Sample Answers

of the Textbook
PHYSICS XI

1. Highlight
   First underline words / terms to be emphasized in the question.
2. Core Answer
   Before answering the question, note down core / main points of the answer on your rough side of your answer sheet.
3. Format
   Start should be noticeable and end with noticeable concluding remarks.
   In the body, there should be a diagram and some mathematical equation.
4. Text Book
   Always give preference to the words, statements & format of the textbook.
5. Words / Terms
   Answer the words / terms used in the question accordingly.
   e.g. 'State' & 'Define' need only statements; but 'Explain, 'What' & 'Why' need statements along with brief explanation.
6. Technical Answer
   Answer the question only to the point, i.e. first understand the sense of the wording used in the question, then answer accordingly.
7. Time management:
   Proportional time for the distribution of marks is: 1 mark $\approx$ 2 minutes
   e.g. for 8 marks question, you will have 16 minutes.

Ross Nazir Ullah

Q. 3. a) <u>What</u> is projectile motion. <u>Deduce</u> equations for maximum height, range and total time of flight in case of projectile motion. <u>Write</u> down the application to ballistic missile. (8)

b) If the force of gravity acts on all bodies in proportion to their masses, <u>why</u> does not a heavier body fall faster than a lighter body? (2)

Ans. 3 (a)

Definitions:

"Projectile motion is two dimensional motion under constant acceleration due to gravity"

"An object launched in an arbitrary direction in space with the initial velocity having no mechanism of propulsion is called a projectile"

Brief Introduction :

A projectile fired in space may have its motion in horizontal direction, or in vertical direction, or making an angle θ with horizontal.

Projectile initially thrown horizontally will have only horizontal component for its initial velocity, but as it falls downward, it will have both vertical and horizontal components of velocity.

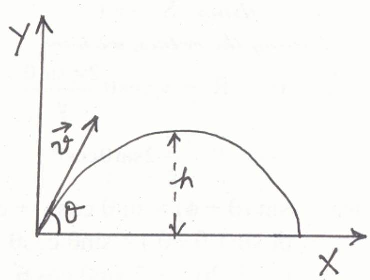
When a projectile is fired at an angle θ with horizontal, it will follow parabolic path, and its motion at different positions depends upon θ.

Calculating Equations:

To deduce equation for maximum height

Height of the projectile is the highest point a projectile attains during its flight.

Consider a projectile motion as shown in the figure



Taking vertical component, we have

$$a = -g$$
$$S = \text{height} = h$$
$$v_f = v_{fy} = 0$$
$$v_i = v_{iy} = v_i \sin\theta$$

Using equation,

$$2aS = v_f^2 - v_i^2$$

putting the values, we have

$$2(-g)h = (0)^2 - (v_i \sin\theta)^2$$

$$\text{or} \quad -2gh = -v_i^2 \sin^2\theta$$

$$\text{or} \quad \boxed{h = \frac{v_i^2 \sin^2\theta}{2g}}$$

which is the equation for maximum height

To deduce equation for total time of flight

Total time of flight is the time taken by body to cover the distance from the place of its projection to the place where it hits the ground.

For vertical component, we have

$$S = h = 0$$
$$v_i = v_{iy} = v_i \sin\theta$$
$$a = -g$$

In initial assumption we have taken v_i as + ve for upward direction, so g will be -ve

Using equation $S = v_i t + \frac{1}{2} a t^2$

Putting the values, we have

$$0 = v_i \sin\theta\, t + \tfrac{1}{2}(-g) t^2$$
$$\text{or} \quad 0 = v_i \sin\theta\, t - \tfrac{1}{2} g t^2$$
$$\text{or} \quad \tfrac{1}{2} g t^2 = v_i \sin\theta\, t$$
$$\text{or} \quad (\tfrac{1}{2} g t) \times t = (v_i \sin\theta) \times t$$
$$\text{or} \quad \tfrac{1}{2} g t = v_i \sin\theta$$
$$\Rightarrow \boxed{t = \frac{2v_i \sin\theta}{g}}$$

which is the equation for total time of flight

To deduce equation for the range of the projectile

The range is the maximum distance, which a projectile covers, in the horizontal direction.

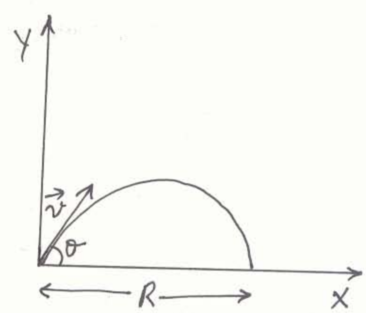
Taking horizontal component, we have

$$S = R$$
$$v = v_{ix} = v_i \cos\theta$$
$$t = \frac{2v_i \sin\theta}{g}$$

Using $S = v\,t$

putting the values, we have

$$\text{or} \quad R = v_i \cos\theta \frac{2v_i \sin\theta}{g}$$
$$\text{or} \quad R = \frac{v_i^2}{g} 2\sin\theta\cos\theta \qquad \ldots\ (\alpha)$$



since $\sin(\theta + \phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$

$$\text{or} \sin(\theta + \theta) = \sin\theta\cos\theta + \cos\theta\sin\theta$$
$$\text{or} \quad \sin 2\theta = 2\sin\theta\cos\theta$$

so from equation (α), we get

$$\boxed{R = \frac{v_i^2}{g}\sin 2\theta}$$

which is the equation for the range of the projectile

Application to Ballistic missile:

An un-powered and un-guided missile is called a ballistic missile, which is like a <u>projectile.</u> The path followed by it called ballistic trajectory is the <u>path of a projectile motion</u>.

If we will fire a ballistic missile at angle of 45° for achieving its maximum range to reach its target in the calculated time, we apply the <u>principle of projectile motion</u>.

Ballistic missiles are <u>useful</u> only for <u>short ranges</u>. For long ranges powered and remote control guided missiles are used, which <u>do not</u> follow the path of a projectile.

