec. gives one watt.

Fission of 1 gm of U^{235} would give 8.2×10^{10} Watt-sec. or 2.3×10^{4} kilowatt-hour or 1 megawatt-day.

If the energy spread out for the period of a day, the fission of 1 kilogram of U^{235} would produce energy in the form of heat at a rate of 1000 megawatts. If this heat could be changed to electricity at a conversion efficiency of 40 % , electrical energy would be supplied at the rate of 400,000 kilowatts. This output is equivalent to that from a large plant which consumes about 3300 tons of coal per day.

8.14 REPROCESSING PLANTS AND THE ATOMIC BOMB

In order to be able to promote a chain reaction it is necessary to remove "impurity" which through absorptions prevents the reproduction factor k from exceeding unity, in other words the isotope of U^{235} must be separated from U^{238}. This explains why the enormous isotope separation plants are developed. However, not only must the purity be high, but in addition the mass of the reacting material must be sufficiently large to ensure that a big enough fraction of the neutrons produced by fissions is retained by the material to initiate new fissions, otherwise k will fall below unity.

The two most practical means of separating U^{235} have turned out to be the electromagnetic separator and the gaseous diffusion plant. The former instrument is identical in principle to the mass spectrometer (see Section 3.35) and depends for its action simply on the fact that U^{235} and U^{238} ions have different ratios of charge to mass. Here a sufficiently large ion current is needed. For example, a beam containing 1 micro-ampere of U^{+} ions will deposit only about 1 microgram of U^{235} per day.

The gaseous-diffusion separator depends on the slight difference in diffusion rate of gases of different molecular weight. If a mixture of U^{235} and U^{238} is pumped through a porous barrier, the fractional enrichment in the light isotope will amount in a single stage to something like 1.003; to produce 99 % pure U^{235} about 4,000 stages are required. A number of such diffusion plants have been constructed, involving thousands of stages, thousands of vacuum pumps to circulate the gas, and some acres of barrier area.

A method for separation, first suggested by Fermi, consists in forcing a chain reaction in natural uranium, in spite of the presence of U^{238}, by the use of certain artificial devices known as moderators. The essential function of the moderator is to reduce effectively the adverse absorption effect of the heavier isotope, by slowing down the fast neutrons of fission very rapidly by elastic collisions to thermal energies, so that the chance of non-fission capture of U^{238} increases, as shown in fig. 8-9. Carbon in the form

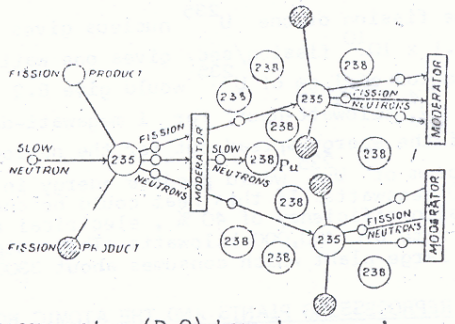


Fig. 8-9: Principle of action of the moderator.

of graphite and deuterium in heavy water (D_2O) have been used as moderators. This method has the additional advantage of producing Pu^{239}.

The fission properties of Pu^{239} have not been published, but it is known that it undergoes fission with thermal neutrons, and it behaves like U^{235}. It is formed from U^{238} through fast moving neutron capture followed by two β-decays. Pu^{239} is stable to further β-decay and would be a fairly long-lived α-emitter which would probably undergo fission when bombarded by slow fast neutrons, as does U^{235}. The important advantage of plutonium is that it can be produced in a reactor and the separation from the parent uranium can be carried out by chemical means. Pu^{239} can be made in sufficiently large amounts, as compared with a separation of U^{235} from U^{238}.

An atomic bomb or fission bomb is a device designed to produce a chain reaction that builds up at an explosive rate. To initiate the chain (i.e. to detonate the reaction) it is only necessary to bring together, by violent impact, two pieces of otherwise safe material each of less than the critical mass. The mechanical act constitutes the detonation since there are always sufficient stray neutrons in such a mass to initiate the chain spontaneously. The mass

reacts in about 10^{-7} seconds. The rate of growthof un-controlled chain reaction is shown in fig.8-10. It would produce an energy equivalent to 2×10^{13} joules. The temperature of the reacting mass rises perhaps to 10^{7}°C, and a pressure of perhaps 10^{12} atmospheres develops, hence it begins to expand so rapidly that the chain collapses long before all the fissile materials react; indeed only a few per cent suceed in reacting. Since radiation is proportional to the fourth power of the temerature, the explosion will be accompanied by a violent and intense blast of visible, ultraviolet X-ray and γ-ray radiations causing a blind flash. The radioactive dusts formed are carried away by air-current and soon disappear if they have short period or get deposited on surrounding objects. The first atomic bomb released energy equivalent of about 20,000 tons of chemical explosive TNT. Many atomic bombs or fission bombs have been exploded, but, for various reasons this application of nuclear physics will not be discussed further in this book.

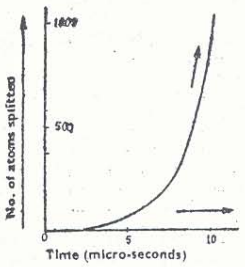


Fig. 8-10: Growth of fission in U^{235}.

8.15 NUCLEAR REACTORS

A nuclear reactor is a device in which the controlled fission of certain substances is used to produce new substances and energy.

Nuclear reactors are of many kinds, sizes, and shapes. They can be classified according to the characteristics of the chain-reacting system. The following features are used to describe them.

1- The neutron energies at which the fission occur.
   a) High energy, that is, due to fast neutrons.
   b) Intermediate energy.
   c) Low energy (thermal).

2- The purpose:
   a) Research to produce radioisotopes.
   b) Production of Plutonium.
   c) Power to produce energy for peaceful purposes.

3- The fuel-moderator;
   a) Homogeneous--the natural uranium in the form of a solution of a salt alongwith liquid moderator, such as, heavy water.
   b) Heterogeneous--the uranium in the form of lumps or rods and graphite is the moderator.

4- The moderator:
   a) Graphite
   b) Water
   c) Heavy water (D_2O)
   d) Beryllium or beryllium oxide

5- The coolant:
   a) Air, CO_2, or He
   b) Water or other liquid
   c) Liquid metal.

Fig. 8-11: Enrico Fermi (1901-1954)

The first successful nuclear reactor was constructed and put in operation in December 1942 in the University of Chicago under the direction of Fermi. After few modifications it is shown in fig.8-12.

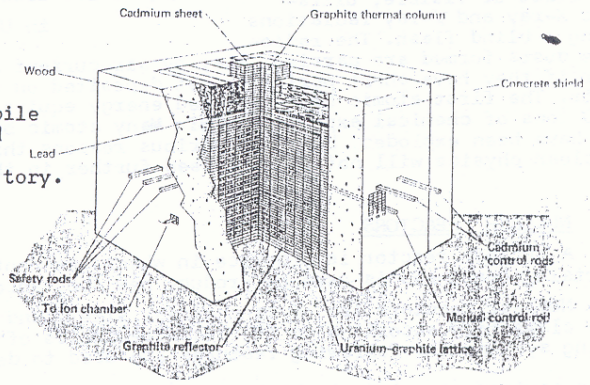


Fig. 8-12: Chicago pile reactor located at the Argonne National Laboratory.

It contains about 52 tons of natural uranium in the form of lumps interspersed with about 472 tons of graphite, which acts as the moderating substance. There are five control rods made of 17 feet long brass strips covered with cadmium. As an automatic safety feature, there are three rods that have weights on them to pull the rods into the reactor in case of an electrical power failure. It produces a low power of 300 watts.