Ans. 3. (b)

$$F = W = mg$$

or $F = gm$

as g is constant $\Rightarrow \quad F \propto m$

Force of gravity, called weight is directly proportional to the mass. Also it is the <u>experimental fact</u> that all bodies fall with the <u>same acceleration (g)</u> at any particular part of the Earth's surface. We use weight as a measure of mass. Although force of gravity (weight) depends upon mass, but 'g' is same for all masses. So <u>heavier bodies does not fall faster than a lighter body.</u>

Core

"**Projectile motion** is two dimensional motion under constant acceleration due to gravity"

Maximum height h:

$$2aS = v_f^2 - v_i^2 \qquad \text{or} \qquad 2(-g)h = (0)^2 - (v_i \sin\theta)^2$$

$$\text{or} \quad -2gh = -v_i^2 \sin^2\theta \Rightarrow \quad h = \frac{v_i^2 \sin^2\theta}{2g}$$

Time of flight t:

$$S = v_i t + \tfrac{1}{2} a t^2 \quad \text{or} \quad 0 = v_i \sin\theta\, t + \tfrac{1}{2}(-g) t^2$$

$$\text{or} \quad 0 = v_i \sin\theta\, t - \tfrac{1}{2} g t^2 \quad \Rightarrow \quad t = \frac{2v_i \sin\theta}{g}$$

Range R:

$$S = v t \quad \text{or} \quad R = v_i \cos\theta \frac{2v_i \sin\theta}{g}$$

$$\text{or} \quad R = \frac{v_i^2}{g} 2\sin\theta\cos\theta \quad \Rightarrow R = \frac{v_i^2}{g}\sin 2\theta$$

An un-powered and un-guided missile is called a ballistic missile, which is like a <u>projectile.</u>

b) $F = mg$ or $F = gm$

as g is constant $\Rightarrow \quad F \propto m$

FORMAN CHRISTIAN COLLEGE
Physics FSc 1st Year
INTERMEDIATE PART I (Session 2006 – 2008)

Estd. 1864

December Examination

Name: ....

Roll No. In figures ....

Roll No. in words ....

December, 2006

Time 20 minutes

Maximum marks [Section I + II] (85)

Marks = 17

OBJECTIVE

Note: Write your Roll no. in the space provided. Cutting overwriting, erasing, or using a lead pencil will have no credit.

Q1. Each question has four possible answers. Select the correct answer and circle it.

(i). One peta is equal to.

(a) 10^{16} (b) 10^{18} (c) 10^{15} (d) 10^{19}

(ii) $\hat{j}\bullet(\hat{i}\times\hat{j}) =$ Zero ________________ $\left[\hat{j}\cdot\hat{k} = 0 \;\left[\hat{i}\times\hat{j} = \hat{k}\right.\right.$

(iii) The minimum number of unequal forces whose resultant will be zero

(a) 2 (b) 3 (c) 4 (d) 5

(iv) The angle between two vectors $\vec{A} = 5\hat{i} + \hat{j}$ and $\vec{B} = 2\hat{i} + 4\hat{j}$

(a) 20° (b) 90° (c) 52° (d) 30°

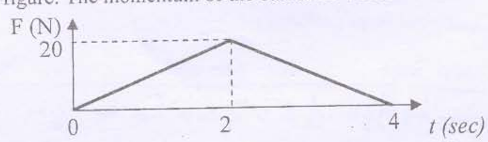
(v) The dimensions of the Gravitational constant G are

(a) $L^3M^{-1}T^{-2}$ (b) $L^3M^{-2}T^{-2}$ (c) $L^3M^2T^{-1}$ (d) $L^2M^{-1}T^{-2}$

(vi) The distance covered by a freely falling body in one second is $\left[S = v_i t + \frac{1}{2}at^2 = 0\times1 + \frac{1}{2}(9.8)(1)^2 = 4.9\right.$

(a) 9.8m (b) 4.9m (c) 19.6m (d) 49.0m

(vii) A Force F acts on a ball initially at rest on a smooth surface for a time t The variation of F with t is shown in the figure. The momentum of the ball after 4 seconds is



(a) $10 kg\,ms^{-1}$ (b) $20 kg\,ms^{-1}$ (c) $30 kg\,ms^{-1}$ (d) $40 kg\,ms^{-1}$

iv) $\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB} = \frac{(5\hat{i}+\hat{j})\cdot(2\hat{i}+4\hat{j})}{\sqrt{5^2+1^2}\times\sqrt{2^2+4^2}} = \frac{10\,\hat{i}\cdot\hat{i}+4\,\hat{j}\cdot\hat{j}}{\sqrt{26}\,\sqrt{20}} = \frac{10+4}{\sqrt{26\times20}} = \frac{14}{22.8}$; $\theta = \cos^{-1}\frac{14}{22.8} = 52°$

v) $F = \cancel{m}g = \frac{G\cancel{m}M}{R^2}$ or $G = \frac{gR^2}{M} = \frac{m s^{-2} m^2}{kg} = \frac{LT^{-2}L^2}{M} = L^3M^{-1}T^{-2}$

vii) $F = \frac{mv_f - mv_i}{t} = \frac{mv_f - m\times0}{t} = \frac{mv_f}{t}$ or $mv_f = F\times t = 20\times2 = 40$

(viii) A moving car X collides head-on with a car Y moving in the opposite direction, the law of conservation of momentum states that

(a) the final momentum of X = the final momentum of Y
(b) the total momentum of X and Y is reserved by the collision
(c) the total momentum of X and Y stays constant
(d) the initial and final momentum of X is the same

(ix) If p is the momentum of an object of mass m, then the expression p^2/m has the same units as

(a) acceleration (b) energy (c) force (d) impulse

(x) Starting from rest, a car of mass 1000kg accelerates steadily to $20ms^{-1}$ in 10 seconds. The average power developed in this time is

(a) 0.2W (b) 4.0W (c) 10kW (d) 20kW

(xi) The range of projectile is the same for the following pair of angles

(a) 30° and 60° (b) 0° and 45° (c) 15° and 60° (d) 30° and 75°

(xii) The consumption of energy by a 60Watt bulb in 2 seconds is

(a) 120J (b) 60J (c) 30J (d) 0.02J

(xiii) The intensity of solar energy reaching the earth surface is about

None of these

(a) $1.2kWm^{-2}$ (b) $1.6kWm^{-2}$ (c) $1.4kWm^{-2}$ (d) $1.8kWm^{-2}$

$[I = 1 kWm^{-2}]$

(xiv) The circumference of a circle subtends an angle at the centre of the circle equal to

$= 2\pi r$

(a) 1 radian (b) Zero radians (c) π radians (d) 2π radians

(xv) The value of 'g' at the centre of the earth is

(a) double (b) zero (c) half (d) Same as at the surface

(xvi) When both the mass and the speed of a moving body are doubles the K.E. will be