The reactor at the Brookhaven National Laboratory was originally a thermal, natural uranium, graphite-moderated, heterogeneous, air-cooled research reactor. The NRX reactor at Chalk River, Canada, is a thermal, natural uranium, heterogeneous, heavy water-moderated, ordinary water-cooled, research reactor.

In a typical nuclear reactor, a mass of uranium slightly greater than the critical mass is taken. The rate of chain reaction is controlled by using neutron absorbing rods. Cadmium or boron is one of materials which absorbs neutrons. If cadmium rods are put into the reactor, the chain reaction will slow down. By controlling the

length of these rods inside a reactor, the chain reaction can be controlled. The rods are inserted in the reactor in such a way that only the desired number of neutrons are left for producing chain reaction, the others are absorbed by it.

8.15.1 THE RESEARCH REACTOR

In one form of this reactor, uranium rods are placed inside a block of graphite. Fast moving neutrons from the fission of U^{235} are slowed down by graphite. The slow neutrons or thermal neutrons are not easily captured by U^{238} instead the isotope U^{235} is used for fission. Cadmium rods absorb rapid production of neutrons, so the controlled chain reaction is obtained. The research reactor is used for producing radioactive isotopes (or radioisotopes), which have very useful applications. (See Section 8.22).

8.15.2 SWIMMING POOL REACTOR

It's another type of research reactor. Here rods of enriched uranium are suspended in a deep uncovered tank of water. The water acts as a moderator. In case the tank is filled with heavy water, the chain reaction can be maintained by using rods of natural uranium.

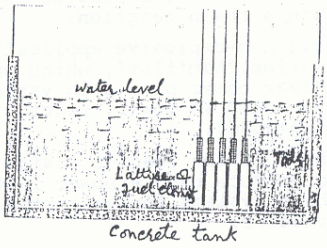


Fig. 8-13: Simplified diagram of Swimming Pool Reactor.

8.15.3 THE PLUTONIUM REACTOR

The plutonium reactor is used to produce plutonium which can be used in the atomic bomb as a fissionable material. The reactor contained a critical mass of uranium. The critical mass of U^{238} is about 25 tons. Here two types of reactions occur. Neutrons cause fission to U^{235}. Then some neutrons strike U^{238} nuclei and produce plutonium.

A breeder reactor is one in which a fissionable material is produced at a greater rate than the fuel is consumed. These reactors are designed so that they make available more fuel than they consume.

8.15.4 POWER REACTOR

A power reactor is a device to make use of the natural heat developed in a uranium reactor as source of huge power.

A kilogram of uranium, when completely utilized in fission reaction, has about the same fuel value as 3×10^6 kg of coal or 1.2×10^7 kg of oil burned in the conventional way. Nuclear power is an important substitute for the world's energy supplies.

Fig. 8-14 shows, the basic difference between a nuclear power plant and a conventional one is in the way in which the steam that turns the turbines is produced. In an ordinary plant, the

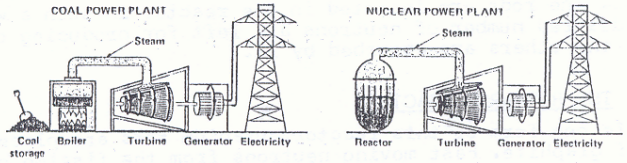


Fig. 8-14: A nuclear power plant differs from other power plants only in the way in which steam is generated for the turbines.

steam comes from a boiler fired with coal, oil, or gas. In a nuclear plant, the steam is generated by the heat released during the fission process in the reactor. The reactor in a nuclear power plant must be able to sustain a chain reaction.

The first non-explosive application of nuclear power was in the American submarine 'Nautilus', which was launched in 1954. Appendix lists the nuclear power plants in various countries in 1981. In Pakistan, we have a 125 MW power reactor in Karachi (and a 5 MW research reactor in Islamabad).

A problem in the reactor design is the extraction of the fission energy to produce usable power in the form of electricity. There are in use three systems for extracting the fission energy from the core of the reactor.

1) BOILING-WATER REACTOR

In it a stream of water circulates through the core, as shown in fig. 8-15. The heat turns the water to steam, which is then used to generate electricity. The drawback of this is that the water can become radioactive.

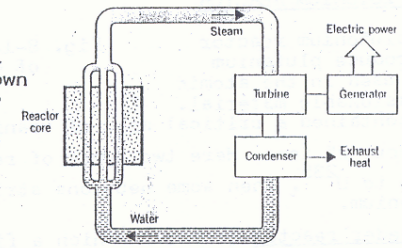


Fig. 8-15: A boiling-water reactor.

2) PRESSURIZED-WATER REACTOR

Here the heat is extracted in two-step process, as shown in the fig. 8-16. Water circulates through the core under great pressure, to prevent its turning to steam. This hot water then in turn heats a second water system, which actually delivers steam to the turbine.

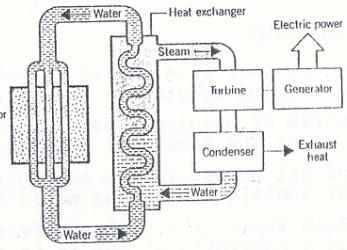


Fig. 8-16: A pressurized-water reactor.

3) LIQUID-METAL REACTOR

The disadvantage of using water is that it has small heat capacity. A metallic medium would be much better for heat transfer. Liquid sodium could replace the pressurized water in fig. 8-16.

8.16 FUSION

Nuclear fusion is a reaction in which light nuclei combine to form a nucleus with medium mass, with the release of nuclear energy.

The binding energy curve in fig. 6.4 shows that a combination of two light nuclei on the steep portion of the curve forms a heavier nucleus, which has a larger binding energy per nucleon than either of the lighter nuclei. Because there is a greater binding energy there is a decrease in nuclear mass, which results in a positive Q and a release in energy.

The sun and other stars are intensely hot. They attribute huge amount of heat. They are emitting heat continuously since billions of years. It was a puzzle until the discovery of nuclear reactions, because all other sources of energy could not possibly generate that amount of radiations.

Fig. 8-17: Hans Albrecht Bethe

When a gas is heated to very high temperature, majority of the gas atoms are ionized. The charged ions and electrons make the behaviour of ionized gases entirely different as compared to neutral gas. Ordinary gas laws are insufficient to describe their behaviour. These very hot and very active ionized gases are called the fourth state of matter which is named plasma. Under these conditions Coulomb repulsion forces are overcome, and the atoms enter into nuclear fusion reactions and liberate large quantities of energy. Hydrogen is the most abundant element. Hans Bethe (1939) developed the theory of carbon cycle as the most probable source of energy production by fusion:

(1) $_1H^1 + {}_6C^{12} \longrightarrow {}_7N^{13} + \gamma$

(2) $_7N^{13} \longrightarrow {}_6C^{13} + {}_{+1}e^{0} + \nu$

(3) $_1H^1 + {}_6C^{13} \longrightarrow {}_7N^{14} + \gamma$

(4) $_1H^1 + {}_7N^{14} \longrightarrow {}_8O^{15} + \gamma$

(5) $_8O^{15} \longrightarrow {}_7N^{15} + {}_{+1}e^{0} + \nu$

(6) $_1H^1 + {}_7N^{15} \longrightarrow {}_6C^{12} + {}_2He^4$

In this chain reaction, four hydrogen atoms are consumed and that two positrons, three γ-rays, two neutrinos, and one helium nucleus, are created. The other nuclei cancel out. C^{12} acts like a catalyst. We can write the net process as

$$4\,{}_1H^1 \longrightarrow 2He^4 + 2\,{}_{+1}e^0 + 2\nu + 3\gamma + Q$$

Since the two positrons disappear in this process, the only masses remaining are four hydrogen atoms and one helium atom, and so the mass difference is

$$4\,{}_1H^1 - ({}_2He^4 + 2\,{}_{+1}e^0)$$

So
$$Q = (m_i - m_f)c^2$$
$$= 4 \times 1.007825 \text{ amu} - (4.002603 + .001098)\text{amu} \times 931.5 \text{ MeV/amu}$$
$$= 25.7 \text{ MeV}$$

Each fusion reaction liberates about 25.7 MeV of energy.