(a) double (b) 4 times (c) 8 times (d) 16 times

(xvii) Centripetal force performs

(a) Max. work (b) Min work (c) Negative work (d) No work

ix) $\frac{p^2}{m} = \frac{m^2v^2}{m} = mv^2 \to kg\,m^2/s^2$ & $E = \frac{1}{2}mv^2 \to kg\,m^2/s^2$

x) $v_f = v_i + at$ or $20 = 0 + a \times 10 \Rightarrow a = \frac{20}{10} = 2$; $F = ma = 1000 \times 2 = 2000$

So $P = \vec{F} \cdot \vec{v}_{av} = 2000 \times \left(\frac{0 + v_f}{2}\right) = 2000 \times \frac{20}{2} = 20,000\,W = 20kW$

xi) $R = \frac{v_i^2}{g}\sin 2\theta$; $\sin(30+30) = \underline{\sin 60^\circ}$ & $\sin(60+60) = \sin(120) = \sin(180-60) = \underline{\sin 60^\circ}$

xii) $P = \frac{\Delta W}{t} = \frac{E}{t}$ or $E = P \times t = 60 \times 2 = 120J$

xvi) $KE = \frac{1}{2}mv^2 = \frac{1}{2}(2m)(2v)^2 = \frac{1}{2}2m \times 4v^2 = 8\frac{1}{2}mv^2 = 8 \times KE$

FORMAN CHRISTIAN COLLEGE

Physics FSc 1st Year

INTERMEDIATE PART I (Session 2006 – 2008)

Estd. 1864

December Examination

Name: ....

Roll No. In figures ....

Roll No. in words ....

December, 2006

Time 2 hrs 10 minutes **Marks = 68**

SUBJECTIVE

Note: Attempt any TWENTY TWO (22) questions from section I and any THREE questions from Section II

Section –I

Q.No.2 Write short answer to any twenty two of the following questions.

(2×22=44) Marks

i) Name several repetitive phenomenon occurring in nature which could serve as reasonable time standards.

ii) Write the dimension of (i) Pressure (ii) Density

iii) Express the following quantities using the prefixes:(i) 3×10^{-4} m (ii) 5×10^{-5} sec

iv) Find the dimension of η in the relation $F = 6\pi\eta r v$, v=velocity, r=radius

v) What is meant by precise and accurate measurement?

vi) Write the rules for finding total uncertainty in the final result for addition and subtraction and for power factor.

vii) What is the unit vector in the direction of the vector $\vec{A} = 4\hat{i} + 3\hat{j}$.

viii) Prove that $(\vec{A}\times\vec{B})^2 + (\vec{A}\bullet\vec{B})^2 = A^2B^2$

ix) Under what circumstances would a vector have components that are equal in magnitude.

x) Show that the sum and difference of two perpendicular vectors of equal lengths are also perpendicular and of the same length.

xi) Name three different conditions that could make $(\vec{A}_1\times\vec{A}_2)=0$

xii) A picture is suspended from a wall by two strings. Prove that the tension in the string will be minimum for which configuration?

xiii) Given that $\vec{A} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{B} = 3\hat{i} - 4\hat{k}$ find the length of the projection of A on B.

xiv) Prove that the magnitude of relative velocity of approach is equal to the magnitude of relative velocity of separation for elastic collision.

xv) Define impulse and show that how it is related to linear momentum?

xvi) Find the angle of projection of projectile for which its maximum height and horizontal range are equal.

xvii) Define elastic collision and discuss the collision when a massive body collides with a light stationary body.

xviii) Find the change in momentum for an object subjected to a given force for a given time and state law of motion in terms of momentum.

xix) What an athlete should do to take a long jump?

xx) At what point or points in the path of a projectile does it have its minimum speed, its maximum speed?

xxi) Show that the rocket accelerates when its fuel is burned and ejected.

xxii) Calculate the work done in kilo Joules in lifting a mass of 10kg (at a steady velocity) through a vertical height of 10m.

xxiii) Describe the negative work with an example.

xxiv) A 1200kg car moving at 15 ms^{-1} collides head-on with a 2000kg truck, initially at rest and sticks to the truck after the collision, what is their velocity just after the collision?

xxv) When a rocket re-enters the atmosphere its nose cone becomes very hot. Where does this heat energy come from?

xxvi) Prove that work done on a body is equal to the gain in K.E.

xxvii) A force of 6N acts horizontally on a stationary mass of 2kg for 4s.What is the gain in kinetic energy by the mass in J.

xxviii) How would the values of 'g' and 'G' be affected if the mass of the earth becomes four times?

xxix) A stone is taken to the bottom of a tunnel. Does the stone possess any potential energy?

xxx) How solar energy as non conventional source, is contributing to the world energy.

xxxi) A 1000kg car travelling with a speed of 144kmh^{-1} round a curve of radius 10^{20}attometre.Find the necessary centripetal force.

xxxii) What is meant by moment of Inertia? Explain its significance.

xxxiii) Why mud flies off the tyre of a moving bicycle, in what direction does it fly? Explain

Section-II

Note: Attempt any three questions. All questions carry equal marks.

($3\times8=24$) marks

Q.No.3(a) State scalar product of two vectors and write down its characteristics?

(b) The line of action of force, $\vec{F}=\hat{i}+2\hat{j}$ passes through the point whose position vector is $\left(-\hat{i}+\hat{k}\right)$, find the moment of F about the point of which the position vector is $(\hat{i}+\hat{k})$.

Q.No.4(a) A projectile is thrown with initial velocity vi making an angle θ with horizontal axis. Find its maximum Height and Range.

(b) A hose pipe ejects water at a speed of $0.3ms^{-1}$ through a hole of area $50cm^2$.If the water strikes a wall normally, calculate the force on the wall, assuming the velocity of the water normal to the wall is zero after striking. [Density of water = $1000kgm^{-3}$]

Q.No.5(a) Drive the formula for Absolute Potential Energy in gravitational field.

(b) Ten bricks, each 6cm thick and mass 1.5kg, lie flat on a table. How much work is required to stack them one on the top of another?

Q.No.6(a) Define escape velocity. Derive its relation.

(b) Derive a relation for the time period of a simple pendulum using dimensional analysis. The various possible factors on which the time period T may depend are: Length of Pendulum (l), Mass of the bob (m), angle θ which the thread makes with the vertical, and acceleration due to gravity (g).

Q.No.7(a) Define centripetal force and derive its relation?

(b). Find the value of "q" for which two vectors will become perpendicular to each other. $\vec{a}=2\hat{i}-4\hat{j}+5\hat{k};\ \vec{b}=13\hat{i}+q\hat{j}+2\hat{k}$

Rough Work

SUBJECTIVE

Ans. 2 : Short answers to the questions.

2 (i) 1) Natural periodic movements, e.g. stars at night.
2) Sun's different periodic directions making shadow at different times.
3) Human pulse rate.
4) Revolution of the moon around the earth.

2 (ii) dimensions of pressure $= \dfrac{\text{dimensions of force}}{\text{dimensions of area}}$

$$[P] = \frac{[F]}{[A]} = \frac{[m\,a]}{[L^2]} = \frac{[M\,L\,T^{-2}]}{[L^2]} = [M\,L^{-1}\,T^{-2}]$$

$P = \frac{F}{A} = \frac{ma}{A}$

dimensions of density $= \dfrac{\text{dimensions of mass}}{\text{dimensions of volume}}$

$$[D] = \frac{[M]}{[L^3]} = [M\,L^{-3}]$$

$\eta = \frac{F}{rv} = \frac{ma}{rv}$

2(iii) 3×10^{-4} m = 30 mm
& 5×10^{-5} sec = 50 x 10 sec = 50 μs

2(iv) 6π is a number having no dimensions. So we have

$$[F] = [\eta r v] \quad \text{or} \quad [\eta] = \frac{[F]}{[r][v]}$$

Substituting the dimensions of F, r & v,

$$[\eta] = \frac{[MLT^{-2}]}{[L][LT^{-1}]} = [ML^{-1}T^{-1}] \quad (\text{SI unit of } \eta \text{ is kg m}^{-1}\text{s}^{-1})$$

2(v) A <u>precise measurement</u> is the one, which has less absolute uncertainty.

< abs. uncertain

<u>Precision (or absolute uncertainty):</u> In measurements considering the magnitude of error. The less magnitude of error gives more precise measurement; it is equal to the least count of the measuring instrument.

< fractional uncertainty

An <u>accurate measurement</u> is the one, which has less fractional uncertainty.

<u>Accuracy</u> in measurements considers the relative error. The less relative error gives more accurate result.

<u>Precision</u> depends upon instrument and <u>accuracy</u> depends upon fractional error.

Rough Work

2(vi) **Rules** *for finding total uncertainty in final result*

For addition & subtraction

Absolute uncertainties are added.

e.g. for $x_1 = 10.5 \pm 0.1$ & $x_2 = 26.8 \pm 0.1$ *is recorded as*

$x = x_1 - x_2 = 16.3 \pm 0.2$ cm

For power factor

Multiply the percentage uncertainty by that power.

e.g. for $V = \frac{4}{3}\pi r^3$ $[r = 2.25 \pm 0.01 \text{ cm}]$

% *uncertainty in* V = 3 x % *age uncertainty in radius* r

Total % *age uncertainty in* V = 3 x 0.4 $\left[\frac{0.01}{2.25} \times 100 = 0.4\ \%\right]$

2(vii) *The unit vector in the direction of* $\vec{A}$ *will be:*

$$\hat{A} = \frac{\vec{A}}{A} = \frac{4\hat{i}+3\hat{j}}{\sqrt{4^2+3^2}} = \frac{4\hat{i}+3\hat{j}}{\sqrt{25}} = \frac{4\hat{i}+3\hat{j}}{5} = \frac{4}{5}\hat{i} + \frac{3}{5}\hat{j}$$

& its magnitude:

$$|\hat{A}| = \left|\frac{4}{5}\hat{i} + \frac{3}{5}\hat{j}\right| = \sqrt{\left(\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2} = \sqrt{\frac{16}{25} + \frac{9}{25}} = 1$$

2(viii) $(\vec{A}\times\vec{B})^2 + (\vec{A}\cdot\vec{B})^2 = (AB\sin\theta\,\hat{n})^2 + (AB\cos\theta)^2$

$$= (A^2B^2\sin^2\theta\,\hat{n}^2) + (A^2B^2\cos^2\theta) = A^2B^2\sin^2\theta + A^2B^2\cos^2\theta$$

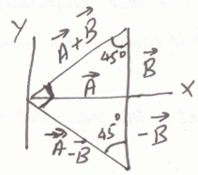
$$= A^2B^2(\sin^2\theta + \cos^2\theta) \qquad [(\hat{n})^2 = \hat{n}\cdot\hat{n} = 1]$$

$$= A^2B^2\sin^2\theta + A^2B^2\cos^2\theta = A^2B^2(\sin^2\theta + \cos^2\theta)$$

$$= A^2B^2 \qquad [(\sin^2\theta + \cos^2\theta) = 1]$$

2(ix) When θ = 45°,

the components will have equal magnitude for a vector making *angle* **45°** *with X-axis.*



2(x) *In the figure,*

Vectors A and B are perpendicular to each other having equal lengths. From the configuration of the figure, we have

$(\vec{A} + \vec{B})$ is $\perp$ to $(\vec{A} - \vec{B})$

i.e. sum and difference of the vectors are perpendicular to each other.

+, added

X, %age X power

$\hat{A} = \frac{\vec{A}}{A}$

$A \times B = AB\sin\theta\,\hat{n}$

$A \cdot B = AB\cos\theta$

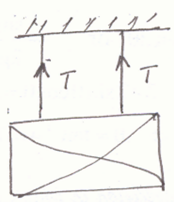
K

2(xi) *The cross product of two vectors can be expressed as:*

a)
$$\vec{A}_1 \times \vec{A}_2 = A_1A_2 \sin 0^0\ \hat{n} = 0 \ \Rightarrow \text{ both vectors are parallel } [\theta = 0^0]$$
b)
$$\vec{A}_1 \times \vec{A}_2 = A_1A_2 \sin 180^0\ \hat{n} = 0 \ \Rightarrow \text{ both are anti-parallel } [\theta = 180^0]$$
c) $\vec{A}_1 \times \vec{A}_2 = (0)A_2 \sin\theta\ \hat{n} = 0 \ \Rightarrow A_1$ is zero

d) $\vec{A}_1 \times \vec{A}_2 = A_1(0) \sin\theta\ \hat{n} = 0 \ \Rightarrow A_2$ is zero