Another fusion cycle, proton-proton cycle given below has been suggested to explain the creation of solar and steller energy

(1) ${}_1H^1 + {}_1H^1 \longrightarrow {}_1H^2 + {}_{+1}e^0 + \nu$ (twice)

(2) ${}_1H^2 + {}_1H^1 \longrightarrow {}_2He^3 + \gamma$ (twice)

(3) ${}_2He^3 + {}_2He^3 \longrightarrow {}_2He^4 + 2\,{}_1H^1$

The net result of these reactions is that four hydrogen nuclei (protons) are combined to produce one helium nucleus (α-particle), two positrons, and two γ-rays. We can write the net process as

$$4\,{}_1H^1 \longrightarrow {}_2He^4 + 2\,{}_1e^0 + 2\nu + 2\gamma \quad \ldots\ldots (8.14)$$

The energy released in this cycle is 26.2 MeV.

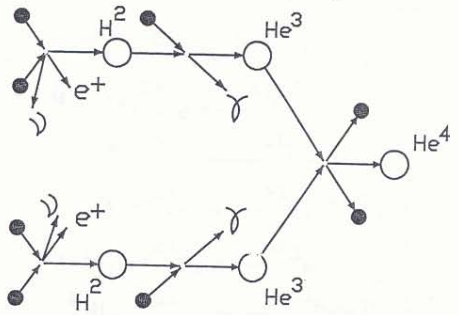


Fig. 8-18: The processes in fusion of protons to form helium.

The energy liberated in the fusion of light nuclei into heavier ones is called thermonuclear energy. Controlled fusion reactors are probably the most promising sources of energy for the future. The Temperatures required to produce fusion can be achieved with the help of high energy particles or with a high-power-laser. A difficult problem is that of confining the heated plasma during the fusion process. Another problem is to overcome Coulomb repulsion forces.

In a star the plasma is confined by the enormous gravitational forces. For a reactor on our planet earth, two confinement schemes are under development. One is magnetic confinement often called "magnetic bottle", in which external magnetic forces on the moving charges of the plasma prevent the plasma from dispersing. The other confinement scheme is called inertial confinement. In this case the fuel is heated to the fusion temperature by an array of high power lasers that bombard it from all directions in a few nanoseconds. The fusion process would be completed before the particles could disperse.

8.17 THERMONUCLEAR OR HYDROGEN BOMB

The explosion of nuclear bomb using fission chain reaction develops a high temperature. This has led to the development of hydrogen bomb in which hydrogen nuclei are fused to form helium nuclei at extremely high temperature produced by the explosion of nuclear bomb.

The essential conditions required for the operation of the hydrogen bomb are the very high temperatures and pressures, which are present in the stars. Thus the Sun and stars are natural hydrogen bombs. In hydrogen bomb there is no limitation of a critical size. Any amount of active material can be used for the thermonuclear energy.

The first hydrogen bomb, exploded in November 1952 in the Marshall Islands, America. Its explosive power was 3 million tons of TNT. In hydrogen bombs probably deuterium or a mixture of deuterium and tritium chemically combined with lithium are used. Details are not a matter of public information. The reactions are given below:

fusion of two deutrons:

$$_1H^2 + {}_1H^2 \longrightarrow {}_2He^3 + {}_0n^1 + 3.3 \text{ MeV}$$

or

$$_1H^2 + {}_1H^2 \longrightarrow {}_1H^3 + {}_1H^1 + 4.0 \text{ MeV}$$

Deutron-tritium combination:

$$_1H^3 + {}_1H^2 \longrightarrow {}_2He^4 + {}_0n^1 + 17.6 \text{ MeV}$$

In the atom bomb, energy is released by fission. In hydrogen bomb the reverse process is occurred. That is the fusion of light nuclei into a heavy nucleus. In order to start fusion, very high temperature is needed. The high temperature is obtained by exploding a fission bomb inside the thermonuclear bomb. The fusion reaction produces a large number of fast moving neutrons which can be used for the fission

of U^{238}. So the natural uranium may be used as a layer on fusion explosive. A hydrogen bomb is thus fission-fusion-fission weapen.

Thermonuclear bombs yielding an energy release equivalent to 10^7 tons of TNT have been successfully detonated.

8.18 NEUTRON BOMB

It has been found that when hydrogen bomb is encased in a special metallic cover, still worse effects are produced by the giving off of lethal radiations sometimes for months and years.

The cobalt bomb is one of this kind. It is made by covering hydrogen bomb with metallic cobalt. When it is exploded, neutrons emitted react with cobalt cover and form Co^{60} which is three hundred times powerful than radium. It does not confine to a particular area, the winds will take it to distant parts, thousands of miles away, carrying death wherever it reaches upto a period of about five years.

8.19 PARTICLE DETECTORS

Modern techniques have developed so far beyond those of the earliest workers in radioactivity that it is now relatively easy to detect single atomic particles, e.g. α-particles, β-particles, fast moving protons, γ-rays, etc.

Five distinctive methods have been developed for the detection of single particles. These can be classified as photographic, cloud chamber, bubble chamber, optical and electrical method. Two of them viz. Geiger-Muller Counter and Solid State Detector will be discussed in detail.

1- Photographic

Nuclear photographic emulsions are up to 1mm in thickness. These can be stacked in layers. When such emulsions are exposed to high energy nuclear beams, the complete track of a high energy particle can be observed after developing the emulsions.

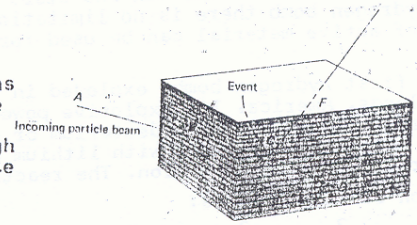


Fig.8-19: Stack of photographic emulsions.

2- Cloud Chamber

Wilson Cloud Chamber is a device that makes the paths of ionizing particles visible. Here a supersaturated vapour condenses into liquid droplets around the ions. It consists of a glass-fronted cylinder containing a mixture of air and water vapour. When the piston is moved back rapidly, the vapours around incoming ions cools adiabatically and a trail of particles can be observed.

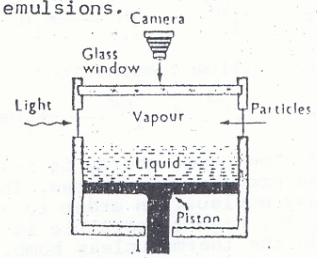


Fig.8-20: Wilson Cloud Chamber.

3- Bubble Chamber

Liquid Bubble Chamber consists of a thick walled glass box filled with a superheated liquid, and the tracks of particles become visible as a series of bubbles of vapour form around the incoming ions or particles.

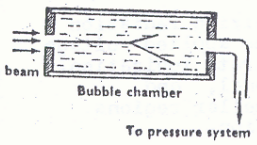


Fig. 8-21: Bubble Chamber with track illumination.

4- Optical:

Spark Chamber is a set of conducting plates, usually aluminium, alternately connected to a source of high dc voltage(about 20 kV). Sudden application of very high voltages to alternate plates, while the others are left at ground potential, results in very high electrical fields across the gaps. Electrical breakdown then occurs along the trails of ions, so the trajectory of a given particle is marked by a series of sparks.

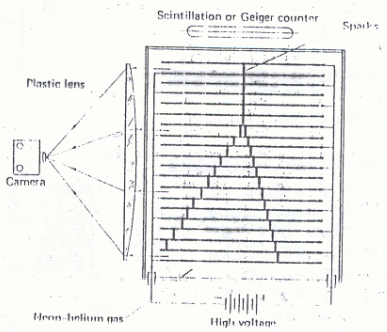


Fig. 8-22: Cross-sectional view of a typical spark chamber.

5- Electrical:

i) Ionization Chamber consists of a gas-filled container with fixed electrodes. When the gas in the chamber becomes ionized because of the passage of a high-speed particle, the electrons and ions drift to the electrodes varying with their nature and velocity.

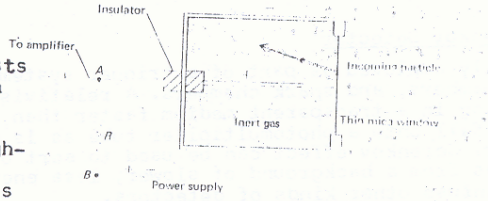


Fig. 8-23: Typical ionization Chamber.

ii) Proportional counter is a type of ionization chamber. In this device (shown in fig. 8-24), a voltage of 250 to 800 volts is applied to the electrodes so that the original gas ions produced by the particle are accelerated to sufficient energy to generate other ions by collision with neutral molecules of the gas. The total charge collected for each particle is larger than the original charge produced and is proportional to it. The use of this counter permits both the counting and the energy determination of particles.

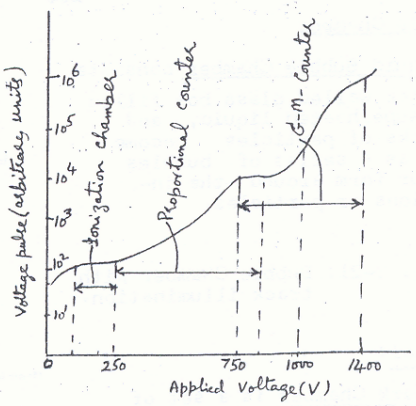


Fig. 8-24: Ionizing radiation verses the applied voltage to illustrate ionization, proportional, and Geiger-Mueller regions of operation.

iii) Geiger-Muller Counter

See Section 8.20.

iv) Scintillation Detector

registers the flashes of light produced when subatomic particles hit a fluorescent screen. (Like TV picture tube). It provides energy and nature of the particles. Photomultiplier tube amplifies the initial output of photoelectrons.

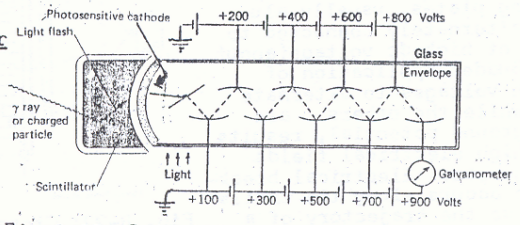


Fig.8-25: Scintillation detector used with a photomultiplier tube.

v) Cerenkov Detector

is usually employed as part of a trigger system for bubble chambers, cloud chambers, and spark chambers. A relativistic charged particle travelling in a transparent medium faster than light is collected and made to fall onto a photomultiplier tube as is done in scintillation detector. Cerenkov effect can be used to sort out highly energetic particles from a background of slower, less energetic particles that might confuse other kinds of detectors.

Definition: Cerenkov (Cherenkov) radiation effect:

Light emitted when charged particles pass through a transparent medium at a speed greater than the speed of light in that medium.

vi) Solid-State Detector

See Section 8.21.

8.20 GEIGER-MUELLER COUNTER

The most widely used detector of single particles is the Geiger-Mueller Counter. Here the principle of ionization chamber is used. This counter, developed by Geiger and Mueller (1913), is very simple to construct and is extremely sensitive to the passage of charged particles. This operates in the unstable region (800-1000) of fig.8-24. It is usually worked with about 1,200 volts applied between the electrodes.

Fig.8-26: Geiger-Mueller counter tube

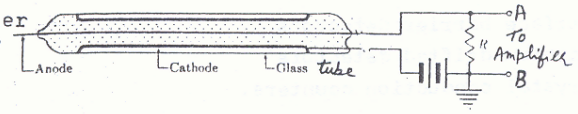


Fig. 8-26 shows the essential parts of a Geiger-Mueller counter. It consists of a long glass tube containing two electrodes. The central wire is very thin and is the anode. The counter is usually filled with 90 % argon and 10 % ethyl alcohol at a pressure of about 2 to 10 cm of Hg.

Geiger-Mueller counter or G.-M. counter has been designed for the measurement of α-, β-, γ-rays, and X-rays. In the form described, they are used mainly for β - and γ-rays.

The discharge which takes place on entry of a particle is a corona discharge along the whole length of the wire. For high accuracy the temperature of the counter should be controlled.

The difference between the proportional counter and the Geiger-Mueller counter is that whereas in the proportional counter an ion which enters produces in its own neighborhood an avalanche of ions, in the Geiger-Mueller counter avalanches which are formed spread rapidly along the whole length of the central wire. The amplification does not therefore depend on the ionising power of the incident particle. Amplification of the order of 10^8 is produced and this great amplification leads to a sufficiently large ion current to be recorded.

Definition: Corona discharge:

Small regions of glowing air usually accompanied by small sounds and is caused by the ionization of the air.

8.21 SOLID-STATE DETECTORS

When some non-conducting solids are exposed to nuclear radiations, tracks are produced. It is due to distortion of crystal lattices. Cosmic rays and other radioactive decay products have left tracks in naturally occuring solids in the solar system. In fig.8-27, by etching the solids with acid, we see the tracks on lunar samples.

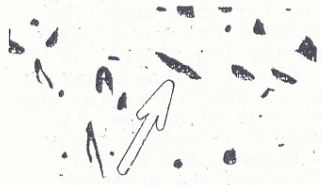


Fig. 8-27: By etching fragments of lunar rocks, they are marked with tracks left by cosmic rays.

First solid-state detector was made in 1959 by two British scientists. It was a thin sheet of mica.

Solid-state detector or the semiconductor detector consists of a semiconducting crystal in which electrons are not free to move, even though a voltage is applied to various parts. The passage of ionizing radiation through the crystal provides mobile electrons which are collected by the wires that apply the external voltage. Then the electrical pulse is amplified.

Three types of solid-state detectors will be discussed.

1- Surface barrier detectors
2- Lithium drifted detectors
3- Crystal conduction counters.

1) Surface barrier detector

The surface barrier detector is used for charged particles. When a charged particle is passed through the semiconductor, its conductivity increases due to formation of "holes" and "free electrons". It consists of silicon layer which is of N-type material. On the surface a thin P-type layer of gold is deposited. The two conducting surfaces are connected with resistance R and to a battery.

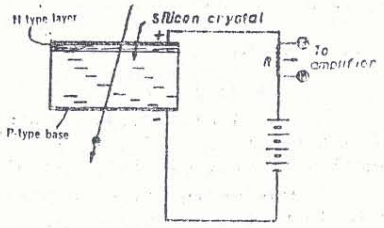


Fig. 8-28: A semiconductor detector.

When some ion is entered, a pulse of current is produced across R and is amplified and then recorded.

2) Lithium drifted detector

Lithium drifted semiconductor detector is used for γ-rays.