2(xii) *The configuration shown in the figure will have minimum tension.*



For T minimum, $\theta = 90^o$

$\Sigma F_y = 0$

$T_y + T_y - w = 0$

$2T_y - w = 0$

$2\,T \sin\theta = w$

$T = w / 2 \sin\theta$

For minimum T, $\theta = 90^o$

i.e. $T = w / 2 \sin 90^o = w / 2$

2(xiii) *By definition* $\vec{A}\cdot\vec{B} = AB\cos\theta$

& the projection of $\vec{A}$ on $\vec{B}$ = $A\cos\theta$,
$$A\cos\theta = \frac{\vec{A}\cdot\vec{B}}{B} = \frac{(\hat{i} - 2\hat{j} + 3\hat{k})\cdot(3\hat{i} - 4\hat{k})}{\sqrt{3^2 + (-4)^2}}$$

$$A\cos\theta = \frac{(1\times3)\hat{i}\cdot\hat{i} + (3\times-4)\hat{k}\cdot\hat{k}}{\sqrt{25}} = \frac{(1\times3)\times1 + (3\times-4)\times1}{5} = -\frac{9}{5} = -1.8$$

2(xiv) *For elastic collision. Consider two smooth, non-rotating balls, law of conservation of momentum gives*
$$m_1\,v_1 + m_2\,v_2 = m_1\,v_1' + m_2\,v_2'$$
$$\text{or} \quad m_1(v_1 - v_1') = m_2(v_2' - v_2) \qquad \text{.....} \ (1)$$
from law of conservation of KE
$$\tfrac12\,m_1\,v_1^2 + \tfrac12\,m_2\,v_2^2 = \tfrac12\,m_1\,v_1'^2 + \tfrac12\,m_2\,v_2'^2$$
$$\text{or} \quad m_1(v_1^2 - v_1'^2) = m_2(v_2'^2 - v_2^2) \qquad \text{.....} \ (2)$$
Dividing eq. (2) by eq. (1) gives
$$(v_1 + v_1') = (v_2' + v_2) \ \text{ or } \ (v_1 - v_2) = (v_2' - v_1')$$
$$\text{or} \quad (v_1 - v_2) = -(v_1' - v_2')$$
The above equation shows that the magnitude of relative velocity of approach is equal to the magnitude of relative velocity of separation

Rough Work

both //

$A_1 \times A_2 = 0$

$T = \frac{w}{2} \sin 90^o$

$A\cos\theta = \frac{\vec{A}\cdot\vec{B}}{B}$

$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$

$\frac12 m_1v_1^2 + \frac12 m_2v_2^2 = \frac12 m_1v_1'^2 + \frac12 m_2v_2'^2$

$(v_1 - v_2) = -(v_1' - v_2')$

Rough Work

2(xv) *Impulse is the product of force and time for which it acts on a body.*

Impulse = F x t = m a t = $\frac{m(v_f - v_i)}{t}$ x t = m (v_f - v_i)

It shows the impulse equals the change in linear momentum of a body.

$I = F x t$
$= \frac{m v_f - m v_i}{t} t$

2(xvi) *From the given conditions,*

$$h = \frac{v_i^2 \sin^2\theta}{2g} = R = \frac{v_i^2}{g}\sin 2\theta \quad \text{or} \quad \frac{v_i^2 \sin^2\theta}{2g} = \frac{v_i^2}{g}\sin 2\theta$$

$$2\sin 2\theta = \sin^2\theta \quad \text{or} \quad 2\times 2\sin\theta\cos\theta = \sin\theta\sin\theta$$

$$\text{or} \quad \frac{\sin\theta}{\cos\theta} = \tan\theta = 4 \quad \Rightarrow \quad \theta = \tan^{-1}4 = 76^0$$

$h = R$
$\frac{v_i^2 \sin^2\theta}{2g} = \frac{v_i^2 \sin 2\theta}{g}$
$\Rightarrow \theta = ---$

2(xvii) *Elastic collision is the collision in which the momentum and the kinetic energy of the system is conserved.*

When $m_1 \gg m_2$ & $v_2 = 0$

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 + \frac{2m_2}{m_1 + m_2} v_2 \qquad \text{..... (3)}$$

$$v_2' = \frac{2m_1}{m_1 + m_2} v_1 + \frac{m_2 - m_1}{m_1 + m_2} v_2 \qquad \text{..... (4)}$$

From equations (3) & (4) *we get*

$$v_1' \simeq v_1 \quad \& \quad v_2' \simeq 2v_1$$

We conclude that the incident particle keeps on moving without loosing much energy. while the target particle moves with the double velocity.

$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 + \frac{2m_2}{m_1 + m_2} v_2$

$v_2' = \frac{2m_1}{m_1 + m_2} v_1 + \frac{m_2 - m_1}{m_1 + m_2} v_2$

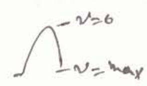
2(xviii) F = m a = $\frac{m(v_f - v_i)}{t}$ = $\frac{mv_f - mv_i}{t}$ = *time rate of change of momentum*

so 2[nd] law of motion in terms of momentum:

"Time rate of change of momentum of a body equals the applied force".

2(xix) *An athlete should take an angle of* $\theta = 45^0$ *to have a long jump. For getting maximum value of the range.*

Rough Work



2(xx) *A projectile will have its minimum speed at the highest point (maximum height).*

It has its maximum speed at the start and end of the projectile motion.

2(xxi) *We have*

$$F = \frac{mv_f - mv_i}{t} \qquad [\,(v_f - v_i) = v, \text{ is relative velocity}]$$

or $F = \dfrac{m(v_f - v_i)}{t} = \dfrac{mv}{t}$ …..(1) also $F = Ma$ ….. (2)

from eqs. (1) & (2) *we get*

$Ma = \dfrac{mv}{t}$ or velocity of mass m per sec. $Ma = mv$

or $a = \dfrac{mv}{M}$

when the fuel in the rocket is burned and ejected, the mass M of rocket decreases and hence the acceleration increases.