Here a P-type cylindrical crystal is taken, Li is diffused on the surface by painting and then heating for some time. The finished crystal has three parts as shown in the fig. There is a central pole of P-type material followed by Li drifted region and an outside layer of N-type Li diffused region. Electric field is applied between the central P-type region and the outside surface. The purpose of drifting Li is to compensate the P-type

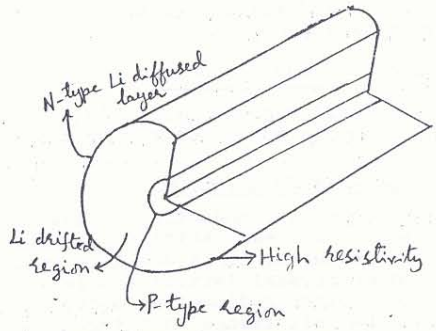


Fig. 8-29: Lithium drifted detector.

impurities originally present and produce a high resistivity region.

The detection of γ-ray and measurement of γ-ray energy are done by trnasfer of γ-energy to charged particles i.e., electrons. These electrons are produced in photoelectric effect, Compton effect and pair production. The electrons loose energy in the active region of the semiconductor detector producing ion pairs and a pulse is formed on the collection of these ions. Thus not only the γ-rays are detected but also the energy is determined from the pulse height.

3) Crystal conduction detectors

Certain crystals, like gases, become conducting when exposed to radiation and this property was first used for counting by Van Heerden (1945) who found that a crystal of silver chloride at very low temperature register a pulse of current when certain radiation passes through it. Some types of diamond operates at normal temperature for β- and γ-radiations.

This counter is rapid in action, so it is attractive. Presently it is not widely used.

8.22 RADIOISOTOPES

A radioisotope is radioactive isotope of an element.

In Section 7.11 we have seen few radioisotopes as a result of natural radioactive decay. In research nuclear reactors radioisotopes of nearly all the elements have been prepared artificially.

During the fission process in the reactor neutrons are emitted. Radioisotopes can be prepared by inserting elements into the reactor during its operation. Elements will absorb a neutron and emit a proton. So the mass number remain same but the atomic number decreases. For example

$$_{7}N^{14} + {}_{0}n^{1} \longrightarrow {}_{6}C^{14} + {}_{1}H^{1}$$

The cost of producing radioisotopes in a research reactor is very small as compared for producing in cyclotrons.

Uses of Radioisotopes

The radioisotopes possess the same chemical properties as non-radioactive isotopes of the same element. Radioisotopes are widely used in various fields. We will consider in the fields of Physics, Chemistry, Biology and medicine.

1- PHYSICS

a) Tracing materials with radioisotopes

Radioactive Fe^{59} in the piston rings of an engine is transferred to the oil through friction in motor which is running. Negligible amount of iron in the oil can be detected by the counter at the other side. In this way inspection of gear-boxes etc. are done without dismantling the parts.

b) Identification of the presence of radioisotopes

Very very small amount of radioactive sodium solved in the water can be detected which is difficult to check from ordinary methods.

c) Effects of radioisotopes

The radioisotope is used as a source of radiation, just as X-rays and radium are used. The γ-rays from Co^{60} are used to kill bacteria in food products. Properties of the plastic can be changed from the radiations.

2- CHEMISTRY

a) Identification of atoms in exchange reactions

By heating we exchange iodine atoms occuring between CH_3I and I_2. There appears to be no exchange of iodine atoms between CH_3I and NaI under all circumstances. But the exchange of atoms can be detected by tracer method.

b) Szilard Chalmers reactions

When neutrons are bombarded in the mixture of Bromine and ethyl bromide, Br atom is knocked out from CH_3Br and become radioactive, but CH_3Br catch another Br atom. By this method we can increase the concentration of radioactive atoms.

3- BIOLOGY

a) Tracing the biological changes in animals

The feed and the radioisotope are mixed and given, for example, to a cow. It is eaten and converted into milk inside the body of a cow. Then the detection instruments are used for the biological changes to be noted.

b) Tracing chemical changes in plants

Radioactive phosphorous ($_{15}P^{32}$) are feeded through the roots of plants and flowers. Then through radio auto-graph or other methods the leaves or branches show the distribution of phosphorous fed to them.

c) Developing genetic changes

A "gene" is an entity for the determination of hereditary characters. Certain high radiations ,e.g., β-particles do serious changes. It may be used for producing mutations.

4- MEDICINE

a) Diagnosis

Sodium which is important in body fluids and tissues is easily traced by means of its radioisotope Na^{24}. If a small quantity of radioactive iodine is taken with the food, the position of the iodine can be followed with the help of G.-M. counter. So the detector tells the position of the food in the digestive system.

b) Metabolic processes

Radioactive calcium or other radioisotopes which have an affinity for a certain biological system, is introduced in the body. With sensitive detectors, it is possible to reconstruct, using computers, a three-dimensional view of the system that can reveal abnormalities, tumors, defective bones, or other difficulties.

8.23 OTHER APPLICATIONS OF RADIOISOTOPES

Radioisotopes are also used in the fields of agriculture, industry, archaeology, and research.

1- AGRICULTURAL

a) Labelling fertilizers

Radioisotopes are chemically combined with the compounds of fertilizer. The radio-compound that results is said to be tagged or labelled. Which are used for experimental growth.

b) Feeding and growth of plants

Some plants are unable to form sufficient amount of chlorophyl to maintain their growth. Radio iron (Fe^{59}) alongwith radiophosphorus (P^{32}) has been used to determine the insufficiency. The system of feeding through the leaves have provided a method for studying the plants feeding.

c) Productivity of food grains

Labelled fertilizer of radio phosphorous is placed at several depths and distances from the plant. The relationship between the root growth and taking of phosphorous from the soil determine the percentage productivity of food grains.

2- INDUSTRIAL

a) Radiographical inspection

Usually polish or wax is worn away from the hood of a polished car. It can be checked. Inspection of castings, molds, etc. is possible by photographing the interior picture. It is done with the help of a sensitive film placed against that object.

b) Detection of dimensions

By placing the material between the source of radiations and detector, position, density or thickness can be measured.

c) Labelling the elements

Labelled radioactive carbon (C^{13}) mixed in certain compound provide a simple test of leaks in pipes and the flow of rates of liquids without effecting the actual flow.

3- RADIO CARBON DATING

a) Age of organic compounds

See Section 7.11.

b) Age of earth, solar system and our galaxy

With the help of radioactive series (Section 7.10) and measurement of half-life (Section 7.13) we are able to estimate the age of our planet earth.

We estimate the age of solar system and our galaxy from analysis of cosmic rays, the radiations received by the sun, and from the theories of thermo-nuclear reactions.

c) Geographical origin of prehistoric people

Chemical composition of various types of pottery can help us to trace the geographical origin of the clay from which they were made. They help to trace the trading routes of prehistoric people. (See Table IV in section 7.11).

4- RESEARCH

a) Neutron activation analysis

Nearly every radioactive isotope emits characteristic gamma rays, and many chemical elements can be identified by their gamma ray spectra. The chemical analysis of minute materials such as paint , gunshot residues, soil, or hair can provide great investigations. Samples of hair of Newton , has revealed the chemicals exposed at that time.

b) Synthetic elements

Atoms beyond uranium are all radioactive, with half-lives short compared with the age of the Earth. They can be produced in the laboratory. Their study would be of great interest for the periodic table. Pu^{235} has used as a power source in Voyager Spacecraft, which photographed Jupiter and Saturn.

8.24 HARMFUL AFFECTS OF RADIATIONS

The radiations with which we are concerned are α-, β-, γ-, neutron-and X-radiations. It excludes the infra-red, ultra-violet and high frequency radio-wave radiations which are capable of producing biological damage, e.g., to the eye and skin.

Radioactive isotopes are extremely dangerous because we can receive a harmful dose of radiation without realising it. It probably caused the death of Marie Curie, who first isolated radium. Within one year after the discovery of the X-rays by Roentgen, a manufacturer of X-ray tubes noticed X-ray damage to his hands and sought medical treatment. Becquerel received a radiation burn on his chest from carrying a vial containing radium in his vest pocket.

Strong doses of radiation can burn the skin. But a greater danger is the effect of gamma radiation on body tissues, bones and blood cells deep inside the body γ-radiation may even lead to cancer or leukaemia (cancer of the blood) in later life. Frequently, normal cell replacement is stopped due to the inability of cells to reproduce after being subjected to ionizing radiations. The most sensitive cells are those in the bone marrow, lymph glands, lining of the mouth and intestines, reproductive organs, hair follicles, and skin. Liver and kidney tissues are moderately sensitive, while nerve, brain, and muscle tissues are least sensitive.

Radiations produce genetic changes and mutations, so great protections are needed from intense radioactivity.

8.25 DIFFERENT UNITS

1) Rad:

It is defined as the absorbed dose of any nuclear radiation that results in the absorption of 10^{-2} joules of energy per kilogram of absorbing material.

2) Roentgen (r):

The roentgen is the unit of radiation exposure. It is defined as the quantity of X-ray or γ-radiation such that in the air the associated corpuscular emission per 1.293×10^{-6} kg of air produces a quantity of electricity equal to one electrostatic unit.

3) Rem (Roentgen equivalent man):

This unit is the quantity of ionizing radiation such that the energy imparted to a biological system per gram of living matter by the ionizing particles present in the locus of interest has the same biological effectiveness as one rad of 200 to 250 kilovolt X-rays.

8.26 RADIATION SICKNESS

In Japan, victims of the atomic explosions of 1945 were found to be suffering from a peculiar new illness which did not develop until several hours had passed. Manifested by vomiting and diarrhoea. The pattern of radiation sickness depends on which organ of

the body are most affected. Following something less than a lethal dose (400-600 rem), the victim complains nearly after six hours, of nausea, loss of appetite, vomiting and possibly diarrhoea. By the fifth day, these features have resolved, and he is then well for nearly four weeks. Symptoms then return due to damage to the stomach and intestines. Irritation of the skin with reddening and sweating follows. It results gradual disappearance of most of the circulating blood cells. If death does not occur, a number of other events may follow months or years later, e.g., abnormalities of the blood count, impairment of fertility, and an increased liability to cancer and leukaemia. High doses ($>$ 2,000 rem) result in brain damage, fits, coma and death within a few days of exposure.

8.27 PROTECTION METHODS

In installations, protection against radiations is based upon three factors.

i) the distance between the individual and the radiation source.

ii) the provision of absorbing barriers.

iii) limitation of the exposure period,

The laboratories should be isolated from other working areas, access being through special changing rooms where protective clothing can be donned; continuous environmental and personnel monitoring will be necessary. In hospitals and in industry many safety precautions are taken, such as: protective clothing, lead or concrete shielding and handling with long mechanical tongs.

a) Radiation Monitoring

Site monitoring includes,

i) routine testing for contamination to check that sources have not been damaged or the contamination on benches or floors does not exceed the permissible level.

ii) measurements, by permanently installed dose-rate measuring equipment, in areas where a radiation hazard may develop.

Personnel monitoring is necessary for all radiation workers. It is done with photographic film, small ionization chambers or with scintillation counters.

b) Nuclear plants

The fission products present in the "processable" uranium are extremely dangerous. It must be protected with great care. All the operations have to be carried out by remote controls behind thick and safe shields. The nuclear reactor is surrounded by concrete walls several feet thick. In the power reactors the coolant must be able to withstand high tempertures. Intense neutron radiation effect the metal structures. This effect must be overcome.

c) Waste Disposal

The reactor materials after their use is called "waste". There is a great problem of disposing that "waste" materials. A "monimorillonite clay" is used, as it can absorb radioactive atoms of the waste.

The waste may be in a solid, liquid or gaseous state and in its disposal it is necessary to consider the levels of external radiation and contamination involved, as affecting people in the vicinity. Sometimes the disposal is made in the great depths of sea.

QUESTIONS

8.1 Describe methods by which electrons have been accelerated in the laboratory to greater than 2 MeV energies.

8.2 Describe the construction and action of the cyclotron.

8.3 Compare and contrast the physical principles involved in the operation of a cyclotron and a betatron.

8.4 Give a list of similarities and differences between the phenomena of nuclear fission and fusion.

8.5 Charged particles show their track in a Wilson Cloud Chamber. Neutrons and gamma rays do not, why?

8.6 Distinguish between a cyclotron and a synchrotron.

8.7 What is the function of a drift tube in a linear accelerator? Why are magnets unnecessary in a linear accelerator?

8.8 What conditions are necessary for a chain reaction? What is meant by critical mass?

8.9 How does a nuclear produce electric power? What is meant by critical reactor? How can natural uranium be used in it?

8.10 How could radioisotopes be used to determine the effectiveness of a fertilizer?

PROBLEMS

8-1 a) Compute the Q value for the reaction

$$H^2 + Cu^{63} \longrightarrow n + Zn^{64}$$

Given atomic mass as: $H^2 = 2.014$ amu, $Cu^{63} = 62.929$ amu

$n = 1.009$ amu, $Zn^{64} = 63.929$ amu.

b) Deutrons of energy 12 MeV are incident on a Cu^{63} target, and neutrons are observed with 16.85 MeV of kinetic energy. Find the kinetic energy of Zn^{64}.

Soln: a) We have from eq. (8.10),

$$Q = (M_x + m_x - M_y - m_y)c^2$$

$$\text{or } Q = (2.014 + 62.929 - 1.009 - 63.929) \times \frac{1.66\times10^{-27} \times 9\times10^{16}}{1.6\times10^{-19}}$$

$$= 5.487 \text{ MeV.}$$

$$\left[\begin{array}{l} 1 \text{ amu} = 1.66\times10^{-27} \text{ Kg} \\ 1 \text{ eV} = 1.6\times10^{-19} \text{ Joules} \end{array}\right.$$

b) From eq.(8.11), we have

$$Q = K_y + K_y - K_x$$

$$\text{or } K_y = Q - K_y + K_x$$

$$= (5.487 - 16.85 + 12) \text{ MeV}$$

$$\text{or } K_y = 0.64 \text{ MeV.}$$

8-2 Calculate the Q value for the following fusion reaction, which is the sum of eq. (8.14).

$$4\ {}_1H^1 \longrightarrow {}_2He^4 + 2\ {}_{+1}e^0 + 2\nu + 2\gamma$$

Soln: We have

$$Q = (\text{Mass of } 4\ {}_1H^1 - \text{Mass of } {}_2He^4 - \text{Mass of } 4\ {}_{+1}e^0)\ c^2$$

$$= (4\times 1.008 - 4.003 - 4\times 0.00055)\times \frac{1.66\times 10^{-27}\times 9\times 10^{16}}{1.6\times 10^{-19}}$$

or $Q = 24.7$ MeV

8-3 What are the design parameters for a cyclotron that is to accelerate α particle to a maximum energy of 20 MeV? The dees are to have a radius $r = 0.50$ m.