$a = \dfrac{mv}{M}$

2(xxii) $W = F\,d\cos 0^\circ = F\,d = m\,a\,d = 10 \times 9.8 \times 10 = 980\text{ J} = 0.98\text{ KJ}$

$W = Fd\cos 0^\circ$

2(xxiii) *Example is the Work done by force of friction. As 'f' always act opposite to displacement.*

$$\text{Work} = \vec{F}\cdot\vec{d} = Fd\cos\theta = Fd\cos 180^0 = -Fd$$

air friction

2(xxiv) $v_1' = \dfrac{m_1 - m_2}{m_1 + m_2}v_1$ & $v_2' = \dfrac{2m_1}{m_1 + m_2}v_1$

putting the values, we have

$$v_1' = \frac{1200\text{kg} - 2000\text{kg}}{1200\text{kg} + 2000\text{kg}} \times 15\text{ms}^{-1} = -3.75\text{ms}^{-1}$$

$$v_2' = \frac{2 \times 1200\text{kg}}{1200\text{kg} + 2000\text{kg}} \times 15\text{ms}^{-1} = 11.25\text{ms}^{-1}$$

$Fd = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

2(xxv) *Due to air friction, the nose cone of the rocket becomes very hot.*

2(xxvi) *A moving body having,* m, v_i, v_f, a, & d (=S)

Using

$$2aS = v_f^2 - v_i^2 \quad \text{or} \quad 2ad = v_f^2 - v_i^2 \quad \text{or} \quad d = \frac{1}{2a}(v_f^2 - v_i^2) \; \ldots\ldots \; (1)$$

Also we have $F = ma$ ….. (2)

Multiplying eqs. (1) & (2) *gives*

$$Fd = \frac{1}{2}m(v_f^2 - v_i^2) = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

i.e. Work done on the body equals to the gain in K.E.

Rough Work

2(xxvii) *We have* F= 6N, m = 2kg, t = 4s, $v_i = 0$

$$S = v_i t + \frac{1}{2}at^2 = v_i t + \frac{1}{2}\left(\frac{F}{m}\right)t^2 = 0\times 4 + \frac{1}{2}\left(\frac{6}{2}\right)(4)^2 = 24m$$

from $Fd = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

gain in K.E. = Fd = 6 x 24 = 144J

2(xxviii) *We have* $g = \frac{GM}{R^2}$

from the given conditions: $g_1 = \frac{G(4M)}{R^2} = 4\times\frac{GM}{R^2} = 4g$

$g = \frac{GM}{R^2}$

2(xxix) *Yes, the stone will posses potential energy.*
The work done in moving the stone from the surface to the tunnel bottom will be stored as its potential energy.

P.E. = mg(h – x) = mgh_1 [x = distance moved downward]

& $U_{Absolute} = U_g = -\frac{GMm}{(R-x)}$

$U = -\frac{GMm}{R}$

2(xxx) *The Earth receives* **$1kWm^{-2}$** *intensity of energy on a clear day at noon. It comes through atmosphere and is being reduced due to reflection, scattering and absorption. This energy can be used to heat water using solar reflectors, thermal absorbers. Solar cells are used to power electrical appliances at nights, in satellites and in calculators.*

$1\ kWm^{-2}$

$$F_c = \frac{mv^2}{r} = \frac{1000\times\left(\frac{144\times10^3}{60\times60}\right)^2}{10^{20}\times10^{-18}} = 16000\ N = 1.6\times10^4\ N$$

2(xxxi)

$F_c = \frac{mv^2}{r}$

2(xxxii) *Moment of Inertia is the rotational analogue or corresponding quantity of mass in angular motion.*
or it is defined as the sum of the products of the mass of each particle of the body and the square of its perpendicular distance from the axis. Mathematically $I = \sum_{i=1}^{n} m_i r_i^2$

$I = \sum m_i r_i^2$

Its significance is when Rotational K.E. ($K.E._{rot} = \frac{1}{2}I\omega^2$) *containing moment of inertia is put to practical use by fly wheels, which are essential parts of many engines.*

F_c tangent

2(xxxiii) *The mud will fly in a direction tangent to the wheel. When mud separates from the tyre, centripetal force is ceased from the mud particles.*

Rough Work

Section-II

Ans. 3(a): **Scalar Product (or Dot Product):**

Scalar product of vectors $\vec{A}$ *and* $\vec{B}$ *is the scalar quantity obtained by multiplying the product of the magnitudes of the vectors by the cosine of the angle between them. Mathematically,* $\vec{A}\cdot\vec{B} = AB\cos\theta$

$A \cdot B = AB\cos\theta$

Characteristics:

1. **Commutative**

   According to commutative law: a * b = b * a,

   For the vectors $\vec{A}$ and $\vec{B}$, *applying the law,*

   $\vec{A}\cdot\vec{B} = AB\cos\theta = BA\cos\theta = \vec{B}\cdot\vec{A}$

   or $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$, *proves the commutative law.*

2. **Mutually Perpendicular Vectors**

   The scalar product of two mutually perpendicular vectors is zero.

   $\vec{A}\cdot\vec{B} = AB\cos 90^0 = 0$

3. **Product of their magnitude**

   The scalar product of two parallel vectors is equal to the product of their magnitudes.

   $\vec{A}\cdot\vec{B} = AB\cos 0^0 = AB$

4. **Self Product**

   The self product of a vector $\vec{A}$ *is equal to the square of its magnitude.*

   $\vec{A}\cdot\vec{A} = AA\cos 0^0 = A^2$

5. **Rectangular Components**

   The scalar product of two vectors $\vec{A}$ *and* $\vec{B}$ *in terms of their rectangular components*

   $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$

Commutative
mutually perp
magnitude
self product
rectangular comp.

Ans. 3(b): The moment of $\vec{F}$ about the point will be:

$$\vec{\tau} = \vec{r}\times\vec{F} = (\vec{r}_2 - \vec{r}_1)\times\vec{F} = \{(-\hat{i}+\hat{k}) - (\hat{i}+\hat{k})\}\times(\hat{i}+2\hat{j})$$

$$[\vec{r}_1 = \hat{i}+\hat{k} \quad \& \quad \vec{r}_2 = -\hat{i}+\hat{k}]$$

or $$\vec{\tau} = (-2\hat{i})\times(\hat{i}+2\hat{j}) = -2\hat{i}\times\hat{i} - 4\hat{i}\times j = -4\hat{k}$$

$$[\hat{i}\times\hat{i} = 0 \quad \& \quad \hat{i}\times j = \hat{k}]$$

$\tau = r \times F$

Rough Work

Ans. 4(a): **Maximum height h of the projectile:**

It is the highest point a projectile attains during its flight.