Soln: We have from eq.(8.3)

$$r = \frac{mv}{Bq}$$

$$\text{or } mv = Bqr$$

$$\text{or } m^2v^2 = B^2q^2r^2$$

$$\text{or } \frac{1}{2}mv^2 = K = \frac{B^2q^2r^2}{2m} \quad \text{------(8.3(b))}$$

$$\text{or } B = \sqrt{\frac{2mK}{q^2r^2}}$$

$$\text{or } B = \left(\frac{2\times 4\times 1.67\times 10^{-27}\times 20\times 1.6\times 10^{-13}}{(2\times 1.6\times 10^{-19})^2(0.5)^2}\right)^{1/2}$$

So $B = 1.3$ tesla

Eq.(8.4) is for semicircle, T for one circle will be

$$T = \frac{2\pi m}{Bq}$$

$$\text{or } T = 1/\nu = \frac{2\pi m}{Bq}$$

$$\text{or } \nu = \frac{Bq}{2\pi m}$$

$$= \frac{1.3\times 2\times 1.6\times 10^{-19}}{2\times 3.14\times 4\times 1.67\times 10^{-27}} = 9.9\times 10^6\ Hz$$

$$\text{or } \nu = 9.9\ MHz.$$

UNSOLVED PROBLEMS

8:1 Calculate the Q-value of each reaction in the Carbon cycle and find from the results the amount of energy in MeV, liberated in the formation of one helium atom.

8:2 How much energy must a photon have if it is to split an alpha particle into a :

a) triton $_1H^3$ and a proton,

b) a $_2He^3$ nucleus and a neutron.

8:3 Calculate the cyclotron frequency of an electron and a proton in a magnetic field of 0.004 tesla.

8:4 A cyclotron with a radius of 1 m has a magnetic field of 2 tesla. For deutrons accelerated in this device, calculate frequency of revolution and maximum energy emparted to deutrons.

8:5 The linear accelerator located in Illinois is designed to accelerate protons to 200 BeV. What is the de Broglie wavelength of these protons?

8:6 Assume 200 MeV of available energy per fission. How many nuclei would have to undergo fission per second in order to generate 1 mega watt of power? How many gram of U^{235} required per hour?

8:7 How many grams of U^{235} would be consumed in producing a mega-watt-hour of electricity, if the conversion efficiency from heat to electricity is 25 % ?

8:8 Find the energy released in the fission of 1 kg of uranium that has been enriched to 3 % in the isotope U^{235}.

NUCLEAR POWER PLANTS

Country	*Plants in operation*	*Plants planned or under construction*	*Power output* (Mw)
Argentina	1	2	1,627
Austria	—	1	692
Belgium	3	4	5,450
Brazil	—	3	3,116
Bulgaria	3	1	1,760
Canada	10	14	15,332
Czechoslovakia	2	8	4,400
Egypt	—	2	1,800
Finland	3	1	2,160
France	29	30	52,468
Germany	17	19	31,967
Hungary	4	10	3,884
India	2	6	2,125
Italy	3	3	3,276
Japan	25	10	25,072
Korea	1	8	7,427
Mexico	—	2	1,308
Netherlands	2	—	495
Pakistan	1	—	125
Philippines	—	1	620
Poland	—	2	880
Rumania	—	3	1,840
South Africa	—	2	1,844
Spain	5	12	15,557
Sweden	9	3	9,410
Switzerland	4	3	4,947
Taiwan	4	2	4,924
United Kingdom	32	10	14,388
United States	78	80	147,767
USSR	31	10	24,795
Yugoslavia	1	—	615

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Standard Prefixes

Multiple and Submultiple units; as a result of the Eleventh General Conference on Weights and Measures (1960), as amended in 1962 by the Executive Board.

Prefix	Symbol	Multiplication Factor
atto	a	10^{-18}
femto	f	10^{-15}
pico (or micromicro)		10^{-12}
nano (or millimicro)	n, m	10^{-9}
micro		10^{-6}
milli	m	10^{-3}
centi	c	10^{-2}
deci	d	10^{-1}
'no prefix'	----	1
deka	da	10
hecto	h	10^{2}
kilo	k	10^{3}
mega	M	10^{6}
giga (or kilomega)	G	10^{9}
tera (or megamega)	T, MM	10^{12}
peta	P	10^{15}
exa	E	10^{18}

Energy, Work, Heat

The electron volt (eV) is the kinetic energy an electron gains from being accelerated through the potential difference of one volt in an electric field. The Mev is the kinetic energy it gains from being accelerated through a million-volt potential difference.

The last two items in this table are not properly energy units but are included for convenience. They arise from the relativistic mass-energy equivalence formula $E = mc^2$, and represent the energy released if a kilogram or atomic mass unit (amu) is destroyed completely.

	Btu	erg	ft-lb	hp-hr	JOULES	Cal	kw-hr	ev	Mev	kg	amu
1 British thermal unit =	1	1.055×10^{10}	777.9	3.929×10^{-4}	1055	252.0	2.930×10^{-4}	6.585×10^{21}	6.585×10^{15}	1.174×10^{-14}	7.074×10^{12}
1 erg =	9.481×10^{-11}	1	7.376×10^{-8}	3.725×10^{-14}	10^{-7}	2.389×10^{-8}	2.778×10^{-14}	6.242×10^{11}	6.242×10^{5}	1.113×10^{-24}	670.5
1 foot-pound =	1.285×10^{-3}	1.356×10^{7}	1	5.051×10^{-7}	1.356	0.324	3.766×10^{-7}	8.464×10^{18}	8.464×10^{12}	1.509×10^{-17}	9.092×10^{9}
1 horsepower-hour=	2545	2.685×10^{13}	1.980×10^{6}	1	2.685×10^{6}	6.414×10^{5}	0.7457	1.676×10^{25}	1.676×10^{19}	2.988×10^{-11}	1.800×10^{16}
1 Joule =	9.481×10^{-4}	10^{7}	0.7376	3.725×10^{-7}	1	0.239	2.778×10^{-7}	6.242×10^{18}	6.242×10^{12}	1.113×10^{-17}	6.705×10^{9}
1 calorie =	3.968×10^{-3}	4.186×10^{7}	3.087	1.559×10^{-6}	4.186	1	1.163×10^{-6}	2.613×10^{19}	2.613×10^{13}	4.659×10^{-17}	2.807×10^{10}
1 kilowatt-hour =	3413	3.6×10^{13}	2.655×10^{6}	1.341	3.6×10^{6}	8.601×10^{5}	1	2.247×10^{25}	2.270×10^{19}	4.007×10^{-11}	2.414×10^{16}
1 electron volt =	1.519×10^{-22}	1.602×10^{-12}	1.182×10^{-19}	5.967×10^{-26}	1.602×10^{-19}	3.827×10^{-20}	4.450×10^{-26}	1	10^{-6}	1.783×10^{-36}	1.074×10^{-9}
1 million electron volts=	1.519×10^{-16}	1.602×10^{-6}	1.182×10^{-13}	5.967×10^{-20}	1.602×10^{-13}	3.827×10^{-14}	4.450×10^{-20}	10^{6}	1	1.783×10^{-30}	1.074×10^{-3}
1 kilogram =	8.521×10^{13}	8.987×10^{23}	6.629×10^{16}	3.348×10^{10}	8.897×10^{16}	2.147×10^{16}	2.497×10^{10}	5.610×10^{35}	5.610×10^{29}	1	6.025×10^{26}
1 atomic mass unit =	1.415×10^{-13}	1.492×10^{-3}	1.100×10^{-10}	5.558×10^{-17}	1.492×10^{-10}	3.564×10^{-11}	4.145×10^{-17}	9.31×10^{8}	931.0	1.660×10^{-27}	1