Using equation, $2aS = v_f^2 - v_i^2$

or $2(-g)h = (0)^2 - (v_i \sin\theta)^2$

or $-2gh = -v_i^2 \sin^2\theta$

or $\boxed{h = \dfrac{v_i^2 \sin^2\theta}{2g}}$

$$\left\{\begin{array}{l} a = -g \\ S = \text{height} = h \\ v_f = v_{fy} = 0 \\ v_i = v_{iy} = v_i \sin\theta \end{array}\right.$$

$h = \dfrac{v_i^2 \sin^2\theta}{2g}$

Range *of the projectile is the maximum distance which a projectile covers in the horizontal direction.*

Using $S = vt$

or $R = v_i \cos\theta \dfrac{2v_i \sin\theta}{g}$

or $R = \dfrac{v_i^2}{g} 2\sin\theta\cos\theta$

or $\boxed{R = \dfrac{v_i^2}{g} \sin 2\theta}$

$$\left\{\begin{array}{l} S = R \\ v = v_{ix} = v_i \cos\theta \\ t = \dfrac{2v_i \sin\theta}{g} \end{array}\right.$$

$R = \dfrac{v_i^2}{g} \sin 2\theta$

$\rho = \dfrac{m}{V}$

$F = \dfrac{m}{t} v$

For Maximum Range R_{max}: *We know that maximum value of* $\sin\theta$ is $\sin 90^o = 1$.

So $\sin 2\theta = 1$ or $2\theta = 90^o$ or $\theta = 45^o$

$\Rightarrow \boxed{R_{max} = \dfrac{v_i^2}{g}}$

Ans. 4(b):

Volume: $[V = \text{area} \times \text{length}]$

$V = 50\text{cm}^2 \times 0.3\text{ms}^{-1} = 50 \times 10^{-4}\text{m}^3 \times 0.3\text{ms}^{-1} = 0.0015\text{m}^3$

Mass per sec. $= \dfrac{m}{t} = \dfrac{(0.0015) \times (1000)\text{kg}}{1\text{ sec.}} = 1.5 \text{ kgs}^{-1}$

$\left[\text{density} = \rho = \dfrac{m}{V} \text{ or } m = \rho \times V\right]$

Using the equation:

$F = \dfrac{m}{t} v = (1.5\text{kgs}^{-1}) \times (0.3\text{ms}^{-1}) = 0.45 \text{ kgms}^{-2} = 0.45 \text{ N}$

Rough Work

Ans. 5(a): **Absolute potential energy**, U_g: *"Energy required to move a mass from earth up to an infinite distance".*

To calculate U_g, *consider a body of mass m which moves from point 1 to point N in gravitational field. Divide distance between 1 to N into small length* Δr.

We have. Mean distance $= r = \dfrac{r_1 + r_2}{2}$ & $r_2 - r_1 = \Delta r$

or $r_2 = r_1 + \Delta r$ or $r^2 = \left(\dfrac{r_1 + r_1 + \Delta r}{2}\right)^2$ or $r^2 = \left(\dfrac{2r_1}{2} + \dfrac{\Delta r}{2}\right)^2$

or $r^2 = r_1^2 + \dfrac{(\Delta r)^2}{4} + r_1 \Delta r \;= r_1 r_2$

We have

$$W_{1\to 2} = \vec{F}\cdot\Delta\vec{r} = F\Delta r\cos 180^\circ = -F\Delta r \quad \left[\; F = G\frac{Mm}{r^2} = G\frac{Mm}{r_1 r_2}\right]$$

or $W_{1\to 2} = -G\dfrac{Mm}{r_1 r_2}(r_2 - r_1)$ $\quad [\; \Delta r = r_2 - r_1 \;\&\; r^2 = r_1 r_2 \;]$

or $W_{1\to 2} = -GMm\left(\dfrac{1}{r_1} - \dfrac{1}{r_2}\right)$ $\quad \left[\; W_{1\to 2} = -GMm\dfrac{r_2 - r_1}{r_1 r_2} \;\right]$

similarly $W_{2\to 3} = -GMm\left(\dfrac{1}{r_2} - \dfrac{1}{r_3}\right)$

\- - - - - - - - - - - - - - - -

$W_{N-1\to N} = -GMm\left(\dfrac{1}{r_{N-1}} - \dfrac{1}{r_N}\right)$

So $\quad W_{total} = W_{1\to 2} + W_{2\to 3} + W_{3\to 4} + \ldots\ldots + W_{N-1\to N}$

$$= -GMm\left[\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \left(\frac{1}{r_2} - \frac{1}{r_3}\right) + \left(\frac{1}{r_3} - \frac{1}{r_4}\right) + \ldots\ldots + \left(\frac{1}{r_{N-1}} - \frac{1}{r_N}\right)\right]$$

$$= -GMm\left[\frac{1}{r_1} - \cancel{\frac{1}{r_2}} + \cancel{\frac{1}{r_2}} - \cancel{\frac{1}{r_3}} + \ldots\ldots + \cancel{\frac{1}{r_{N-1}}} - \frac{1}{r_N}\right]$$

or $\quad W_{total} = -GMm\left(\dfrac{1}{r_1} - \dfrac{1}{r_N}\right)$

If the point N is at an infinite distance, then

$W_{total} = \dfrac{-GMm}{r_1}$ *& general expression is,*

$\boxed{U = \dfrac{-GMm}{r}}$ *Taking* $r_1 = R$, $\boxed{U_g = -\dfrac{GMm}{R}}$

$r = \dfrac{r_1 + r_2}{2}$

$\Delta r = r_2 - r_1$

$r^2 = r_1 r_2$

$W = F\cdot\Delta r$

$= -\dfrac{GMm}{r_1 r_2}(\Delta r)$

$W_T = -GMm\left(\dfrac{1}{r_1} - \dfrac{1}{r_N}\right)$

$U_g = -\dfrac{GMm}{R}$

Rough Work

Ans. 5(b): *We have, number of bricks* = 10

Mass of each brick = 1.5 kg

& *Height of each brick* = 6.0 cm = 6/100 m

To find work required to stack them.