1 m-kgf = 9.807 joules 1 watt-sec = 1 joule = 1 nt-m 1 cm-dyne = 1 erg

Electromagnetic Spectrum

Rays	Frequency	Wave-length	Photon Energy
Cosmic rays	10^{23} Hz	10^{-11} cm	below 10^{8} to 10^{20} eV
γ-rays	6×10^{20} to 10^{18} Hz	10^{-10} to 10^{-8} cm	10^{5} to 10^{7} eV
X - rays	6×10^{19} to 6×10^{15} Hz	10^{-9} to 10^{-5} cm	10 to 10^{5} eV
Ultra-violet	2×10^{16} to 8×10^{14} Hz	1.4×10^{-6} to 4×10^{-5} cm	1 to 10^{2} eV
Visible	8×10^{14} to 4×10^{14} Hz	4×10^{-5} to 8×10^{-5} cm	1 eV
Infra-red (heat radiation)	4×10^{14} to 3×10^{11} Hz	8×10^{-5} to 0.04 cm	10^{-3} to 1 eV
Microwaves	10^{13} to 10^{9} Hz	10^{-6} to 10^{-4} cm	10^{-2} to 10^{-3}
Electrical radio waves	10^{13} to 10^{3} Hz	0.01 cm to 100 Km	10^{-10} to 10^{-1} eV
TV, Radar	7×10^{9} to 2×10^{6} Hz	4×10^{1} to 3.5×10^{-4} m	10^{-3} to 10^{-10} eV
Micropulsation	-----	1×10^{8} to 5×10^{6} m	10^{-14} to 10^{-17} eV

Spectrum of visible portion

Colour of light	Wavelength $\times10^{-7}$ m	Frequency $\times10^{14}$ Hz
Red	6.470 to 7.000	4.634 to 4.284
Orange	5.850 to 6.470	5.125 to 4.634
Yellow	5.750 to 5.850	5.215 to 5.125
Green	4.912 to 5.750	6.104 to 5.215
Blue	4.240 to 4.912	7.115 to 6.104
Violet	4.000 to 4.240	7.495 to 7.115

Radio and TV waves

	Range	Frequency	Wave-length
TV	UHF	2.1×10^{8} to 6.9×10^{9}	.1 to 1 m
TV	VHF	5×10^{7} to 1.2×10^{8}	1 to 10 m
TV	HF (or VL)	2×10^{6} to 4×10^{7}	10 to 100 m
Radio	FM (frequency modulation)	8.8×10^{7} to 1.08×10^{8}	1 to 10 m
Radio	MW (MF)	5.3×10^{5} to 1.605×10^{6}	200 to 550 m
Radio	SW 1 (MF)	2.3×10^{6} to 7.0×10^{6}	49 to 120 m
Radio	SW 2 (MF)	7.0×10^{6} to 22.0×10^{6}	13 to 41 m
	LF	3×10^{4} to 3×10^{5}	1000 to 10,000 m
	VLF	3×10^{3} to 3×10^{4}	10,000 to 100,000 m
	ELF (extremely)	--- to 3×10^{3}	100,000 to ----- m

Fundamental Constants

Velocity of light, c and radio waves	$= 2.9979 \times 10^{8} ms^{-1} = 186{,}000$ miles/s
Elementary charge, e	$= 1.6021 \times 10^{-19} C$
Electron rest mass, m_e	$= 9.1091 \times 10^{-31} kg$
Proton rest mass, m_p	$= 1.6725 \times 10^{-27} kg = 1.008$ amu $= 1836$ electron masses
Neutron rest mass, m_n	$= 1.6748 \times 10^{-27} kg = 1837$ electron masses
Planck's constant, h	$= 6.6256 \times 10^{-34}$ J.s.
e/m for electron, e/m_e	$= 1.7588 \times 10^{11} kg^{-1} C$
Rydberg constant, R	$= 1.0974 \times 10^{7} m^{-1}$
Avogadro constant, N_o, No. of molecules/gm-mol.	$= 6.0225 \times 10^{23} mol^{-1}$
Boltzmann constant, $k=R/N_o$	$= 1.3805 \times 10^{-23} J\ K^{o-1}$
Universal gas constant, R	$= 8.3143\ J\ K^{o-1} mol^{-1}$
Vacuum permittivity,	$= 8.8544 \times 10^{-12} N^{-1} m^{-2} C^{2}$
Vacuum permeability,	$= 1.3566 \times 10^{-6} m\ kg\ C^{-2}$
Acceleration of gravity, g	$= 9.7805\ ms^{-2}$
Gravitational constant, G	$= 6.673 \times 10^{-11}\ N\text{-}m^{2}/kg^{2}$
The Faraday, F	$= 96{,}520\ C\ gm^{-1} mol^{-1}$
One atomic mass unit $(c^{12})/\mu$	$= 1.66 \times 10^{-27} kg = 931$ Mev $= 1.49 \times 10^{-10}$ J
Heat of fusion of ice	$= 336000\ J\ kg^{-1}$
Heat of vaporization of water	$= 2263800\ J\ kg^{-1}$
1 electron volt	$= 1.501 \times 10^{-12}$ erg
Ratio of proton mass to electron mass, m_p/m_e	$= 1836.14$
Radius of the Earth	$= 6.4 \times 10^{6} m = 3959$ miles
Mass of the Earth	$= 6 \times 10^{24} kg$
Stefan-Boltzmann const.	$= 5.6697 \times 10^{-8}\ W\ m^{-2} K^{o-4}$
Mean orbital speed of earth	$= 29{,}770$ m/sec $= 18.50$ miles/sec
Mean angular speed of rotation of earth	$= 7.29 \times 10^{-5}$ radians/sec
Bohr magneton, $\mu_B = eh/2m_e$	$= 9.274 \times 10^{-24}\ J\ T^{-1}$
nuclear magneton, $\mu_N = \frac{eh}{2m_p}$	$= 5.051 \times 10^{-27} JT^{-1}$
neutron-H atom mass difference $m_n - M_H$	$= 0.782\ MeV/c^{2}$

Luminous Efficiency

Source	Luminous Efficiency (lumens/watt)	lumens
tungsten lamp (watts):		
40	11	440
75	14	1,050
100	15	1,500
mercury vapour (watts):		
150 (low)	13	1,950
400 (High)	30	12,000
sodium vapour, 220 watts	50	11,000
fluorescent lamp (watts):		
10	40	400
30	50	1,500
40	60	2,400

Length of light waves (specific colours)

Very dark red	0.00081 mm	Green	0.00052 mm
Red	0.00065 mm	Bluish green	0.00050 mm
Reddish orange	0.00064 mm	Blue	0.00047 mm
Orange	0.00060 mm	Indigo	0.00043 mm
Yellow	0.00058 mm	Violet	0.00041 mm

The Electromagnetic Spectrum (Specific characteristics)

Name	Type of transition	Characteristic temperature (hν/k, °K)	Artificial production	Detection
Gamma rays	Nuclear	10^9 to 10^{10}	Betatron	Geiger and Scintillation Counters, Ionization Chamber
X-rays	Inner Electron	10^8	X-ray tube	" " "
Ultraviolet radiation	" "	10^6 to 10^7	" "	" " "
Visible light	Outer Electron	10^4 to 10^5	Gas discharge, Arcs, Sparks	Photoelectric and Photomultiplier Cells
Infrared radiation	Molecular vibration	10^2 to 10^3	Hot filaments	Bolometer, Thermopile
Microwaves	Molecular rotation	10	Magnetron, Klystron, Traveling-wave tube	Crystal
FM and TV broadcasting	Electron Spin	10^{-1} to 1	Electrical circuit, AC Generator	Electrical circuit
AM broadcasting	Nuclear Spin			
Micropulsation	Unknown	------	-----	Magnetic

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