No work is done by first brick. & for work done by **9** *bricks, we have*

Total mass = (9 x 1.5) kg

& *Mean height* = $\frac{6\times10}{2}$ cm = $\frac{6\times10}{2\times100}$ m

So Work done = $W = Fd\cos\theta \quad = mgh\cos 0^{o}$

putting the values, we get

$$W \;=\; 9\times1.5\times9.8\times\frac{6\times10}{2\times100} \;=\; 39.69 \;\cong\; 40\text{ J}$$

So work required is: $\boxed{\mathbf{W \;=\; 40\ J}}$

$W = 9\times1.5\times9.8 \times(h)$

$40J$

Ans. 6(b): *From the given conditions, we have*

$$T \propto m^{a}\times l^{b}\times\theta^{c}\times g^{d} \quad \text{or} \quad T = \text{Const.}\times m^{a}\times l^{b}\times\theta^{c}\times g^{d}$$

$$\text{or} \quad [T] = \text{Const.}\times[M]^{a}[L]^{b}\left[LL^{-1}\right]^{c}\left[LT^{-2}\right]^{d}$$

Comparing dimensions on both sides:

$$[T]=[T]^{-2d}, \quad [M]^{0}=[M]^{a} \quad \& \quad [L]=[L]^{b+d+c-c}$$

Equating powers on each side gives

$$-2d = 1 \quad\Rightarrow\quad d = -\frac{1}{2} \quad \& \quad a = 0$$

and

$$b + d = 0 \quad \text{or } b = -d = \frac{1}{2} \quad \& \quad \theta = \left[LL^{-1}\right]^{c} = \left[L^{0}\right]^{0} = 1$$

Substituting the values of a, b, θ & d, *we get*

$$T = \text{Const.}\times m^{0}\times l^{1/2}\times1\times g^{-1/2} \quad \text{or} \quad T = \text{cont.}\times\sqrt{\frac{l}{g}}$$

Numerical value of the constant can be found by experiments.

$T \propto m^{a} l^{b} \theta^{c} g^{d}$

$T = \text{const}\sqrt{l/g}$

Rough Work

Ans. 6(a): **Escape velocity**: "*The initial velocity, which a projectile must have at the earth's surface in order to go out of earth's gravitational field.*"

(Initial) $KE = \frac{1}{2} m v_{esc}^2$ [$KE = \frac{1}{2} mv^2 \Rightarrow KE \propto v$]

Energy required to move a mass from the earth up to an infinite distance is Absolute potential energy

$$U = \left| -\frac{GMm}{R} \right| = \frac{GMm}{R}$$

The Energy [Initial KE / Increase in PE] needed to go free from 'g' [earth's gravitational field / infinite distance] implies.

$$KE_{initial} = PE_{absolute}$$

$$\text{or} \quad \frac{1}{2} \cancel{m} v_{esc}^2 = \frac{GM\cancel{m}}{R}$$

$$\text{or} \quad v_{esc}^2 = \frac{2GM \times R}{R \times R} = \frac{2GMR}{R^2}$$

$$\text{or} \quad v_{esc}^2 = 2R \times \frac{GM}{R^2} = 2Rg$$

$$\text{or} \quad \boxed{v_{esc} = \sqrt{2gR}}$$

$$\left\{ \begin{aligned} & F = mg \ \& \ F = \frac{GMm}{R^2} \\ & \Rightarrow \cancel{m} g = \frac{GM\cancel{m}}{R^2} \\ & \text{or} \quad g = \frac{GM}{R^2} \end{aligned} \right.$$

$KE_i = \frac{1}{2} m v_{esc}^2$

$U = \frac{GMm}{R}$

$v_{esc} = \sqrt{2gR}$

Ans. 7(b): *When two vectors are lying perpendicular to each other.*

The condition fulfilled is:

$$\vec{A} \cdot \vec{B} = AB\cos\theta = AB\cos 90^0 = AB(0) = 0$$

Using the condition, we have

$$\vec{a} \cdot \vec{b} = (2\hat{i} - 4\hat{j} + 5\hat{k}) \cdot (13\hat{i} + q\hat{j} + 2\hat{k})$$

$$= (2 \times 13)\hat{i} \cdot \hat{i} + (-4 \times q)\hat{j} \cdot \hat{j} + (5 \times 2)\hat{k} \cdot \hat{k} = 0$$

$$\text{or} \quad (26) + (-4q) + (10) = 36 - 4q = 0$$

$$\text{or} \quad q = \frac{36}{4} = 9$$

$\vec{A} \cdot \vec{B} = AB\cos 90^0$
$= 0$

Rough Work

Ans. 7(a): Centripetal Force:

"The force needed to bend the normally straight path of the particle into a circular path."

Or "A force that causes a body to move in a circular path."

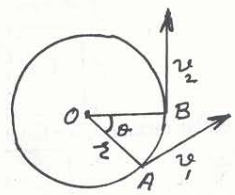
To calculate Centripetal Force

Consider

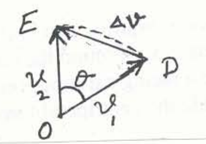
a body revolving with mass m

In the figure

two velocities at points A and B will be same.



so $v_1 = v_2 = v$ ..... (1)



from fig. (b), *we have* $\vec{v}_1 + \Delta\vec{v} = \vec{v}_2$

or $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$ ..... (2)

We have $\sin\theta = \dfrac{\Delta v}{v_2}$ [for small angle & sinθ= θ]

putting the values from equations (1) & (2) *we get*

$\theta = \dfrac{\Delta v}{v}$ or $v\,\Delta\theta = \Delta v$ [for small change; θ = Δθ]

multiplying and dividing by Δt to L.H.S. , *we get*

$$v\frac{\Delta\theta}{\Delta t}\Delta t = \Delta v$$

or $\Delta v = v\,\omega\,t$ or $\dfrac{\Delta v}{\Delta t} = \omega v$

Now we define: $\vec{a} = \dfrac{\Delta\vec{v}}{\Delta t}$ so $a = \omega v$

or $\boxed{a = \omega^2 r = \dfrac{v^2}{r}}$

& we have $\vec{F} = m\vec{a}$ or $\vec{F} = -\dfrac{mv^2}{r^2}\vec{r}$

or $\boxed{F_c = \dfrac{mv^2}{r}}$

or in angular measure,

$$\boxed{F_c = mr\omega^2}$$

$a_c = \dfrac{v^2}{r}$

$F_c = \dfrac{mv^2}{r}$